Sets and Numbers 6

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SINGER MATHEMATICS PROGRAM

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Sets and Numbers 6

PATRICK SUPPES / Stanford University

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Part 1 Sets

Equal and Equivalent Sets

A **set** is a collection of things. We use braces to show sets:

{Helen} { } {1, 3, 5, 7}

The set with no **members** or **elements** is called **the empty set.**

Sets are **equal** when they have the same members or elements.

$\{1, 5, 7\} = \{5, 1, 7\}$ $\{1, 5, 7\} \neq \{2, 4, 6\}$

Sets are **equivalent** when they have the same number of members.

$\{1, 5, 7\} \approx \{2, 4, 6\}$ $\{1, 5, 7\} \not\approx \{4, 8\}$

1. Which of these sets describe the empty set?

a. The set of whole numbers from 0 to 5.

b. The set of goblins in this room.

c. The set of lions in this room.

d. The set of boys in this room.

Choose $=$ or $\neq$.

2. a. $\{a, b\} \bullet \{b, a\}$

b. $\{1, 2, 3\} \bullet \{4, 5, 6\}$

c. $\{Tom\} \bullet \{Bill\}$

d. $\{r, s\} \bullet \{s, t\}$

3. a. $\{\ \} \bullet \{\ \}$

b. $\{32, 40, 63\} \bullet \{42, 30, 63\}$

c. $\{Betty, Connie\} \bullet \{Connie, Betty\}$

d. $\{Joe\} \bullet \{Mary, Joe\}$

Choose $\approx$ or $\not\approx$.

4. a. $\{a, b\} \bullet \{d, b\}$

b. $\{Jack, Jim\} \bullet \{Fred\}$

c. $\{\ \} \bullet \{\ \}$

d. $\{p, q, r\} \bullet \{q, r, p\}$

5. a. $\{1, 2, 3, 4\} \bullet \{a, b, c, d\}$

b. $\{3, 6, 9\} \bullet \{1, 2\}$

c. $\{Mike\} \bullet \{4\}$

d. $\{m, n, r\} \bullet \{s, t\}$

The numbers 2 and 4 are elements or members of the set $\{2, 4\}$.

$2 \in \{2, 4\}$ $\quad$ $4 \in \{2, 4\}$

The set $\{2, 4\}$ is a **proper subset** of the set $\{1, 2, 3, 4\}$ because the set $\{1, 2, 3, 4\}$ contains all the members of the set $\{2, 4\}$ and at least one more member.

$\{2, 4\} \subset \{1, 2, 3, 4\}$

Which statements are true?

1. a. $3 \in \{1, 2, 3, 4\}$

b. $5 \in \{1, 2, 3, 4\}$

c. $0 \in \{1, 2, 3, 4\}$

d. $2 \in \{1, 2, 3, 4\}$

e. $0 \in \{\ \}$

2. a. Joan $\in$ {Joan, Jill, Jan}

b. Marie $\in$ {Flora, Elizabeth}

c. Don $\in$ {Larry, Gary, Don}

d. Laura $\in$ {Laura}

e. Jim $\in$ {Pat, Charles}

3. a. $\{3, 4\} \subset \{2, 3, 4\}$

b. $\{3, 4\} \subset \{3, 4\}$

c. $\{\ \} \subset \{0\}$

d. $\{\ \} \subset \{0, 2\}$

4. a. {Bill, Ted} $\subset$ {Mike, Ted, Bill}

b. {Dora} $\subset$ {Marilyn, Linda}

c. {Nancy, Suzanne} $\subset$ {Suzanne}

d. {Clara} $\subset$ {Clara}

Choose $\in$ or $\subset$.

5. a. $4 \bigcirc \{2, 4\}$

b. $\{4\} \bigcirc \{2, 4\}$

c. $\{\ \} \bigcirc \{2, 4\}$

d. $2 \bigcirc \{2, 4\}$

6. a. $0 \bigcirc \{0, 3, 6\}$

b. $\{6\} \bigcirc \{0, 3, 6\}$

c. $\{0\} \bigcirc \{0, 3, 6\}$

d. $6 \bigcirc \{0, 3, 6\}$

7. a. $\{0, 6\} \bigcirc \{0, 3, 6\}$

b. $\{\ \} \bigcirc \{0, 3, 6\}$

c. $7 \bigcirc \{1, 7\}$

d. $\{7\} \bigcirc \{1, 7\}$

Disjoint Sets and Overlapping Sets

Disjoint sets have no members in common.

{Scott, Mark} {Marie}

Overlapping sets have at least one member in common.

{Mary, Wendy, Mike} {Mary, Henry}

1. Is each pair of sets disjoint or overlapping?

a. {1, 3, 5} {5, 7, 9, 11}

b. {4, 8, 12} {8, 12}

c. {0, 1, 2, 3} {5, 10, 15}

d. {Frank, Mel} {Sam, Frank}

e. {Jean, Joan} {Jenny}

f. {7, 4, 3} {1, 3, 4, 7, 11}

g. {1, 2, 3, 4, 5} {0}

h. {Earth, Mars} {Venus, Mars}

We can perform the union operation on sets.
If a member is in more than one set, we list it only once.

$$\{\text{April, May}\} \cup \{\text{May, June}\} = \{\text{April, May, June}\}$$

$$\{1, 2\} \cup \{2, 3, 4\} = \{1, 2, 3, 4\}$$

2. Find each answer set.

a. {Buffalo, Hartford, Pierre} ∪ {Sacramento, Atlanta} = ■

b. {Utah, Texas, Ohio, Idaho, Iowa} ∪ {Ohio, Texas, Iowa} = ■

c. {Pat, Kit} ∪ {Pat, Kit Helen} = ■

d. {1, 2, 3} ∪ {4, 5} = ■

e. {a, b, c} ∪ {c, b, a} = ■

f. {April, May, June} ∪ {March, April} ∪ {June, July} = ■

We can find the intersection of two sets.
Only the members common to both sets are in the answer set.

$\{4, 5, 6, 7\} \cap \{9, 8, 7, 4\} = \{7, 4\}$

1. Give each answer set.

a. $\{2, 3, 4, 5\} \cap \{2, 3\} =$ ▢

b. $\{\text{Alex, Bert}\} \cap \{\text{Bert}\} =$ ▢

c. $\{\text{Hilda, Diane, Ada}\} \cap \{\text{Hilda}\} =$ ▢

d. $\{p, q, r, s\} \cap \{q, r\} =$ ▢

e. $\{2, 4, 6, 8\} \cap \{1, 2, 3\} =$ ▢

f. $\{a, b, c\} \cap \{b, c, d\} =$ ▢

g. $\{6, 8, 9\} \cap \{4, 8, 7\} =$ ▢

h. $\{w, y, z\} \cap \{x, y\} =$ ▢

The intersection of two disjoint sets is the empty set.

$\{4, 5, 6\} \cap \{0, 1\} = \{\ \}$

2. Give each answer set.

a. $\{\text{Venus, Mars, Saturn}\} \cap \{\text{Jupiter, Pluto, Neptune}\} =$ ▢

b. $\{\text{Cabot, Cook, Columbus}\} \cap \{\text{DeSoto, Hudson}\} =$ ▢

c. $\{4, 5, 6\} \cap \{5, 6, 7, 9\} =$ ▢

d. $\{a, b, c, d\} \cap \{b, d, a, c\} =$ ▢

e. $\{\text{June, May}\} \cap \{\text{August, April}\} =$ ▢

f. $\{\text{Tom, Jeff}\} \cap \{\text{Ben, Jeff, Al}\} =$ ▢

g. $\{0, 1, 2, 3, 4, 5\} \cap \{0, 2, 4, 6\} =$ ▢

h. $\{6, 12, 18, 24\} \cap \{12, 24, 36, 48\} =$ ▢

i. $\{\text{Chicago, Los Angeles, New York}\} \cap \{\text{Chicago, Philadelphia}\} =$ ▢

j. $\{\text{Wyoming, Nevada, Montana}\} \cap \{\text{Vermont, Indiana}\} =$ ▢

We can find the difference of two sets.

$\{\text{April, May, June}\} - \{\text{June}\} = \{\text{April, May}\}$

Sometimes there are elements in the second set that are not in the first set. We can take away only the elements that are in both sets.

$\{\text{April, May, June}\} - \{\text{June, July}\} = \{\text{April, May}\}$

$\{\text{June, July}\} - \{\text{April, May, June}\} = \{\text{July}\}$

Give the answer set.

1. a. $\{\text{Washington, Jefferson, Adams}\} - \{\text{Jefferson, Monroe}\} =$ ■
   b. $\{\text{Monday, Tuesday, Wednesday}\} - \{\text{Monday, Friday}\} =$ ■
   c. $\{\text{June, July, August, September}\} - \{\text{September, October}\} =$ ■
   d. $\{\text{September, October}\} - \{\text{June, July, August, September}\} =$ ■
   e. $\{3, 4, 5\} - \{5, 6, 7\} =$ ■
   f. $\{\text{Rose, Mary, Kate}\} - \{\text{Rose}\} =$ ■
   g. $\{\text{Rose}\} - \{\text{Rose, Mary, Kate}\} =$ ■
   h. $\{a, b, c, d\} - \{c, d, e\} =$ ■
   i. $\{c, d, e\} - \{a, b, c, d\} =$ ■

2. a. $\{a, e, i, o, u\} - \{e, i\} =$ ■
   b. $\{\text{Karl, Ted}\} - \{\text{Ted}\} =$ ■
   c. $\{59, 58, 57, 56\} - \{56\} =$ ■
   d. $\{a, b, c, d\} - \{x, y, z\} =$ ■
   e. $\{39, 45, 87\} - \{39, 40, 41\} =$ ■
   f. $\{0, 2, 4\} - \{\ \} =$ ■

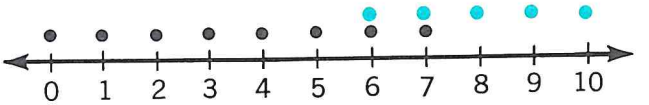
3. a. $\{\text{Terry, Joe}\} - \{\text{Bill}\} =$ ■
   b. $\{a, d, g, r, k\} - \{a, g, k\} =$ ■
   c. $\{t, i, w, m, j\} - \{s, i, o, b\} =$ ■
   d. $\{0, 2, 4, 6\} - \{1, 3, 5\} =$ ■
   e. $\{\text{Anna, Ruth}\} - \{\text{Anna}\} =$ ■
   f. $\{\ \} - \{3, 5\} =$ ■

Sets of Whole Numbers

The set of whole numbers n such that $n < 3$ is $\{0, 1, 2\}$.

The set of whole numbers n such that $n > 3$ is $\{4, 5, 6, \ldots\}$. We name three numbers and then use three dots to show that the number pattern continues. When the three dots appear at the end, the dots indicate there is no last number. Such a set is an **infinite** set.

The set of whole numbers n such that $n < 8$ and $n > 5$ is $\{6, 7\}$.



1. Find the set of whole numbers n such that:

   a. $n < 4$ c. $n < 2$ e. $n < 0$

   b. $n < 7$ d. $n < 10$ f. $n < 1$

2. Find the set of whole numbers n such that:

   a. $n > 4$ c. $n > 10$ e. $n > 75$

   b. $n > 7$ d. $n > 25$ f. $n > 100$

3. Find the set of whole numbers n such that:

   a. $n < 4$ and $n > 2$

   b. $n < 8$ and $n > 5$

   c. $n < 10$ and $n > 6$

   d. $n < 2$ and $n > 1$

   e. $n < 5$ and n is odd

   f. $n < 9$ and n is even

   g. $n > 7$ and n is even

   h. $n > 4$ and n is odd

4. Which answer sets in Exercise 3 are infinite sets?

5. Which answer sets in Exercise 3 are not infinite sets?

In place of:	We write:
The set of all whole numbers n such that $n < 3$	$\{n: n < 3\}$
The set of all whole numbers n such that $n > 3$	$\{n: n > 3\}$
The set of all whole numbers n such that $n > 4$ and $n < 8$	$\{n: 4 < n < 8\}$

Find the members of each set.

1. $\{n: n < 5\}$
2. $\{n: n < 8\}$
3. $\{n: n < 7\}$
4. $\{n: n < 6\}$
5. $\{n: n < 10\}$
6. $\{n: n > 0\}$
7. $\{n: n > 3\}$
8. $\{n: n > 8\}$
9. $\{n: n > 1\}$
10. $\{n: n > 11\}$
11. $\{n: 3 < n < 6\}$
12. $\{n: 3 < n < 4\}$
13. $\{n: 25 < n < 31\}$
14. $\{n: 102 < n < 112\}$
15. $\{n: 104 < n < 110\}$

Perform the operation and list the members of each answer set.

16. $\{n: 1 < n < 4\} \cup \{n: 5 < n < 10\}$
17. $\{n: 2 < n < 5\} \cup \{n: 3 < n < 8\}$
18. $\{n: 5 < n < 9\} \cup \{n: 5 < n < 11\}$
19. $\{n: 4 < n < 10\} - \{n: 5 < n < 8\}$
20. $\{n: 21 < n < 25\} - \{n: 23 < n < 29\}$
21. $\{n: 18 < n < 24\} \cap \{n: 22 < n < 25\}$
22. $\{n: n < 4\} \cap \{n: 2 < n < 8\}$
23. $\{n: n < 7\} \cap \{n: 9 < n < 16\}$
24. $\{n: n < 8\} - \{n: n < 6\}$
25. $\{n: 4 < n < 10\} \cap \{n: n < 6\} \cap \{n: 2 < n < 7\}$

Sometimes we use capital letters to stand for sets.

$A = \{\ \}$ $\quad E = \{Al, Ed\}$

$B = \{Al\}$ $\quad F = \{Al, Ann\}$

$C = \{Ed\}$ $\quad G = \{Ed, Ann\}$

$D = \{Ann\}$ $\quad H = \{Al, Ed, Ann\}$

We can use capital letters in mathematical sentences to stand for sets.

$\{Ed, Ann\} \cup \{Al\} = \{Al, Ed, Ann\}$

$G \cup B = H$

1. Copy each problem and give the answer set. Use capital letters as shown in the example above.

   a. $\{Al, Ed\} - \{Ed\} =$ ■

   b. $\{Ann\} \cup \{Al, Ed\} =$ ■

   c. $\{Ed\} \cup \{Al, Ed\} =$ ■

   d. $\{Al, Ed, Ann\} \cap \{Al\} =$ ■

   e. $\{Al, Ann\} - \{Al, Ed, Ann\} =$ ■

   f. $\{Al, Ed\} \cap \{Ed, Ann\} =$ ■

Copy each problem. Using the sets in the box above, list the members in each answer set.

$C \cup D = \{Ed, Ann\}$

2. a. $A \cup B =$ ■
   b. $H - E =$ ■
   c. $G \cap C =$ ■
   d. $D \cap F =$ ■
   e. $G - F =$ ■
   f. $B \cup E =$ ■
   g. $H - G =$ ■

3. a. $C - G =$ ■
   b. $E \cup D =$ ■
   c. $H \cup G =$ ■
   d. $A \cap D =$ ■
   e. $D - B =$ ■
   f. $H \cap F =$ ■
   g. $A \cup G =$ ■

4. a. $E \cap B =$ ■
   b. $G \cup D =$ ■
   c. $F - A =$ ■
   d. $E - H =$ ■
   e. $C \cap H =$ ■
   f. $F \cup C =$ ■
   g. $B \cap G =$ ■

Each capital letter names a set. Copy and solve each problem as in the example below.

$\{\text{April, May}\} \cap \{\text{May}\} = T$

$T = \{\text{May}\}$

1. a. $\{1, 2, 4, 5\} \cup \{3, 0\} = Y$
   b. $\{a, b, c, d\} \cap \{c, d, e\} = M$
   c. $\{\text{Tim, Pete}\} - \{\text{Mike, Pete}\} = X$
   d. $\{\text{Bill, Sam}\} \cup \{\text{Sam, Bill}\} = G$
   e. $\{10, 20, 30\} \cap \{30\} = Q$

2. a. $\{t, r, i\} \cap \{b, i\} = W$
   b. $\{4, 9, 0\} \cup \{0, 4, 8, 9\} = P$
   c. $\{w, y\} - \{x, y, z\} = N$
   d. $\{\text{Don, Bert}\} - \{\text{Bert}\} = S$
   e. $\{\text{Mary}\} \cap \{\text{Helen, Mary}\} = Z$

Using sets A through I given below, complete each problem.

$A = \{\ \}$
$B = \{z\}$
$C = \{y\}$
$D = \{v, y\}$
$E = \{x, y\}$
$F = \{x, z\}$
$G = \{v, y, z\}$
$H = \{x, y, z\}$
$I = \{v, x, y, z\}$

3. a. $C \cup F = H$
   b. $E \cap H =$ ■
   c. $B \cup D =$ ■
   d. $G - B =$ ■
   e. $B - G =$ ■

4. a. $C - I =$ ■
   b. $D \cap E =$ ■
   c. $H - C =$ ■
   d. $D \cup F =$ ■
   e. $A - E =$ ■

5. a. $G \cup H =$ ■
   b. $B \cap I =$ ■
   c. $F \cap G =$ ■
   d. $I - F =$ ■
   e. $G \cup A =$ ■

Using sets A through I given above find all possible answers.

6. a. $C \cup$ ■ $= H$
   b. $D \cap$ ■ $= C$
   c. ■ $\cup G = I$
   d. $E -$ ■ $= E$

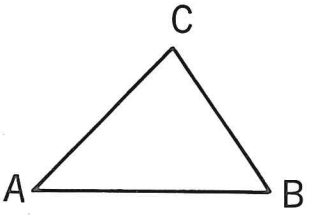
7. a. $H -$ ■ $= B$
   b. ■ $\cup F = I$
   c. $B \cup$ ■ $= H$
   d. ■ $\cap I = E$

8. a. $H \cup$ ■ $= H$
   b. ■ $\cap C = A$
   c. ■ $\cup H = I$
   d. $B \cap$ ■ $= A$

Sets in Geometry

Any figure may be described as a set of points.

The line segment AB is the set of points whose elements are A and B and all the points on the line between A and B. We write "$\overline{AB}$" for segment AB.



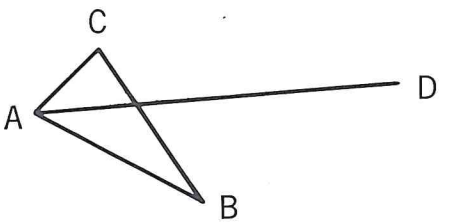
A triangle is the union of three line segments.

Triangle ABC = $\overline{AB} \cup \overline{BC} \cup \overline{CA}$

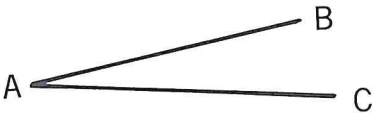
Match the set description to the figure.

1. $\overline{AB} \cup \overline{AC}$
2. $\overline{AB} \cup \overline{AC} \cup \overline{AD}$
3. $\overline{AB} \cup \overline{BC} \cup \overline{CA}$
4. $\overline{AB} \cup \overline{BC} \cup \overline{CA} \cup \overline{AD}$

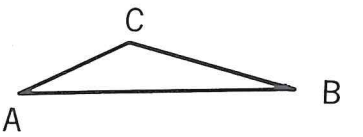
A.



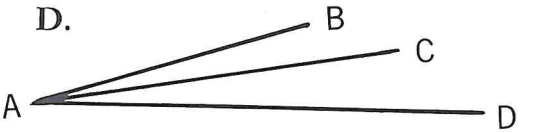
B.



C.

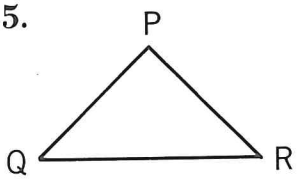


D.



Write a set description for each figure.

5.



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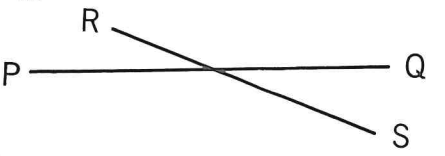

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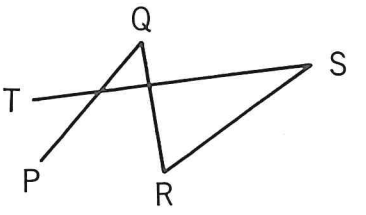


Checkout

Give the set described by listing the members.

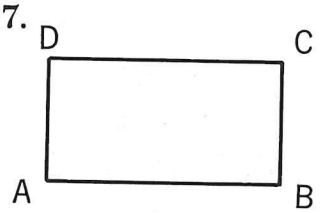
1. $\{n: 136 < n < 141\}$
2. $\{n: 18 < n < 22\}$
3. $\{n: 40 < n < 47\}$
4. $\{n: 259 < n < 264\}$

Perform the operation and find the answer set.

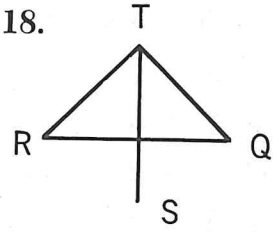
5. $\{\text{April, May, June}\} \cup \{\text{March, April}\} \cup \{\text{June, July, August}\} =$ ■
6. $\{\text{Venus, Mars, Pluto, Mercury}\} \cup \{\text{Earth, Pluto, Mars}\} =$ ■
7. $\{2, 4, 6\} - \{0, 2, 3, 4\} =$ ■
8. $\{\text{Sue, Amy}\} - \{\text{Amy}\} =$ ■
9. $\{104, 103, 102, 101\} \cap \{103, 113\} \cap \{93, 103, 113\} =$ ■
10. $\{\text{Iowa, Idaho, Utah}\} \cap \{\text{Utah, Texas}\} \cap \{\text{Ohio, Utah}\} =$ ■
11. $\{n: 13 < n < 15\} - \{n: 11 < n < 17\} =$ ■
12. $\{n: n < 6\} \cup \{n: 4 < n < 8\} =$ ■
13. $\{n: n < 8\} \cup \{0, 2, 4\} \cup \{n: 3 < n < 6\} =$ ■
14. $\{n: 3 < n < 7\} - \{n: 4 < n < 8\} =$ ■
15. $\{n: 8 < n < 11\} \cap \{n: 7 < n < 12\} =$ ■
16. $\{6, 5, 4\} \cap \{n: 4 < n < 7\} \cap \{n: 3 < n < 8\} =$ ■

Write a set description for each figure.

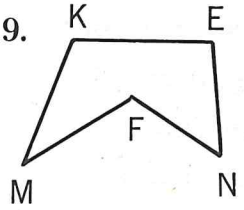
17.



18.



19.



Part 2 Laws of Arithmetic

The operations on sets that we have studied are union, intersection, and difference.

The operations on numbers that we have studied are addition, subtraction, multiplication, and division.

These operations on sets and on numbers are all **binary** operations. **Bi** means two. In all binary operations on numbers, we obtain a third number from **two** numbers.

$28 + 36 = 64$
addends sum

$64 - 28 = 36$
difference or remainder

$9 \times 38 = 342$
factors product

$342 \div 9 = 38$
quotient

Complete each problem. Tell in each case whether you have found a sum, a difference, a product, or a quotient.

1. $9 \times 56 =$ ■
2. $84 + 69 =$ ■
3. $864 \div 8 =$ ■
4. $984 - 679 =$ ■
5. $1765 + 8493 =$ ■
6. $6 \times 746 =$ ■
7. $966 \div 6 =$ ■
8. $8 \times 46 =$ ■
9. $9462 - 1495 =$ ■
10. $165 + 308 =$ ■
11. $2469 \div 3 =$ ■
12. $19 + 38 =$ ■

Choose $+$, $-$, $\times$, or $\div$. Give all possible operations.

13. $4 \bigcirc 4 = 16$
14. $6 \bigcirc 0 = 0$
15. $6 \bigcirc 0 = 6$
16. $1 \bigcirc 1 = 1$
17. $0 \bigcirc 4 = 0$
18. $2 \bigcirc 2 = 4$
19. $7 \bigcirc 1 = 8$
20. $7 \bigcirc 1 = 6$
21. $7 \bigcirc 1 = 7$
22. $9 \bigcirc 3 = 6$
23. $9 \bigcirc 3 = 3$
24. $9 \bigcirc 3 = 27$

Complete each problem.

1. a. $8 \times 9 = \square$
   b. $6 \times 7 = \square$
   c. $3 + 8 = \square$
   d. $9 + 4 = \square$
   e. $36 \div 6 = \square$
   f. $36 \div 4 = \square$
   g. $15 - 8 = \square$
   h. $12 - 3 = \square$

2. a. $\square \times 7 = 28$
   b. $\square \times 8 = 64$
   c. $\square + 2 = 11$
   d. $\square + 9 = 17$
   e. $\square \div 9 = 9$
   f. $\square \div 5 = 6$
   g. $\square - 6 = 7$
   h. $\square - 4 = 6$

3. a. $4 \times \square = 12$
   b. $6 \times \square = 48$
   c. $6 + \square = 11$
   d. $9 + \square = 16$
   e. $25 \div \square = 5$
   f. $24 \div \square = 3$
   g. $12 - \square = 6$
   h. $11 - \square = 7$

Solve each equation.

$6 + 5 = a$	$b + 4 = 10$	$9 + t = 14$
$a = 11$	$b = 6$	$t = 5$

4. a. $7 + 5 = m$
   b. $5 + 9 = y$
   c. $7 \times 8 = c$
   d. $9 \times 6 = f$
   e. $10 - 3 = a$
   f. $16 - 9 = j$
   g. $45 \div 5 = b$
   h. $20 \div 4 = k$

5. a. $d + 7 = 14$
   b. $n + 8 = 12$
   c. $p \times 7 = 35$
   d. $s \times 8 = 32$
   e. $a - 5 = 8$
   f. $r - 0 = 7$
   g. $q \div 9 = 3$
   h. $t \div 6 = 4$

6. a. $9 + e = 15$
   b. $8 + g = 16$
   c. $7 \times v = 49$
   d. $8 \times w = 0$
   e. $10 - h = 5$
   f. $14 - x = 8$
   g. $63 \div d = 9$
   h. $32 \div z = 8$

Write an equation for each problem and solve the equation.

7. Bob collected some rocks and put them in a box. The empty box weighed 10 pounds. The box and rocks weighed 76 pounds. How much did the rocks weigh?

8. It took George 12 minutes to solve a problem. It took Harry 3 times as long. How long did it take Harry to solve the problem?

Commutativity

Two questions are often asked about binary operations.

Question 1: Does the order in which we operate on two numbers change the result?

If the order in which we operate on two numbers does not change the result, the operation is **commutative**.

Solve each equation and answer each question.

1. **a.** $m + 9 = 18$
b. $9 + n = 18$
c. Does $m = n$?

2. **a.** $m \times 9 = 72$
b. $9 \times n = 72$
c. Does $m = n$?

3. **a.** $m - 8 = 6$
b. $8 - n = 6$
c. Does $m = n$?

4. **a.** $m \div 3 = 3$
b. $3 \div n = 3$
c. Does $m = n$?

5. **a.** $m \times 6 = 54$
b. $6 \times n = 54$
c. Does $m = n$?

6. **a.** $m + \frac{1}{2} = 1$
b. $\frac{1}{2} + n = 1$
c. Does $m = n$?

7. **a.** $m \times 8 = 56$
b. $8 \times n = 56$
c. Does $m = n$?

8. **a.** $m \div 4 = 2$
b. $4 \div n = 2$
c. Does $m = n$?

9. **a.** $m + 7 = 16$
b. $7 + n = 16$
c. Does $m = n$?

10. **a.** $m - 4 = 2$
b. $4 - n = 2$
c. Does $m = n$?

11. **a.** $m \times 4 = 0$
b. $4 \times n = 0$
c. Does $m = n$?

12. **a.** $m - 5 = 3$
b. $5 - n = 3$
c. Does $m = n$?

13. **a.** $m + 0 = 8$
b. $0 + n = 8$
c. Does $m = n$?

14. **a.** $m \div 6 = 2$
b. $6 \div n = 2$
c. Does $m = n$?

15. **a.** $m \times \frac{1}{2} = 6$
b. $\frac{1}{2} \times n = 6$
c. Does $m = n$?

16. **a.** Which operation is used in each Exercise above?
b. In which Exercises above does $m = n$?

For any two numbers m and n, $m + n = n + m$.
Addition is commutative.

For any two numbers m and n, if $m \neq n$, $m - n \neq n - m$.
Subtraction is not commutative.

For any two numbers m and n, $m \times n = n \times m$.
Multiplication is commutative.

For any two non-zero numbers m and n, if $m \neq n$, $m \div n \neq n \div m$.
Division is not commutative.

Choose = or ≠.

1. $84 + 97 \bigcirc 97 + 84$
2. $98 - 49 \bigcirc 49 - 98$
3. $8 \times 36 \bigcirc 36 \times 8$
4. $28 \div 4 \bigcirc 4 \div 28$
5. $74 - 38 \bigcirc 38 - 74$
6. $37 + 65 \bigcirc 65 + 37$
7. $729 + 986 \bigcirc 986 + 729$
8. $64 \div 8 \bigcirc 8 \div 64$
9. $1046 - 832 \bigcirc 832 - 1046$
10. $6 \times 93 \bigcirc 93 \times 6$
11. $642 + 859 \bigcirc 859 + 642$
12. $96 - 38 \bigcirc 38 - 96$

Choose **is** or **is not**.

13. Since the order in which we add two numbers does not change the sum, addition ■ commutative.
14. Since the order in which we multiply two numbers does not change the product, multiplication ■ commutative.
15. Since the order in which we subtract two numbers may affect the difference, subtraction ■ commutative.
16. Since the order in which we divide two numbers may affect the quotient, division ■ commutative.

Give a reason for the truth of each equation.

17. $84 + 69 = 69 + 84$ because **addition is commutative.**
18. $98 \times 65 = 65 \times 98$ because ■
19. $\frac{3}{5} + \frac{4}{5} = \frac{4}{5} + \frac{3}{5}$ because ■
20. $4 \times \frac{2}{3} = \frac{2}{3} \times 4$ because ■

Associativity

Question 2: If we operate on more than two numbers, does the way in which we pair the numbers change the result?

If the way in which we pair the numbers does not change the result, the operation is **associative**.

Solve each equation and answer each question.

1. a. $(8 + 5) + 5 = m$
   b. $8 + (5 + 5) = n$
   c. Does $m = n$?

2. a. $(8 \times 4) \times 6 = m$
   b. $8 \times (4 \times 6) = n$
   c. Does $m = n$?

3. a. $(32 - 8) - 4 = m$
   b. $32 - (8 - 4) = n$
   c. Does $m = n$?

4. a. $(64 \div 8) \div 8 = m$
   b. $64 \div (8 \div 8) = n$
   c. Does $m = n$?

5. a. $\left(\frac{3}{8} + \frac{4}{8}\right) + \frac{5}{8} = m$
   b. $\frac{3}{8} + \left(\frac{4}{8} + \frac{5}{8}\right) = n$
   c. Does $m = n$?

6. a. $(46 - 28) - 15 = m$
   b. $46 - (28 - 15) = n$
   c. Does $m = n$?

7. a. $(38 + 64) + 15 = m$
   b. $38 + (64 + 15) = n$
   c. Does $m = n$?

8. a. $(9 \times 8) \times 7 = m$
   b. $9 \times (8 \times 7) = n$
   c. Does $m = n$?

9. a. $(48 \div 4) \div 2 = m$
   b. $48 \div (4 \div 2) = n$
   c. Does $m = n$?

10. a. $\left(\frac{11}{12} - \frac{6}{12}\right) - \frac{4}{12} = m$
    b. $\frac{11}{12} - \left(\frac{6}{12} - \frac{4}{12}\right) = n$
    c. Does $m = n$?

11. a. Which operation is used in each Exercise above?
    b. In which Exercises above does $m = n$?

For any numbers m, n, and p, $(m + n) + p = m + (n + p)$.
Addition is **associative.**
For any numbers m, n, and p, $(m \times n) \times p = m \times (n \times p)$.
Multiplication is **associative.**
For any numbers m, n, and p, if $p \neq 0$, $(m - n) - p \neq m - (n - p)$.
Subtraction is not associative. We must use parentheses to show how to pair the numbers.
For any positive numbers m, n, and p, if $p \neq 1$,
$(m \div n) \div p \neq m \div (n \div p)$.
Division is not associative. We must use parentheses to show how to pair the numbers.

Choose = or ≠. The parentheses tell how to pair the numbers.

1. $(4 + 2) + 2 \bigcirc 4 + (2 + 2)$
2. $(4 - 2) - 2 \bigcirc 4 - (2 - 2)$
3. $(4 \times 2) \times 2 \bigcirc 4 \times (2 \times 2)$
4. $(4 \div 2) \div 2 \bigcirc 4 \div (2 \div 2)$
5. $(16 + 4) + 2 \bigcirc 16 + (4 + 2)$
6. $(16 - 4) - 2 \bigcirc 16 - (4 - 2)$
7. $(16 \times 4) \times 2 \bigcirc 16 \times (4 \times 2)$
8. $(16 \div 4) \div 2 \bigcirc 16 \div (4 \div 2)$
9. $(12 + 6) + 2 \bigcirc 12 + (6 + 2)$
10. $(12 - 6) - 2 \bigcirc 12 - (6 - 2)$
11. $(12 \times 6) \times 2 \bigcirc 12 \times (6 \times 2)$
12. $(12 \div 6) \div 2 \bigcirc 12 \div (6 \div 2)$
13. $(8 + 4) + 4 \bigcirc 8 + (4 + 4)$
14. $(8 - 4) - 4 \bigcirc 8 - (4 - 4)$
15. $(8 \times 4) \times 4 \bigcirc 8 \times (4 \times 4)$
16. $(8 \div 4) \div 4 \bigcirc 8 \div (4 \div 4)$

Choose **is** or **is not**.

17. Since the pairing of numbers does not change the result, addition ■ associative.
18. Since the pairing of numbers may change the result, subtraction ■ associative.
19. Since the pairing of numbers does not change the result, multiplication ■ associative.
20. Since the pairing of numbers may change the result, division ■ associative.

Checking an Answer

We can check a product by interchanging the factors because multiplication is commutative.

$$\begin{array}{r} 84 \\ \times\ 19 \\ \hline 756 \\ 840 \\ \hline 1596 \end{array} \qquad \textbf{Check:} \quad \begin{array}{r} 19 \\ \times\ 84 \\ \hline 76 \\ 1520 \\ \hline 1596 \end{array}$$

Find each product and check each result as shown above.

1. $\begin{array}{r} 28 \\ \times 15 \\ \hline \end{array}$	2. $\begin{array}{r} 38 \\ \times 20 \\ \hline \end{array}$	3. $\begin{array}{r} 67 \\ \times 36 \\ \hline \end{array}$	4. $\begin{array}{r} 49 \\ \times 54 \\ \hline \end{array}$	5. $\begin{array}{r} 87 \\ \times 65 \\ \hline \end{array}$
6. $\begin{array}{r} 95 \\ \times 29 \\ \hline \end{array}$	7. $\begin{array}{r} 278 \\ \times\ 46 \\ \hline \end{array}$	8. $\begin{array}{r} 689 \\ \times\ 34 \\ \hline \end{array}$	9. $\begin{array}{r} 509 \\ \times 268 \\ \hline \end{array}$	10. $\begin{array}{r} 817 \\ \times 249 \\ \hline \end{array}$

We can check a sum by interchanging the addends because addition is commutative and associative.

$$\begin{array}{r} 97 \\ 68 \\ 45 \\ \hline 210 \end{array} \qquad \textbf{Check:} \quad \begin{array}{r} 68 \\ 45 \\ 97 \\ \hline 210 \end{array} \qquad\qquad \begin{array}{r} 846 \\ 958 \\ 654 \\ \hline 2458 \end{array} \qquad \textbf{Check:} \quad \begin{array}{r} 654 \\ 846 \\ 958 \\ \hline 2458 \end{array}$$

Find each sum. Check each result as shown above.

11. $\begin{array}{r} 884 \\ 957 \\ 276 \\ \hline \end{array}$	12. $\begin{array}{r} 938 \\ 305 \\ 624 \\ \hline \end{array}$	13. $\begin{array}{r} 714 \\ 586 \\ 920 \\ \hline \end{array}$	14. $\begin{array}{r} 8290 \\ 1078 \\ 6347 \\ \hline \end{array}$	15. $\begin{array}{r} 4132 \\ 7596 \\ 8029 \\ \hline \end{array}$
16. $\begin{array}{r} 493 \\ 786 \\ 934 \\ \hline \end{array}$	17. $\begin{array}{r} 188 \\ 729 \\ 706 \\ \hline \end{array}$	18. $\begin{array}{r} 319 \\ 465 \\ 206 \\ \hline \end{array}$	19. $\begin{array}{r} 4373 \\ 9448 \\ 5198 \\ \hline \end{array}$	20. $\begin{array}{r} 5827 \\ 9659 \\ 7069 \\ \hline \end{array}$

Inverse Operations

We can check subtraction by addition. Subtraction and addition are **inverse** operations. Inverse operations undo each other.

$$\begin{array}{r} 8146 \\ -2328 \\ \hline 5818 \end{array}$$

We add these numbers to check.

Check: $5818 + 2328 = 8146$

$$7 - 3 = 4$$

Check: $3 + 4 = 7$

Find each difference. Check by using the inverse operation.

1. $\begin{array}{r} 859 \\ -146 \\ \hline \end{array}$	**2.** $\begin{array}{r} 708 \\ -149 \\ \hline \end{array}$	**3.** $\begin{array}{r} 945 \\ -296 \\ \hline \end{array}$	**4.** $\begin{array}{r} 8456 \\ -4188 \\ \hline \end{array}$	**5.** $\begin{array}{r} 3009 \\ -1463 \\ \hline \end{array}$
6. $\begin{array}{r} 954 \\ -387 \\ \hline \end{array}$	**7.** $\begin{array}{r} 863 \\ -149 \\ \hline \end{array}$	**8.** $\begin{array}{r} 900 \\ -278 \\ \hline \end{array}$	**9.** $\begin{array}{r} 7283 \\ -2087 \\ \hline \end{array}$	**10.** $\begin{array}{r} 4000 \\ -1496 \\ \hline \end{array}$

We can check division by multiplication. Division and multiplication are **inverse** operations.

$$\begin{array}{r} 624 \\ 7\overline{)4368} \\ 4200 \\ \hline 168 \\ 140 \\ \hline 28 \\ 28 \\ \hline 0 \end{array}$$

Check: $\begin{array}{r} 624 \\ \times\ 7 \\ \hline 4368 \end{array}$

Find each quotient. Check by using the inverse operation.

11. $6\overline{)7386}$	**12.** $8\overline{)8616}$	**13.** $4\overline{)4628}$	**14.** $8\overline{)7384}$	**15.** $6\overline{)9402}$
16. $9\overline{)5490}$	**17.** $7\overline{)4585}$	**18.** $8\overline{)9368}$	**19.** $5\overline{)7900}$	**20.** $9\overline{)8658}$
21. $3\overline{)7674}$	**22.** $5\overline{)4825}$	**23.** $9\overline{)8577}$	**24.** $7\overline{)8645}$	**25.** $4\overline{)7368}$

Using two Binary Operations

We can use two binary operations in a problem. Parentheses show how to pair the numbers.

$$28 \div (9 + 5) = 28 \div 14$$
$$= 2$$

Copy and complete each problem.

1. $18 + (24 - 9) =$ ■
2. $(48 + 85) - 28 =$ ■
3. $(274 + 182) \div 6 =$ ■
4. $8 \times (49 + 54) =$ ■
5. $5 \times (92 - 37) =$ ■
6. $(6 \times 74) \div 3 =$ ■
7. $(44 - 27) - 15 =$ ■
8. $(76 - 49) + 83 =$ ■
9. $(939 - 480) \div 9 =$ ■
10. $984 - (624 + 309) =$ ■
11. $9 \times (64 + 78) =$ ■
12. $(785 + 255) \div 5 =$ ■
13. $7 \times (956 - 329) =$ ■
14. $67 - (45 - 29) =$ ■

Copy each equation and put in parentheses to make the equation true.

15. $72 \div 8 + 4 = 13$
16. $14 - 2 \times 3 = 8$
17. $16 - 9 - 7 = 14$
18. $12 - 4 \times 0 = 0$
19. $28 \div 8 \div 4 = 14$
20. $92 - 12 - 9 = 71$
21. $46 + 18 \div 8 = 8$
22. $85 - 45 \div 9 = 80$
23. $69 \div 3 \div 3 = 69$
24. $6 \times 8 + 4 = 52$
25. $10 \times 0 + 8 = 8$
26. $94 \div 2 + 3 = 50$

Choose $=$ or $\neq$.

27. $6 \times (50 + 8)$ ● $(6 \times 50) + (6 \times 8)$
28. $9 \times (42 + 8)$ ● $(9 \times 42) + (9 \times 8)$
29. $7 \times (46 - 38)$ ● $(7 \times 46) - (7 \times 38)$
30. $5 \times (82 - 49)$ ● $(5 \times 82) - (5 \times 49)$
31. $4 \times (32 + 83)$ ● $(4 \times 32) + (4 \times 83)$
32. $8 \times (63 - 48)$ ● $(8 \times 63) - (8 \times 48)$

The Distributive Laws

Solve each equation and answer each question.

1. a. $7 \times (18 + 38) = m$
   b. $(7 \times 18) + (7 \times 38) = n$
   c. Does $m = n$?

2. a. $9 \times (64 - 53) = m$
   b. $(9 \times 64) - (9 \times 53) = n$
   c. Does $m = n$?

3. a. $(19 + 34) \times 8 = m$
   b. $(19 \times 8) + (34 \times 8) = n$
   c. Does $m = n$?

4. a. $(82 - 75) \times 6 = m$
   b. $(82 \times 6) - (75 \times 6) = n$
   c. Does $m = n$?

5. a. $8 \times (49 + 63) = m$
   b. $(8 \times 49) + (8 \times 63) = n$
   c. Does $m = n$?

6. a. $9 \times (89 - 75) = m$
   b. $(9 \times 89) - (9 \times 75) = n$
   c. Does $m = n$?

There are two forms of the **distributive law for multiplication over addition.** For any numbers m, n, and p,

$$m \times (n + p) = (m \times n) + (m \times p)$$
$$(n + p) \times m = (n \times m) + (p \times m)$$

There are two forms of the **distributive law for multiplication over subtraction.** For any numbers m, n, and p,

$$m \times (n - p) = (m \times n) - (m \times p)$$
$$(n - p) \times m = (n \times m) - (p \times m)$$

Copy and complete each problem.

7. $6 \times (4 + 5) = (6 \times 4) + (6 \times \blacksquare)$
8. $8 \times (9 - 6) = (\blacksquare \times 9) - (\blacksquare \times 6)$
9. $(7 - 6) \times 3 = (7 \times 3) - (\blacksquare \times 3)$
10. $(8 + 9) \times 4 = (\blacksquare \times 4) + (9 \times 4)$
11. $5 \times (6 + 7) = (\blacksquare \times 6) + (\blacksquare \times 7)$
12. $(9 - 4) \times 8 = (9 \times \blacksquare) - (4 \times \blacksquare)$

Solve each equation and answer each question.

1. a. $(95 + 70) \div 5 = m$
   b. $(95 \div 5) + (70 \div 5) = n$
   c. Does $m = n$?

2. a. $(174 - 72) \div 6 = m$
   b. $(174 \div 6) - (72 \div 6) = n$
   c. Does $m = n$?

3. a. $(182 + 238) \div 7 = m$
   b. $(182 \div 7) + (238 \div 7) = n$
   c. Does $m = n$?

4. a. $(972 - 810) \div 9 = m$
   b. $(972 \div 9) - (810 \div 9) = n$
   c. Does $m = n$?

There is only one form of the **distributive law for division over addition.**
For any numbers m, n, and p, if $p \neq 0$,

$$(m + n) \div p = (m \div p) + (n \div p).$$

There is only one form of the **distributive law for division over subtraction.**
For any numbers m, n, and p, if $p \neq 0$,

$$(m - n) \div p = (m \div p) - (n \div p).$$

Copy and complete each problem.

5. $(26 + 19) \div 5 = (\square \div 5) + (19 \div 5)$
6. $(73 - 49) \div 2 = (73 \div \square) - (49 \div \square)$
7. $(94 + 56) \div 6 = (94 \div 6) + (\square \div 6)$
8. $(87 - 39) \div 8 = (\square \div 8) - (39 \div 8)$
9. $(65 - 38) \div 7 = (65 \div 7) - (\square \div 7)$
10. $(89 + 94) \div 4 = (89 \div \square) + (94 \div \square)$

Solve each problem.

11. Barbara read 85 pages in one book and 110 pages in another book during a 5-day period. If she read the same number of pages each day, how many pages did she read each day?

12. At a school party 96 red balloons and 48 yellow balloons were divided equally among 8 team captains. How many balloons did each team captain receive for his team?

Identity Element in Addition

Complete each problem.

1. $0 + \square = 27$
2. $46 + \square = 46$
3. $\square - 0 = 96$
4. $28 - \square = 28$
5. $\frac{4}{5} + \square = \frac{4}{5}$
6. $87 + \square = 87$
7. $\square + 54 = 54$
8. $\square - 0 = 71$
9. $0 + 39 = \square$
10. $\frac{2}{9} - \square = \frac{2}{9}$
11. $57 + \square = 57$
12. $82 - 0 = \square$
13. $\square + 46 = 46$
14. $92 - \square = 92$
15. $\square + 0 = \frac{5}{6}$

Choose $=$ or $\neq$.

16. $0 - 7 \bigcirc 7$
17. $9 - 0 \bigcirc 9$
18. $8 + 0 \bigcirc 8$
19. $0 + 4 \bigcirc 4$
20. $0 - 0 \bigcirc 0$
21. $4 - 0 \bigcirc 4$
22. $6 + 0 \bigcirc 6$
23. $0 + 6 \bigcirc 6$
24. $8 - 0 \bigcirc 8$

If we add zero to any number, the sum is that number. **Zero** is the **identity element** in addition. For any number n,

$$n + 0 = n \text{ and } 0 + n = n.$$

If we subtract zero from any number, the difference is that number. For any number n,

$$n - 0 = n.$$

If we subtract any non-zero number from zero, the difference is not that number. For any non-zero number n,

$$0 - n \neq n.$$

Complete each problem.

25. $93 \times 0 = \square$
26. $\square \times 93 = 0$
27. $0 \div 84 = \square$
28. $49 \times 0 = \square$
29. $83 \times \square = 0$
30. $\square \div 19 = 0$

For any number n, $n \times 0 = 0$ and $0 \times n = 0$.
For any non-zero number n, $0 \div n = 0$.
We **cannot** divide by zero.

Identity Element in Multiplication

Complete each problem.

1. $1 \times \square = 36$
2. $42 \times \square = 42$
3. $86 \div \square = 86$
4. $\square \times 57 = 57$
5. $75 \div 1 = \square$
6. $84 \times \square = 84$
7. $1 \times 0 = \square$
8. $\square \times 1 = 91$
9. $\square \div 1 = 43$
10. $69 \div \square = 69$
11. $1 \div \square = 1$
12. $24 \times 1 = \square$
13. $96 \div \square = 1$
14. $\square \times 1 = 51$
15. $67 \div 67 = \square$

Choose $=$ or $\neq$.

16. $\frac{6}{7} \times 1 \bigcirc \frac{6}{7}$
17. $79 \times 1 \bigcirc 79$
18. $1 \times 53 \bigcirc 53$
19. $\frac{3}{4} \div 1 \bigcirc \frac{3}{4}$
20. $86 \div 1 \bigcirc 86$
21. $1 \div 45 \bigcirc 45$
22. $\frac{7}{8} \div \frac{7}{8} \bigcirc 1$
23. $64 \div 64 \bigcirc 1$
24. $0 \div 1 \bigcirc 0$

If we multiply **one** by any number, the product is that number. **One** is the **identity element** in multiplication. For any number n,

$$1 \times n = n \text{ and } n \times 1 = n.$$

If we divide any number by one, the quotient is that number. For any number n,

$$n \div 1 = n.$$

If we divide one by any number except one, the quotient is not that number. For any number n, if $n \neq 1$,

$$1 \div n \neq n.$$

If we divide any non-zero number by itself, the quotient is one. For any number n, if $n \neq 0$,

$$n \div n = 1.$$

Complete each statement.

25. $\square$ is the identity element in addition.
26. $\square$ is the identity element in multiplication.
27. If we divide any number p by one, the quotient is $\square$.
28. If we divide any non-zero number q by itself, the quotient is $\square$.

Giving Reasons

Pair each equation with the reason why it is true.

1. $a + 0 = a$
2. $(a + b) \div c = (a \div c) + (b \div c)$
3. $(a \times b) \times c = a \times (b \times c)$
4. $a \times 1 = a$
5. $(a + b) + c = a + (b + c)$
6. $(a - b) \div c = (a \div c) - (b \div c)$
7. $a \times b = b \times a$
8. $a \times (b - c) = (a \times b) - (a \times c)$
9. $a + b = b + a$
10. $a \times (b + c) = (a \times b) + (a \times c)$

A. Addition is associative.
B. Multiplication is commutative.
C. Addition is commutative.
D. Multiplication is distributive over addition.
E. Multiplication is distributive over subtraction.
F. One is the identity element in multiplication.
G. Zero is the identity element in addition.
H. Multiplication is associative.
I. Division is distributive over addition.
J. Division is distributive over subtraction.

Give the reason why each equation is true.

11. $964 \times 1 = 964$
12. $58 + 46 = 46 + 58$
13. $8 \times 6 = 6 \times 8$
14. $1 \times 400 = 400$
15. $926 + 0 = 926$
16. $28 + 54 = 54 + 28$
17. $0 + 87 = 87$
18. $3 \times 45 = 45 \times 3$
19. $(16 + 15) + 28 = 16 + (15 + 28)$
20. $4 \times (20 + 13) = (4 \times 20) + (4 \times 13)$
21. $(81 - 63) \div 9 = (81 \div 9) - (63 \div 9)$
22. $7 \times (4 + 5) = (7 \times 4) + (7 \times 5)$
23. $(49 + 42) \div 7 = (49 \div 7) + (42 \div 7)$
24. $6 \times (45 + 38) = (6 \times 45) + (6 \times 38)$
25. $(42 + 83) + 15 = 42 + (83 + 15)$
26. $17 \times (28 - 15) = (17 \times 28) - (17 \times 15)$

Checkout

Match the name of each law with the mathematical sentence describing it.

1. associative law for addition	A. $m \times (n + p) = (m \times n) + (m \times p)$
2. associative law for multiplication	B. $a + b = b + a$
3. distributive law for multiplication	C. $(e + f) + g = e + (f + g)$
4. distributive law for division	D. $j \times k = k \times j$
5. commutative law for addition	E. $(a + b) \div c = (a \div c) + (b \div c)$
6. commutative law for multiplication	F. $(p \times q) \times r = p \times (q \times r)$

Choose $=$ or $\neq$.

7. $(216 - 147) - 65 \bigcirc 216 - (147 - 65)$
8. $(18 \times 24) \times 36 \bigcirc 18 \times (24 \times 36)$
9. $(789 + 234) + 111 \bigcirc 789 + (234 + 111)$
10. $(108 \div 12) \div 3 \bigcirc 108 \div (12 \div 3)$
11. $(a + b) + 34 \bigcirc 34 + (a + b)$
12. $(52 + 26) \div 13 \bigcirc (52 \div 13) + (26 \div 13)$

Find each sum.

13.	14.	15.	16.	17.
2	10	68	357	703
4	15	73	106	498
6	3	45	829	340
8	18	32	112	271

Checkout

1. $\{Al, Ed\} \cup \{Joe, Ed, Ian\} =$ ■
2. $\{a, b, c, d\} \cap \{d, b, y\} =$ ■
3. $\{9, 8, 6\} \cap \{n: n < 9\} =$ ■
4. $\{Utah, Ohio\} \cup \{Utah\} =$ ■
5. $\{May, June\} - \{April\} =$ ■
6. $\{n: n < 3\} - \{n: n < 5\} =$ ■
7. $\{Joy, Ann, Pat\} - \{Ann\} =$ ■

8. $\{n: 3 < n < 9\} \cap \{n: 4 < n < 8\} \cap \{n: 1 < n < 9 \text{ and } n \text{ is odd}\} =$ ■
9. $\{10, 20, 30, 40\} \cup \{10, 11, 12\} \cup \{n: 12 < n < 17\} =$ ■

10. Choose $=$ or $\neq$.
    a. $\{1, 2\}$ ● $\{2, 3\}$
    b. $\{Ted\}$ ● $\{Ken, Ted\}$
    c. $\{3, 5\}$ ● $\{5, 3\}$

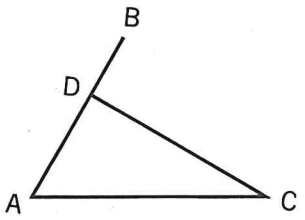
11. Choose $\approx$ or $\not\approx$.
    a. $\{1, 2\}$ ● $\{2, 3\}$
    b. $\{Ted\}$ ● $\{Ken, Ted\}$
    c. $\{3, 5\}$ ● $\{5, 3\}$

12. Choose $\in$ or $\subset$.
    a. 3 ● $\{2, 3, 4\}$
    b. $\{2\}$ ● $\{2, 3, 4\}$

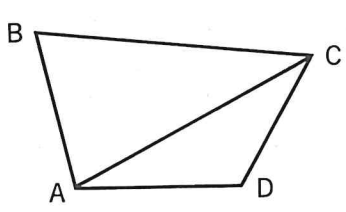
13. If $A = \{1, 2, 3, 4\}$ and $B = \{2, 4, 6, 8\}$, find $A \cap B$.

14. Write a set description for each figure.

a.



b.



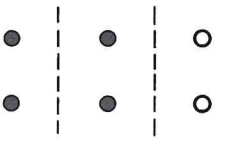
Part 3 Fractions

The **denominator** of a fraction tells into how many equal parts a whole has been divided.

The **numerator** of a fraction tells how many of the parts we are talking about.

numerator → $\frac{2}{3}$ ← denominator

$\frac{2}{3}$ refers to 2 of 3 equal parts.



1. Give the numerator of each fraction.

a. $\frac{3}{5}$	c. $\frac{11}{4}$	e. $\frac{8}{12}$	g. $\frac{6}{6}$	i. $\frac{10}{9}$
b. $\frac{7}{8}$	d. $\frac{6}{3}$	f. $\frac{4}{4}$	h. $\frac{1}{8}$	j. $\frac{3}{7}$

2. Give the denominator of each fraction in Exercise 1.

If the numerator is less than the denominator, the fraction is less than 1.

$\frac{2}{3} < 1$ $\frac{7}{8} < 1$ $\frac{31}{36} < 1$

If the numerator is equal to the denominator, the fraction is equal to 1.

$\frac{3}{3} = 1$ $\frac{8}{8} = 1$ $\frac{36}{36} = 1$

If the numerator is greater than the denominator, the fraction is greater than 1.

$\frac{5}{3} > 1$ $\frac{11}{8} > 1$ $\frac{40}{36} > 1$

3. Which fractions in Exercise 1 are less than 1?
4. Which fractions in Exercise 1 are greater than 1?

The whole numbers are 0, 1, 2, 3, 4, 5, 6, 7, . . .
We can add whole numbers and fractions.

$3 + \frac{1}{7} = 3\frac{1}{7}$ $\qquad$ $11 + \frac{4}{5} = 11\frac{4}{5}$

Numbers like $3\frac{1}{7}$ and $11\frac{4}{5}$ are often called **mixed numbers.**

1. Which of these numbers are mixed numbers?

a. 12	c. $4\frac{1}{4}$	e. 2	g. $\frac{3}{11}$	i. $7\frac{1}{3}$
b. $6\frac{1}{2}$	d. $\frac{3}{4}$	f. $6\frac{9}{10}$	h. 5	j. $9\frac{2}{5}$

If a fraction is less than 1, it is sometimes called a proper fraction.
Otherwise, it is called an improper fraction.

2. Which of these fractions are proper fractions?

a. $\frac{3}{4}$	c. $\frac{2}{7}$	e. $\frac{4}{4}$	g. $\frac{2}{9}$	i. $\frac{19}{20}$
b. $\frac{8}{3}$	d. $\frac{9}{4}$	f. $\frac{9}{3}$	h. $\frac{5}{16}$	j. $\frac{7}{7}$

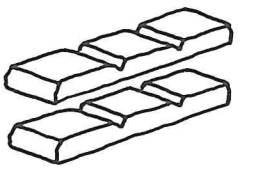
3. Which of these fractions are equal to 1?

a. $\frac{11}{8}$	e. $\frac{6}{6}$	i. $\frac{1}{1}$	m. $\frac{1}{9}$	q. $\frac{9}{9}$
b. $\frac{5}{7}$	f. $\frac{0}{5}$	j. $\frac{15}{15}$	n. $\frac{0}{4}$	r. $\frac{16}{15}$
c. $\frac{3}{8}$	g. $\frac{2}{2}$	k. $\frac{9}{6}$	o. $\frac{3}{3}$	s. $\frac{18}{19}$
d. $\frac{8}{8}$	h. $\frac{1}{4}$	l. $\frac{14}{3}$	p. $\frac{5}{5}$	t. $\frac{8}{18}$

4. Choose **equal to** or **less than** or **greater than.**
   a. If the numerator is equal to the denominator, the fraction is ■ 1.
   b. If the numerator is greater than the denominator, the fraction is ■ 1.
   c. If the numerator is less than the denominator, the fraction is ■ 1.

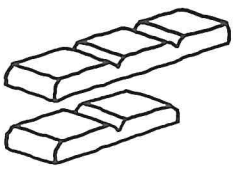
Some fractions equal whole numbers.
We have 2 whole candy bars.
We cut the bars to serve 6 people equal amounts.
We have 6 thirds.

$\frac{6}{3} = 2$



Some fractions equal mixed numbers.
We have 1 whole candy bar and 2 thirds.
One piece of the candy bar is gone.
We have 5 thirds left.

$\frac{5}{3} = 1\frac{2}{3}$



We divide in order to find the whole number or mixed number equal to an improper fraction.

$\frac{6}{3} = 6 \div 3 = 2$ $\qquad$ $\frac{5}{3} = 5 \div 3 = 1\frac{2}{3}$

Find the whole number or mixed number equal to each improper fraction.

1. $\frac{18}{4}$
2. $\frac{21}{3}$
3. $\frac{19}{6}$
4. $\frac{20}{4}$
5. $\frac{13}{2}$
6. $\frac{11}{6}$
7. $\frac{60}{12}$
8. $\frac{19}{7}$
9. $\frac{16}{3}$
10. $\frac{27}{10}$
11. $\frac{19}{2}$
12. $\frac{23}{9}$
13. $\frac{56}{7}$
14. $\frac{36}{9}$
15. $\frac{27}{4}$
16. $\frac{387}{100}$
17. $\frac{230}{100}$
18. $\frac{41}{7}$
19. $\frac{187}{100}$
20. $\frac{59}{6}$

Choose $<$ or $=$ or $>$.

21. $\frac{5}{5}$ ○ 1
22. $\frac{2}{5}$ ○ 1
23. $\frac{10}{5}$ ○ 1
24. 1 ○ $\frac{9}{6}$
25. 1 ○ $\frac{5}{6}$
26. 1 ○ $\frac{6}{6}$
27. $\frac{3}{4}$ ○ $\frac{4}{4}$
28. $\frac{5}{4}$ ○ $\frac{4}{4}$
29. $\frac{8}{8}$ ○ $\frac{4}{4}$

Fractions whose denominators are the same are often called **like fractions.** When we add or subtract like fractions we add or subtract their numerators.

$$\frac{3}{8} + \frac{2}{8} = \frac{5}{8} \qquad \frac{7}{16} - \frac{2}{16} = \frac{5}{16}$$

Find each sum. If possible, express the answer as a mixed number or a whole number.

1. $\frac{1}{5} + \frac{3}{5} =$ ■
2. $\frac{5}{2} + \frac{4}{2} =$ ■
3. $\frac{1}{3} + \frac{1}{3} =$ ■
4. $\frac{5}{4} + \frac{1}{4} =$ ■
5. $\frac{5}{8} + \frac{2}{8} =$ ■
6. $\frac{11}{12} + \frac{2}{12} =$ ■
7. $\frac{3}{7} + \frac{5}{7} =$ ■
8. $\frac{2}{9} + \frac{6}{9} =$ ■
9. $\frac{1}{17} + \frac{4}{17} =$ ■
10. $\frac{4}{39} + \frac{3}{39} =$ ■
11. $\frac{12}{10} + \frac{6}{10} =$ ■
12. $\frac{7}{8} + \frac{7}{8} =$ ■

Find each difference.

13. $\frac{3}{4} - \frac{1}{4} =$ ■
14. $\frac{5}{8} - \frac{2}{8} =$ ■
15. $\frac{6}{7} - \frac{1}{7} =$ ■
16. $\frac{7}{9} - \frac{6}{9} =$ ■
17. $\frac{19}{10} - \frac{6}{10} =$ ■
18. $\frac{15}{7} - \frac{4}{7} =$ ■
19. $\frac{3}{5} - \frac{3}{5} =$ ■
20. $\frac{19}{8} - \frac{9}{8} =$ ■
21. $\frac{17}{20} - \frac{14}{20} =$ ■
22. $\frac{15}{17} - \frac{12}{17} =$ ■
23. $\frac{16}{31} - \frac{2}{31} =$ ■
24. $\frac{70}{47} - \frac{10}{47} =$ ■

Copy and complete each problem.

25. $\frac{2}{4} +$ ■ $= \frac{3}{4}$
26. $\frac{1}{5} +$ ■ $= \frac{4}{5}$
27. $\frac{1}{7} +$ ■ $= \frac{3}{7}$
28. $\frac{2}{8} +$ ■ $= \frac{5}{8}$
29. $\frac{10}{8} -$ ■ $= \frac{7}{8}$
30. $\frac{4}{5} -$ ■ $= \frac{3}{5}$
31. $\frac{9}{7} -$ ■ $= \frac{6}{7}$
32. $\frac{11}{10} -$ ■ $= \frac{8}{10}$
33. ■ $- \frac{4}{6} = \frac{5}{6}$
34. ■ $- \frac{7}{100} = \frac{9}{100}$
35. ■ $- \frac{8}{14} = \frac{3}{14}$
36. ■ $- \frac{9}{11} = \frac{7}{11}$

Solve each problem. When possible give answers either as mixed numbers or as whole numbers.

1. Each side of a square measures $\frac{1}{4}$ foot. What is the perimeter of the square?

$$\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{4}{4} = \square$$

2. Penny has $\frac{3}{4}$ pound of candy. Dottie has $\frac{3}{4}$ pound of candy more than Penny. How many pounds of candy has Dottie?

3. Leon has $\frac{3}{5}$ dollar; Hugh has $\frac{1}{5}$ dollar. How much more money has Leon?

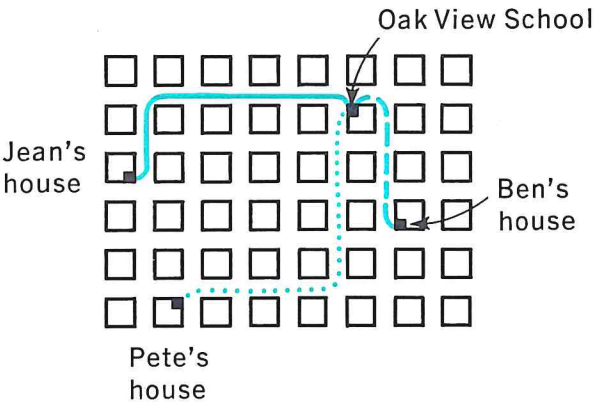
4. The side of an equilateral triangle measures $\frac{7}{4}$ inches. What is the perimeter of the triangle?

5. Patty and Janet each spend $\frac{7}{6}$ of an hour every night doing homework. If Patty has spent $\frac{3}{6}$ of an hour and Janet $\frac{5}{6}$ of an hour, how much more time will each girl study?

6. The perimeter of a rectangular field is $\frac{8}{12}$ mile. If the width of the field is $\frac{1}{12}$ mile, what is its length?

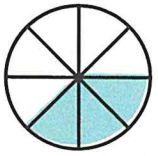
7. The map shows Jean's, Pete's, and Ben's routes to school. Each block is $\frac{1}{10}$ mile long. How far does each one walk?

8. Who walks the greatest distance to the Oak View School, Ben, Pete, or Jean?

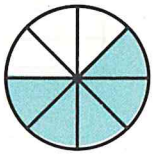


9. Claire has $\frac{3}{4}$ of an hour each day for lunch. How much time does she have for lunch in five days?

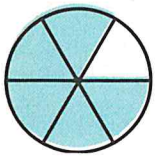
If two fractions have the same denominator, then the fraction with the lesser numerator is the lesser.



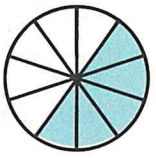
$\frac{3}{8}$ $\frac{3}{8} < \frac{5}{8}$ $\frac{5}{8}$



If two fractions have the same numerator, then the fraction with the greater denominator is the lesser.



$\frac{5}{6}$ $\frac{5}{10} < \frac{5}{6}$ $\frac{5}{10}$



An improper fraction is greater than a proper fraction.

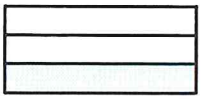
Choose < or >.

1. $\frac{3}{7}$ ● $\frac{7}{3}$
2. $\frac{5}{8}$ ● $\frac{5}{6}$
3. $\frac{6}{5}$ ● $\frac{3}{5}$
4. $\frac{1}{2}$ ● $\frac{1}{3}$
5. $\frac{7}{8}$ ● $\frac{4}{3}$
6. $\frac{1}{4}$ ● $\frac{1}{5}$
7. $\frac{15}{16}$ ● $\frac{11}{10}$
8. $\frac{3}{5}$ ● $\frac{3}{2}$
9. $\frac{7}{6}$ ● $\frac{9}{10}$
10. $\frac{6}{7}$ ● $\frac{5}{7}$
11. $\frac{1}{7}$ ● $\frac{1}{9}$
12. $\frac{3}{4}$ ● $\frac{2}{4}$
13. $\frac{4}{6}$ ● $\frac{5}{6}$
14. $\frac{5}{8}$ ● $\frac{5}{9}$
15. $\frac{8}{12}$ ● $\frac{11}{12}$
16. $\frac{8}{11}$ ● $\frac{5}{4}$
17. $\frac{7}{5}$ ● $\frac{5}{7}$
18. $\frac{9}{10}$ ● $\frac{8}{10}$
19. $\frac{2}{7}$ ● $\frac{2}{6}$
20. $\frac{17}{18}$ ● $\frac{4}{2}$

Choose < or = or >.

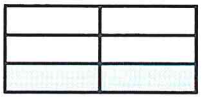
21. $\frac{4}{4}$ ● 1
22. $\frac{3}{5}$ ● $\frac{3}{7}$
23. $\frac{2}{5}$ ● $\frac{3}{5}$
24. $\frac{10}{15}$ ● $\frac{2}{15}$
25. $\frac{8}{12}$ ● $\frac{8}{2}$
26. $\frac{3}{5}$ ● $\frac{5}{5}$
27. 1 ● $\frac{5}{5}$
28. $\frac{19}{20}$ ● 1
29. $\frac{7}{4}$ ● $\frac{7}{7}$
30. $\frac{7}{7}$ ● $\frac{4}{4}$
31. $\frac{3}{8}$ ● $\frac{7}{8}$
32. $\frac{3}{10}$ ● $\frac{3}{2}$
33. $\frac{3}{4}$ ● $\frac{4}{4}$
34. $\frac{3}{2}$ ● $\frac{3}{4}$
35. $\frac{2}{2}$ ● $\frac{5}{5}$
36. 1 ● $\frac{3}{9}$

Finding Other Fraction Names



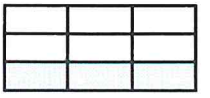
$\frac{1}{3}$ The given fraction

$\frac{1 \times 2}{3 \times 2}$



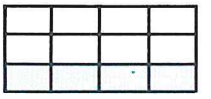
$\frac{2}{6}$ The equal parts each divided into 2 parts

$\frac{1 \times 3}{3 \times 3}$



$\frac{3}{9}$ The equal parts each divided into 3 parts

$\frac{1 \times 4}{3 \times 4}$



$\frac{4}{12}$ The equal parts each divided into 4 parts

$$\frac{1}{3} = \frac{2}{6} = \frac{3}{9} = \frac{4}{12}$$

We can find an equal fraction by multiplying the numerator and the denominator by the same number.

$$\frac{2}{3} = \frac{2}{3} \times \frac{4}{4} \qquad \frac{4}{4} = 1$$

$$= \frac{2 \times 4}{3 \times 4}$$

$$= \frac{8}{12}$$

$$\frac{4}{5} = \frac{4}{5} \times \frac{3}{3} \qquad \frac{3}{3} = 1$$

$$= \frac{4 \times 3}{5 \times 3}$$

$$= \frac{12}{15}$$

Find another name for each fraction. Choose different names for one.

$$\frac{3}{4} = \frac{3 \times 7}{4 \times 7} = \frac{21}{28}$$

1. $\frac{1}{9}$
2. $\frac{7}{10}$
3. $\frac{4}{5}$
4. $\frac{7}{8}$
5. $\frac{3}{11}$
6. $\frac{7}{9}$
7. $\frac{1}{10}$
8. $\frac{6}{7}$
9. $\frac{8}{9}$
10. $\frac{5}{12}$
11. $\frac{2}{11}$
12. $\frac{1}{4}$
13. $\frac{9}{10}$
14. $\frac{11}{12}$
15. $\frac{2}{9}$
16. $\frac{8}{11}$
17. $\frac{3}{22}$
18. $\frac{3}{7}$
19. $\frac{8}{17}$
20. $\frac{5}{13}$

Give 4 fractions equal to the given fraction.

$$\frac{2}{3} = \frac{4}{6} = \frac{6}{9} = \frac{8}{12} = \frac{10}{15}$$

1. $\frac{1}{2}$
2. $\frac{3}{5}$
3. $\frac{4}{11}$
4. $\frac{3}{10}$
5. $\frac{5}{6}$
6. $\frac{2}{7}$
7. $\frac{7}{10}$
8. $\frac{3}{8}$

We can sometimes find other fraction names by dividing the numerator and denominator by the same number. When we divide the numerator and denominator by the same number, we are dividing the fraction by one.

$$\frac{6}{18} = \frac{6}{18} \div \frac{3}{3} \qquad \frac{3}{3} = 1$$
$$= \frac{6 \div 3}{18 \div 3}$$
$$= \frac{2}{6}$$

$$\frac{6}{18} = \frac{6}{18} \div \frac{6}{6} \qquad \frac{6}{6} = 1$$
$$= \frac{6 \div 6}{18 \div 6}$$
$$= \frac{1}{3}$$

Divide to find 2 fractions equal to the given fraction.

9. $\frac{4}{16}$
10. $\frac{8}{16}$
11. $\frac{6}{48}$
12. $\frac{12}{36}$
13. $\frac{8}{28}$
14. $\frac{4}{20}$
15. $\frac{18}{24}$
16. $\frac{10}{60}$
17. $\frac{30}{48}$
18. $\frac{18}{42}$
19. $\frac{24}{30}$
20. $\frac{12}{54}$
21. $\frac{16}{72}$
22. $\frac{20}{50}$
23. $\frac{40}{56}$
24. $\frac{30}{36}$

Choose $=$ or $\neq$.

25. $\frac{2}{3} \bigcirc \frac{4}{6}$
26. $\frac{5}{10} \bigcirc \frac{20}{40}$
27. $\frac{2}{11} \bigcirc \frac{3}{22}$
28. $\frac{9}{12} \bigcirc \frac{2}{3}$
29. $\frac{6}{7} \bigcirc \frac{36}{27}$
30. $\frac{5}{30} \bigcirc \frac{1}{6}$
31. $\frac{8}{10} \bigcirc \frac{4}{5}$
32. $\frac{3}{5} \bigcirc \frac{12}{20}$
33. $\frac{1}{7} \bigcirc \frac{6}{42}$
34. $\frac{5}{6} \bigcirc \frac{25}{40}$
35. $\frac{18}{48} \bigcirc \frac{3}{8}$
36. $\frac{16}{64} \bigcirc \frac{2}{8}$

When no whole number except 1 exactly divides both the numerator and the denominator, we often say the fraction is given in its **lowest terms.**

Only 1 exactly divides both 4 and 7.
$\frac{4}{7}$ is in lowest terms.

1. Which fractions are in lowest terms?

a. $\frac{2}{3}$	c. $\frac{8}{12}$	e. $\frac{6}{7}$	g. $\frac{4}{6}$
b. $\frac{4}{10}$	d. $\frac{3}{5}$	f. $\frac{3}{4}$	h. $\frac{3}{9}$

2. Divide both numerator and denominator by the same number to express each fraction in lowest terms.

a. $\frac{3}{6}$	c. $\frac{12}{15}$	e. $\frac{6}{8}$	g. $\frac{6}{21}$
b. $\frac{9}{12}$	d. $\frac{9}{24}$	f. $\frac{10}{15}$	h. $\frac{7}{14}$

We may divide more than once.

$$\frac{60}{100} = \frac{60 \div 10}{100 \div 10} = \frac{6}{10}$$

$$\frac{6}{10} = \frac{6 \div 2}{10 \div 2} = \frac{3}{5}$$

$\frac{3}{5}$ is $\frac{60}{100}$ expressed in lowest terms.

3. Express in lowest terms.

a. $\frac{4}{16}$	e. $\frac{12}{24}$	i. $\frac{75}{100}$	m. $\frac{18}{48}$
b. $\frac{20}{40}$	f. $\frac{8}{12}$	j. $\frac{20}{32}$	n. $\frac{10}{12}$
c. $\frac{6}{36}$	g. $\frac{24}{30}$	k. $\frac{24}{56}$	o. $\frac{40}{100}$
d. $\frac{12}{20}$	h. $\frac{50}{100}$	l. $\frac{7}{21}$	p. $\frac{12}{60}$

Finding a Missing Numerator

We can find a missing numerator if we are given a fraction and a new denominator which has the first denominator as a factor.

$$\frac{2}{3} = \frac{\square}{12}$$

1. Divide the greater denominator by the lesser. $12 \div 3 = 4$
2. Multiply the numerator and denominator of the given fraction by this quotient.

$$\frac{2 \times 4}{3 \times 4} = \frac{8}{12}$$

3. The numerator of the new fraction is the missing numerator.

$$\frac{2}{3} = \frac{2 \times 4}{3 \times 4} = \frac{8}{12}$$

Find each missing numerator.

1. $\frac{1}{3} = \frac{\square}{9}$
2. $\frac{3}{4} = \frac{\square}{8}$
3. $\frac{2}{5} = \frac{\square}{10}$
4. $\frac{2}{4} = \frac{\square}{8}$
5. $\frac{4}{6} = \frac{\square}{12}$
6. $\frac{3}{7} = \frac{\square}{14}$
7. $\frac{1}{3} = \frac{\square}{6}$
8. $\frac{1}{4} = \frac{\square}{12}$
9. $\frac{2}{3} = \frac{\square}{9}$
10. $\frac{5}{6} = \frac{\square}{18}$
11. $\frac{5}{7} = \frac{\square}{21}$
12. $\frac{3}{5} = \frac{\square}{10}$
13. $\frac{2}{4} = \frac{\square}{12}$
14. $\frac{3}{8} = \frac{\square}{16}$
15. $\frac{2}{3} = \frac{\square}{15}$
16. $\frac{3}{6} = \frac{\square}{18}$
17. $\frac{3}{4} = \frac{\square}{12}$
18. $\frac{2}{9} = \frac{\square}{18}$
19. $\frac{5}{6} = \frac{\square}{24}$
20. $\frac{1}{5} = \frac{\square}{10}$
21. $\frac{3}{4} = \frac{\square}{16}$
22. $\frac{5}{7} = \frac{\square}{14}$
23. $\frac{3}{10} = \frac{\square}{20}$
24. $\frac{1}{3} = \frac{\square}{12}$
25. $\frac{5}{11} = \frac{\square}{55}$
26. $\frac{1}{12} = \frac{\square}{36}$
27. $\frac{7}{10} = \frac{\square}{30}$
28. $\frac{1}{4} = \frac{\square}{36}$

29. Margie's recipe calls for $\frac{3}{4}$ cup of flour.
    a. Betty says that this is the same as $\frac{5}{8}$ cup of flour. Is Betty correct?
    b. Ann says that this is the same as $\frac{7}{8}$ cup of flour. Is Ann correct?

Comparing Fractions

We can compare fractions with the same denominator or the same numerator.

$\frac{3}{5}$ is less than $\frac{4}{5}$ because 3 is less than 4.

$\frac{3}{5}$ is less than $\frac{3}{4}$ because $\frac{1}{5}$ is less than $\frac{1}{4}$.

We can find other fraction names for the same fraction to help us compare fractions.

$\frac{2}{3}$ is less than $\frac{3}{4}$ because $\frac{8}{12}$ is less than $\frac{9}{12}$.

Choose $<$ or $>$.

1. $\frac{4}{6} \bigcirc \frac{4}{7}$
2. $\frac{3}{4} \bigcirc \frac{5}{4}$
3. $\frac{4}{3} \bigcirc \frac{4}{12}$
4. $\frac{6}{5} \bigcirc \frac{6}{7}$
5. $\frac{2}{7} \bigcirc \frac{2}{9}$
6. $\frac{5}{6} \bigcirc \frac{6}{6}$
7. $\frac{9}{7} \bigcirc \frac{4}{7}$
8. $\frac{5}{8} \bigcirc \frac{5}{9}$
9. $\frac{3}{8} \bigcirc \frac{3}{7}$
10. $\frac{9}{10} \bigcirc \frac{9}{12}$
11. $\frac{1}{3} \bigcirc \frac{2}{3}$
12. $\frac{7}{12} \bigcirc \frac{5}{12}$

Choose $<$ or $=$ or $>$.

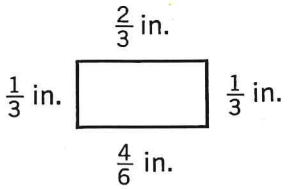
$\frac{3}{4} \bigcirc \frac{5}{8}$ Think: $\frac{3}{4} = \frac{6}{8}$ and $\frac{6}{8} > \frac{5}{8}$

$\frac{3}{4} > \frac{5}{8}$

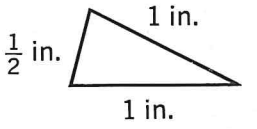
13. $\frac{3}{5} \bigcirc \frac{4}{10}$
14. $\frac{9}{12} \bigcirc \frac{2}{3}$
15. $\frac{2}{14} \bigcirc \frac{2}{7}$
16. $\frac{3}{4} \bigcirc \frac{5}{8}$
17. $\frac{1}{3} \bigcirc \frac{5}{12}$
18. $\frac{7}{12} \bigcirc \frac{2}{3}$
19. $\frac{4}{9} \bigcirc \frac{1}{3}$
20. $\frac{6}{9} \bigcirc \frac{2}{3}$
21. $\frac{5}{6} \bigcirc \frac{7}{18}$

1. Find the perimeter of each of the pictured figures.

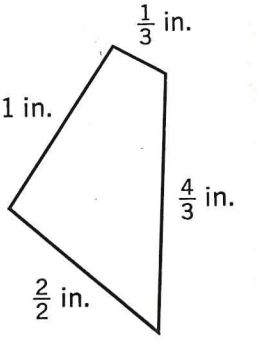
a.



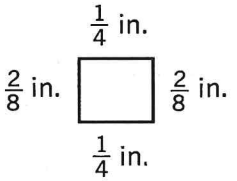
b.



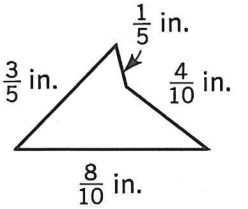
c.



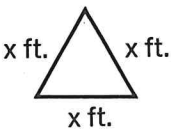
d.



e.



2. The perimeter of an equilateral triangle is $\frac{3}{5}$ foot. How long is each side?



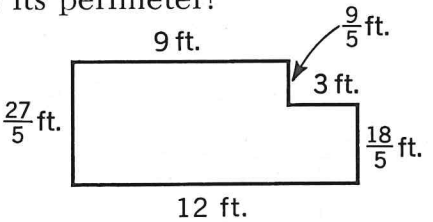
$x + x + x = \frac{3}{5}$

$x =$ ▢

3. Each side of a pentagon measures $\frac{3}{8}$ in. What is its perimeter?

4. Each side of a square measures $\frac{2}{7}$ in. What is its perimeter?

5. How many feet of molding must be bought to go around the base of the walls of the room shown in the picture?



6. Each side of a hexagon measures $\frac{5}{6}$ in. What is its perimeter?

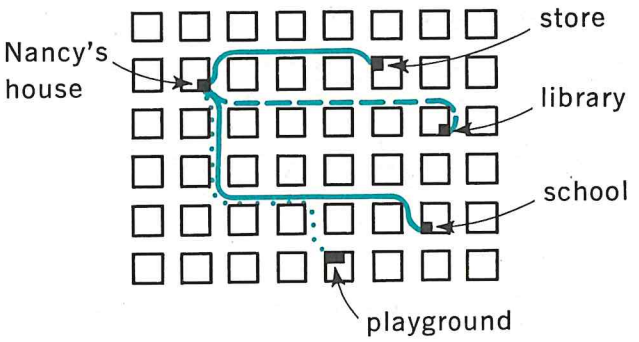
7. Each block measures $\frac{1}{9}$ mile. Each route is marked.

   a. Find the distance from Nancy's house to the store.

   b. Find the distance from Nancy's house to the school.

   c. Find the distance from Nancy's house to the library.

   d. Find the distance from Nancy's house to the playground.



In adding or subtracting fractions we often must find equal fractions with greater denominators.

$$\frac{1}{12} + \frac{1}{2} = \frac{1}{12} + \frac{6}{12} \qquad \text{because } \frac{1}{2} = \frac{6}{12}$$
$$= \frac{7}{12}$$

$$\frac{1}{2} - \frac{1}{12} = \frac{6}{12} - \frac{1}{12} \qquad \text{because } \frac{1}{2} = \frac{6}{12}$$
$$= \frac{5}{12}$$

Rename a fraction in each pair so that the fractions have like denominators.

1. $\frac{2}{5}$ and $\frac{3}{10}$
2. $\frac{5}{3}$ and $\frac{7}{12}$
3. $\frac{2}{3}$ and $\frac{11}{18}$
4. $\frac{1}{3}$ and $\frac{7}{36}$
5. $\frac{1}{5}$ and $\frac{4}{15}$
6. $\frac{3}{8}$ and $\frac{5}{16}$
7. $\frac{5}{6}$ and $\frac{2}{3}$
8. $\frac{7}{12}$ and $\frac{1}{4}$
9. $\frac{2}{3}$ and $\frac{7}{9}$
10. $\frac{1}{12}$ and $\frac{5}{6}$
11. $\frac{5}{18}$ and $\frac{2}{3}$
12. $\frac{7}{16}$ and $\frac{1}{2}$

Complete each problem.

13. $\frac{2}{3} + \frac{1}{12} = \frac{8}{12} + \frac{1}{12} =$ ▢
14. $\frac{7}{8} + \frac{3}{4} = \frac{7}{8} + \frac{\square}{8} =$ ▢
15. $\frac{1}{2} + \frac{7}{10} = \frac{\square}{10} + \frac{7}{10} =$ ▢
16. $\frac{2}{3} + \frac{2}{15} = \frac{\square}{15} + \frac{2}{15} =$ ▢
17. $\frac{3}{8} + \frac{1}{2} = \frac{3}{8} + \frac{\square}{8} =$ ▢
18. $\frac{3}{4} + \frac{7}{16} = \frac{\square}{16} + \frac{7}{16} =$ ▢
19. $\frac{2}{7} - \frac{3}{14} = \frac{\square}{14} - \frac{3}{14} =$ ▢
20. $\frac{1}{2} - \frac{1}{6} = \frac{\square}{6} - \frac{1}{6} =$ ▢
21. $\frac{11}{12} - \frac{5}{6} = \frac{11}{12} - \frac{\square}{12} =$ ▢
22. $\frac{3}{8} - \frac{7}{24} = \frac{\square}{24} - \frac{7}{24} =$ ▢
23. $\frac{8}{9} - \frac{1}{3} = \frac{8}{9} - \frac{\square}{9} =$ ▢
24. $\frac{3}{4} - \frac{7}{20} = \frac{\square}{20} - \frac{7}{20} =$ ▢

25. Larry had $\frac{3}{4}$ pound of nails. If he used $\frac{3}{16}$ pound of nails to build a soapbox racer, what part of a pound of nails does he have left?

Checkout

Find each sum.

1. 7 / 6 / 4 / 4

2. 9 / 15 / 24 / 37

3. 70 / 389 / 461 / 233

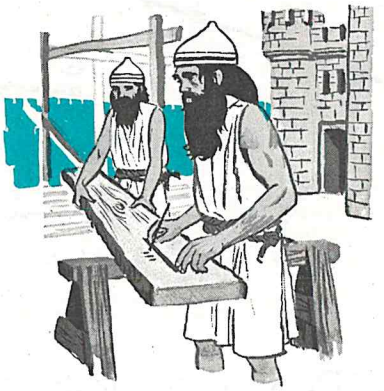
4. 444 / 81 / 29 / 335

Solve each equation.

5. $7 \times 9 = n$
6. $12 \times 7 = x$
7. $8 \times 6 = b$
8. $64 \div 8 = y$
9. $11 \times a = 121$
10. $12 \times d = 96$
11. $3 \times m = 27$
12. $132 \div p = 11$
13. $w \times 12 = 48$
14. $s \div 14 = 0$
15. $t \div 9 = 8$
16. $v \div 6 = 10$

Copy and complete each problem.

17. $\frac{3}{4} = \frac{\square}{8}$
18. $\frac{1}{2} = \frac{\square}{6}$
19. $\frac{3}{4} = \frac{\square}{12}$
20. $\frac{16}{20} = \frac{\square}{5}$
21. $\frac{6}{8} = \frac{\square}{4}$
22. $\frac{5}{5} = \frac{\square}{3}$



23. Mr. Banks drove 258 miles in 6 hours. If he drove at the same speed, how many miles did he drive each hour?

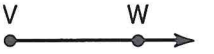
24. Karen mailed 6 letters. If she paid 8 cents for postage to mail each letter, how much postage did she pay?

25. Jane has read 312 pages, Mike 498 pages, Jim 273 pages, and Alice 300 pages.
    a. How many pages have the children read all together?
    b. How many pages must Jim read to catch up with Mike?

Part 4 Geometry

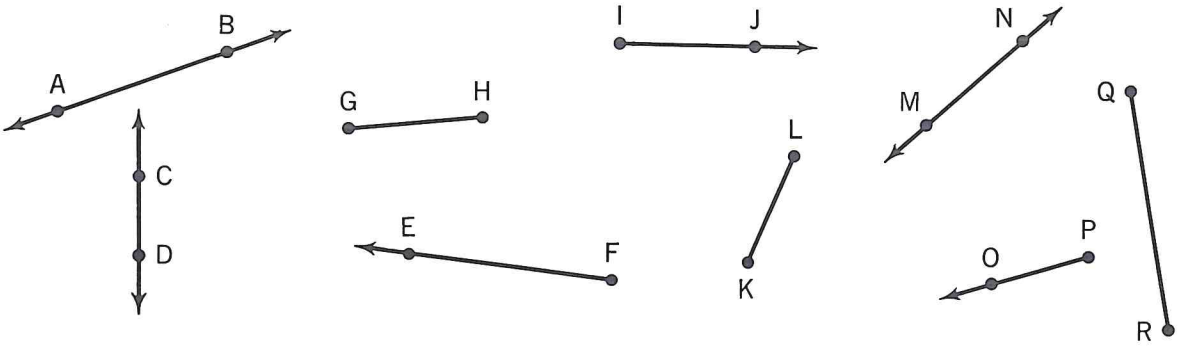
A **line** has no endpoints.
We write "line ST" or "$\overleftrightarrow{ST}$" or "line TS" or "$\overleftrightarrow{TS}$"

A **ray** has one endpoint.
We write "ray VW" or "$\overrightarrow{VW}$".



We name the endpoint first.
A **line segment** has two endpoints.
We write "segment XY" or "$\overline{XY}$" or "segment YX" or "$\overline{YX}$".

1. Tell whether each figure is a line, a ray, or a segment.



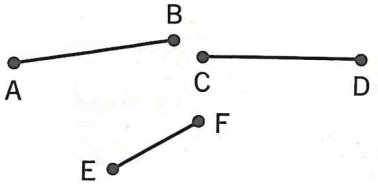
2. Draw two segments. Use your compass to find which is longer.

3. **a.** Draw a line and mark on it a segment JK.
b. With center K and radius the length of $\overline{JK}$, construct a circle which intersects the line at a point other than J.
c. Label the point of intersection "L".
d. $\overline{JL}$ is how many times as long as $\overline{JK}$?

4. Draw a segment. Use the above method to construct a segment 3 times as long.

Congruent Segments

Two segments are **congruent** if they are equal in length.

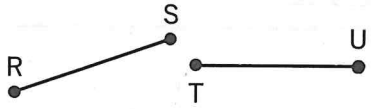


$\overline{AB} \cong \overline{CD}$ because $\overline{AB}$ is equal in length to $\overline{CD}$.
$\overline{AB} \ncong \overline{EF}$ because $\overline{AB}$ is not equal in length to $\overline{EF}$.

We read "$\overline{AB} \cong \overline{CD}$" as "segment AB is congruent to segment CD".
We read "$\overline{AB} \ncong \overline{EF}$" as "segment AB is not congruent to segment EF".

1. Which pairs of segments are congruent?

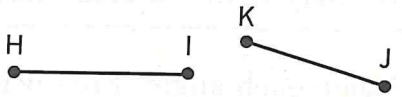
a.



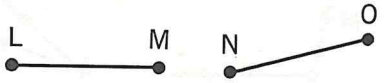
b.

c.

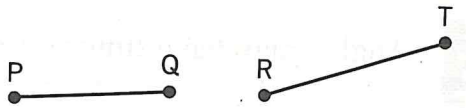
d.



e.

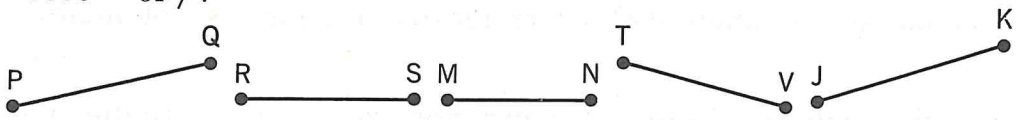


f.



2. Draw 4 pairs of segments. Use your compass to determine whether each pair is congruent.

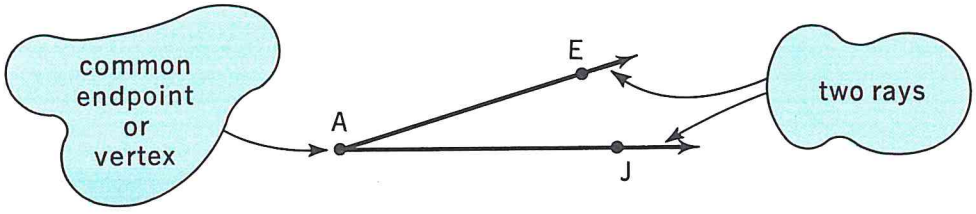
3. Choose $\cong$ or $\ncong$.



a. $\overline{PQ}$ ● $\overline{RS}$
b. $\overline{TV}$ ● $\overline{PQ}$
c. $\overline{MN}$ ● $\overline{JK}$
d. $\overline{RS}$ ● $\overline{TV}$
e. $\overline{JK}$ ● $\overline{PQ}$
f. $\overline{MN}$ ● $\overline{TV}$
g. $\overline{MN}$ ● $\overline{RS}$
h. $\overline{PQ}$ ● $\overline{MN}$
i. $\overline{JK}$ ● $\overline{RS}$

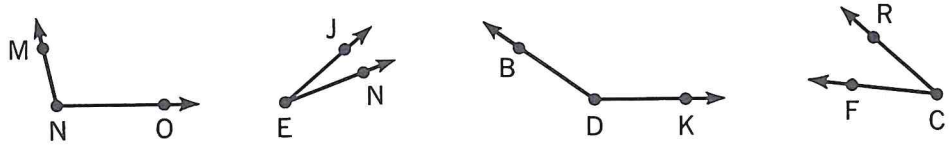
Naming Angles

An **angle** is the union of two rays with a common endpoint.

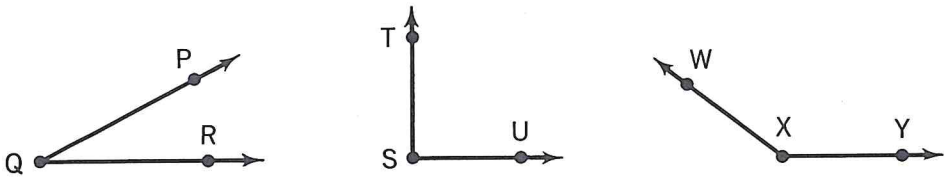


We can name an angle several ways. The above angle is angle A or angle EAJ or angle JAE. We often name 3 points on the angle to avoid confusion. First, name a point on one ray. Next, name the vertex. Then name a point on the other ray.
We may write "∠ JAE" instead of "angle JAE".

1. Name each angle three ways.



Angles may have different measures.

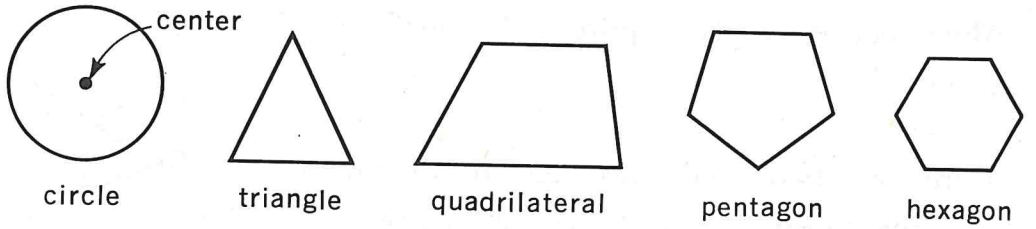


The measure of angle PQR is less than the measure of angle TSU.
The measure of angle WXY is greater than the measure of angle TSU.

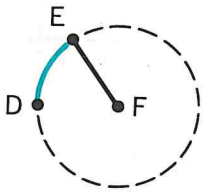
2. List the angles in Exercise 1 in order of size, starting with the one whose measure is least.

3. Draw several angles of different sizes, and label each one.

Circles and Arcs

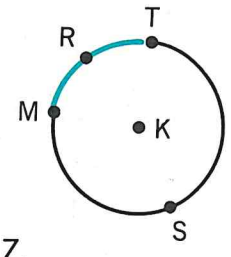


1. Construct four circles with the same center. Label the center "C".
2. Construct a circle. Draw a quadrilateral inside the circle.
3. Draw a pentagon. Construct a circle whose center G is on the pentagon.
4. Construct a circle. Construct another circle whose center J is on the first circle.
5. Mark a point A on your paper. Construct a circle with center A, and mark point B on it. Draw line segment AB. Segment AB is a radius of the circle. Are all radii of a circle equal in length?



An **arc** is a part of a circle.
A **radius of an arc** is a radius of the circle of which the arc is a part.
$\overline{EF}$ is a radius of arc DE.

6. a. Which arc is longer, arc MRT or arc MST?
   b. Name a radius of arc MRT.

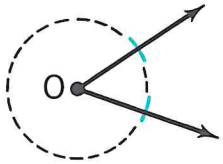


7. On your paper, mark points X, Y, and Z. Construct an arc with center X which passes through Y. Construct an arc with center X which passes through Z.
8. Draw line segment AB. Construct an arc with center A which passes through B. Construct an arc with center B which passes through A.

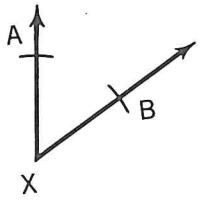
Comparing Angles

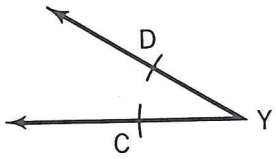
Many constructions require only small arcs of a circle.

Here only two small arcs need be drawn to find the intersections of the two given rays with the circle whose center is O.

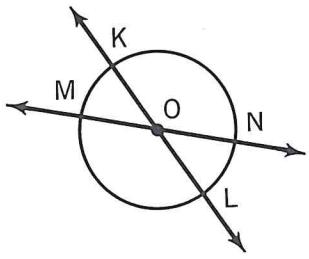


1. a. Draw 2 angles with vertices X and Y.
   b. With center X, construct 2 small arcs of a circle, intersecting the 2 rays of the angle in A and B.
   c. With center Y and using the same radius, construct 2 small arcs of a circle, intersecting the 2 rays of the angle in C and D.
   d. Compare the distances from A to B and from C to D to compare the measures of angles AXB and CYD. Which distance is greater?
   e. The greater distance shows the angle whose measure is greater. Which angle has the greater measure?



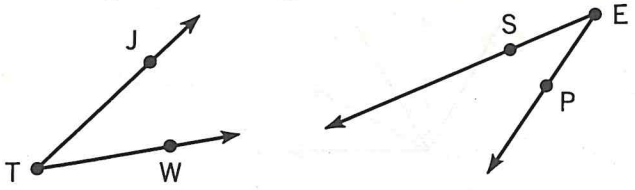


2. a. Draw 2 lines intersecting in O.
   b. With center O, construct a circle to intersect one line in K and L and the other line in M and N.
   c. Compare the distances from K to M and from L to N to compare the measures of angles KOM and LON. The angles should be equal in measure.



Congruent Angles

Two angles are congruent if they have the same measure.

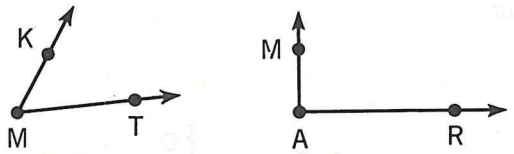


angle JTW ≅ angle SEP
∠ JTW ≅ ∠ SEP

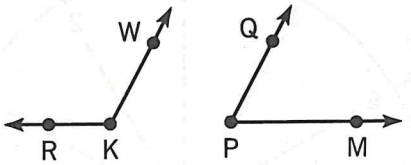
We read "∠ JTW ≅ ∠ SEP" as "angle JTW is congruent to angle SEP".

1. Which pairs of angles are congruent?

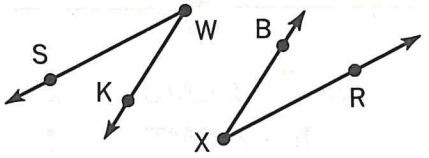
a.



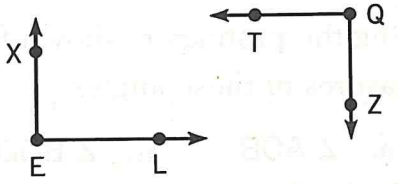
b.



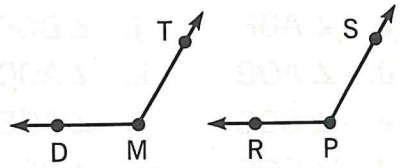
c.



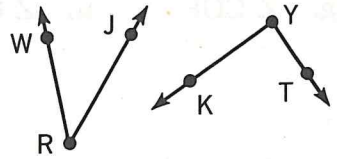
d.



e.



f.



2. Draw 4 pairs of angles. Use a compass to decide whether or not the angles are congruent.

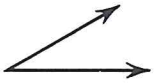
3. a. Draw a large triangle on a sheet of paper and label the vertices "A", "B", and "C".

b. Trace this triangle on another sheet of paper and label the corresponding vertices "D", "E", and "F".

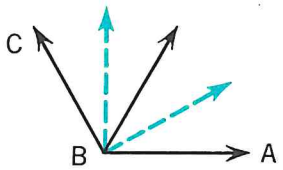
c. Use your compasses to show which pairs of angles are congruent.

Measuring Angles

We have units to measure angles.



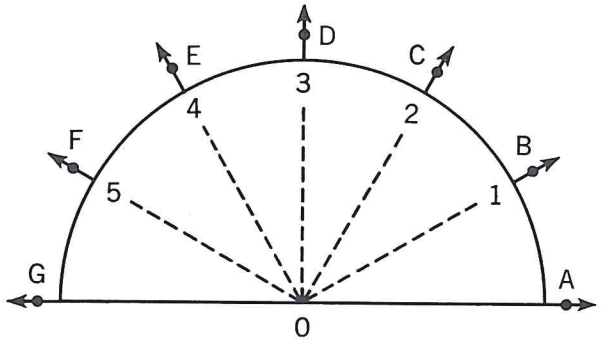
unit

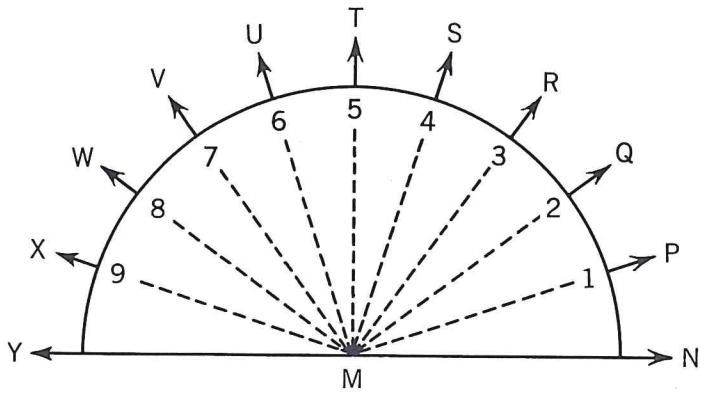


The measure of angle ABC using the unit shown is 4 units.
An instrument used for measuring angles is a **protractor.**

Using the protractors shown find the measures of these angles.

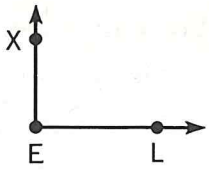
1. **a.** ∠ AOB
b. ∠ AOD
c. ∠ AOF
d. ∠ AOG
e. ∠ COG
f. ∠ COE
g. ∠ COF
h. ∠ BOD
i. ∠ EOB
j. ∠ DOF
k. ∠ AOC
l. ∠ AOE
m. ∠ BOC
n. ∠ FOB





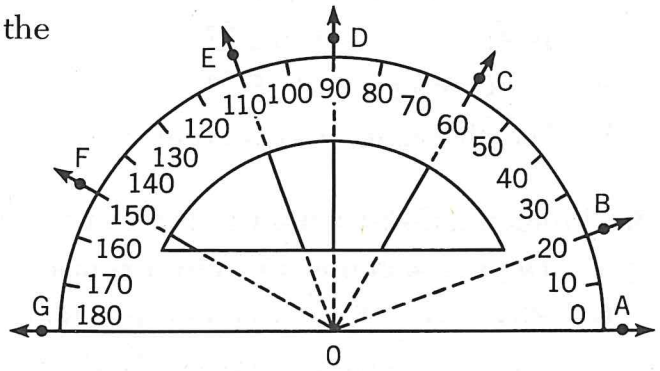
2. **a.** ∠ QMU
b. ∠ SMT
c. ∠ NMQ
d. ∠ SMP
e. ∠ PMT
f. ∠ QMW
g. ∠ UMY
h. ∠ UMR
i. ∠ QMV
j. ∠ VMS
k. ∠ RMX
l. ∠ YMN
m. ∠ NMS
n. ∠ PMW
o. ∠ RMP
p. ∠ SMN
q. ∠ WMY
r. ∠ SMU
s. ∠ TMY
t. ∠ RMN

A **degree** is a common unit for measuring angles.
There are 90 degrees in a **right** angle.
Angle XEL is a right angle. Its measure is 90 degrees.
We write: $m\angle XEL = 90°$



An angle whose measure is less than 90 degrees is an **acute** angle.
An angle whose measure is greater than 90 degrees and less than 180 degrees is an **obtuse** angle.

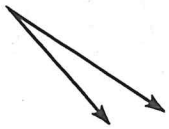
Using the protractor shown, find the measures of these angles.



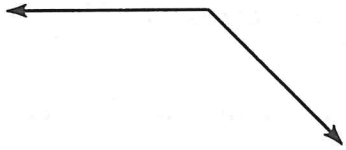
1. **a.** $\angle AOB$ **e.** $\angle GOE$
b. $\angle AOE$ **f.** $\angle GOA$
c. $\angle AOD$ **g.** $\angle GOC$
d. $\angle AOF$ **h.** $\angle GOF$

Use a protractor to find the measures in degrees of these angles.

2.



3.



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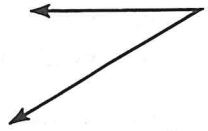
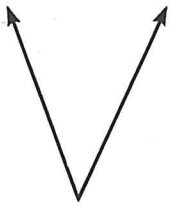
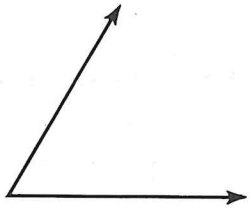


5.



6.



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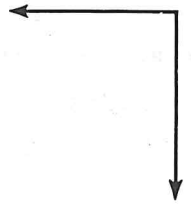
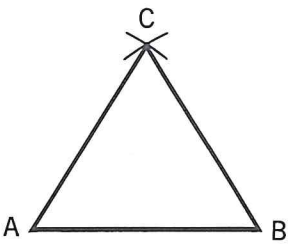


8. Tell whether each angle in Exercises 2-7 is a right angle, an obtuse angle, or an acute angle.

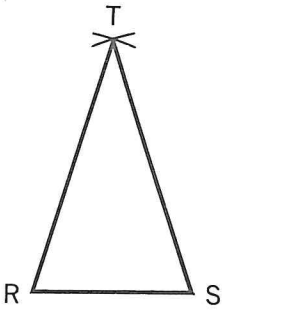
Use a protractor and draw angles that have these measures.

9. 135° **10.** 75° **11.** 30° **12.** 90° **13.** 110°

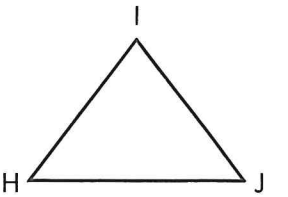
1. Problem: Construct an equilateral triangle.
   a. Draw a segment AB.
   b. With center A and radius the length of $\overline{AB}$, construct an arc above $\overline{AB}$.
   c. With center B and radius the length of $\overline{AB}$, construct an arc that intersects the first arc at C.
   d. Draw $\overline{AC}$ and $\overline{BC}$.
   e. Is $\overline{AB}$ congruent to $\overline{AC}$?
   f. Is $\overline{AB}$ congruent to $\overline{BC}$?
   g. Is $\overline{AC}$ congruent to $\overline{BC}$?
   h. Which angles are congruent?

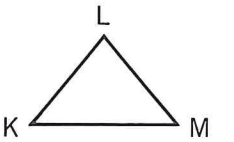


2. Problem: Construct an isosceles triangle.
   a. Draw a segment PQ and a shorter segment RS.
   b. First using R as a center and then S as a center, construct above $\overline{RS}$ 2 arcs whose radii are the same length as $\overline{PQ}$.
   c. Label the point of intersection "T".
   d. Draw $\overline{RT}$ and $\overline{ST}$.
   e. Which segments in triangle RST are congruent?
   f. Which angles are congruent?



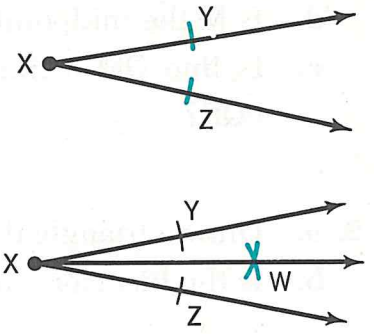
3. a. Construct an isosceles triangle HIJ with base HJ 4 in. long and sides HI and IJ 3 in. long.
   b. Construct an isosceles triangle with base KM 2 in. long, and sides KL and LM $1\frac{1}{2}$ in. long.
   c. Compare the sizes of angles HIJ and KLM. Since triangles HIJ and KLM are the same shape, angles HIJ and KLM are the same size.
   d. Compare the sizes of angles IHJ and LKM.
   e. Compare the sizes of angles HJI and KML.



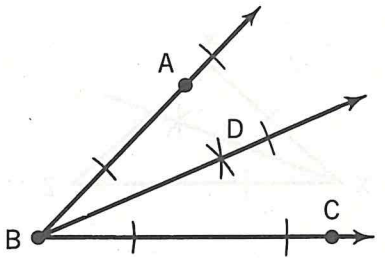


Bisecting an Angle

1. Problem: Bisect an angle.
   - **a.** Draw an angle with vertex X.
   - **b.** Draw 2 small arcs of a circle with center X, intersecting the two rays at Y and Z.
   - **c.** Between the rays draw an arc with center Y.
   - **d.** Using a radius of the same length draw another arc whose center is Z. Let this arc intersect the first arc at W.
   - **e.** Draw the ray from X through W. Angles YXW and ZXW are equal in measure. Ray XW bisects angle YXZ.

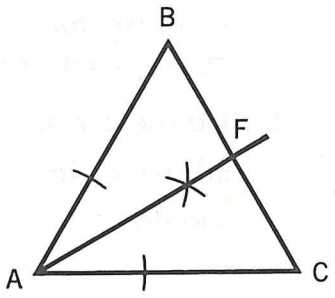


2. **a.** Draw angle ABC.
   - **b.** Bisect it by constructing ray BD.
   - **c.** Show that angle ABD is equal in measure to angle DBC.

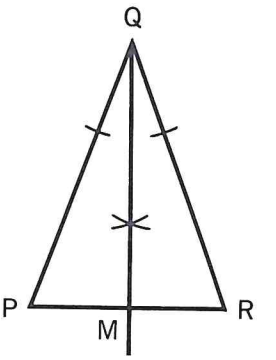


3. Draw several angles whose measures are different, and bisect each one.

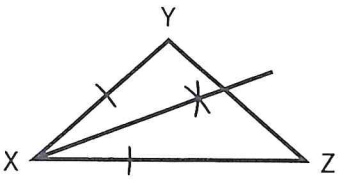
4. **a.** Construct an equilateral triangle ABC.
   - **b.** Bisect angle BAC.
   - **c.** Let F be the point of intersection of side BC and the bisector through A.
   - **d.** Is $\overline{BF} \cong \overline{FC}$?
   - **e.** If a figure is folded on a **line of symmetry,** then its two parts fit together. Is $\overleftrightarrow{AF}$ a line of symmetry?
   - **f.** Can you construct any other lines of symmetry?



1. **a.** Construct an isosceles triangle PQR with base PR.
   b. Bisect angle PQR.
   c. Let the bisector intersect base PR in M.
   d. Is M the midpoint of $\overline{PR}$?
   e. Is line QM a line of symmetry for triangle PQR?

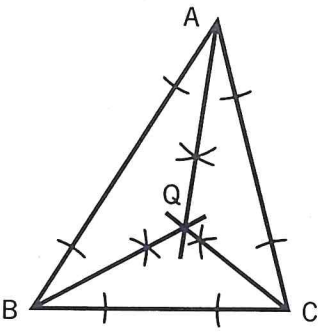


2. **a.** Draw a triangle that is not isosceles, and bisect one interior angle.
   b. Is the bisector a line of symmetry?



3. **a.** Construct an isosceles triangle XYZ with base XZ 3 in. long and sides 2 in. long.
   b. Bisect base angle YXZ.
   c. Does the bisector pass through the midpoint of side YZ?
   d. Is the bisector a line of symmetry?

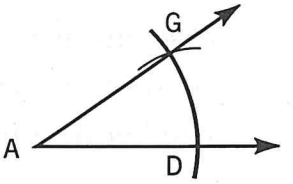
4. **a.** Draw a large triangle ABC.
   b. Construct the bisector of angle ABC.
   c. Construct the bisector of angle BCA.
   d. Let Q be the point of intersection of these two bisectors.
   e. Construct the bisector of angle CAB.
   f. Do the 3 bisectors of the angles of a triangle all pass through one point? They always should.

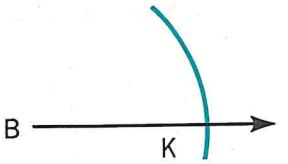


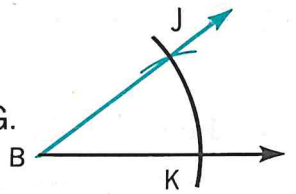
5. Must every triangle have a line of symmetry?

Copying an Angle

1. Problem: Copy angle A.
   - **a.** Using A as center construct an arc intersecting the two rays of angle A at D and G.
   - **b.** Draw a ray and let B be the endpoint.
   - **c.** Use a radius the length of $\overline{AD}$ and use B as center to construct another arc. Let K be the point of intersection.
   - **d.** Construct an arc with K as center and with a radius the length of $\overline{DG}$. Let J be the point of intersection of the two arcs.
   - **e.** Draw ray BJ. Angle KBJ is congruent to angle DAG.

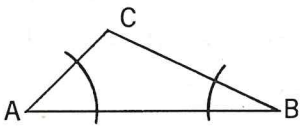


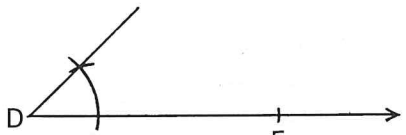


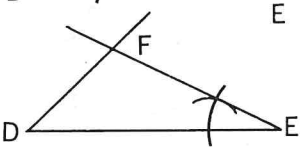


2. **a.** Draw an obtuse angle and copy it.
   b. Draw an acute angle and copy it.

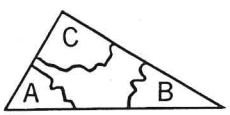
3. Dale says he can copy triangle ABC this way:
   - **a.** Draw a ray with endpoint D.
   - **b.** Lay off $\overline{DE}$ congruent to $\overline{AB}$.
   - **c.** Copy angle A using D as the vertex.
   - **d.** Copy angle B using E as the vertex.
   - **e.** Let F be the point of intersection.
   - **f.** Is Dale's triangle the same size and shape as triangle ABC?

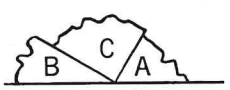




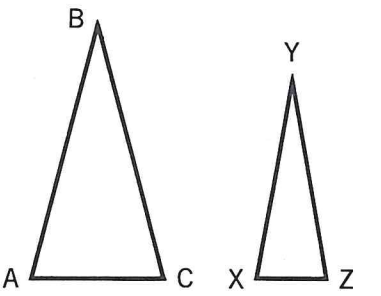


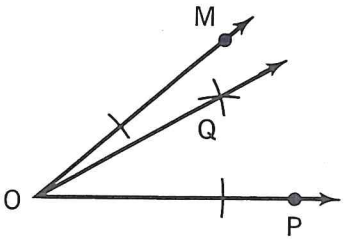
4. Cut a triangular shape from a sheet of paper and label the angles "A", "B", and "C". Tear off each corner as shown and place these pieces along the edge of your paper. Is the sum of the measures of angles A, B, and C equal to 180 degrees?





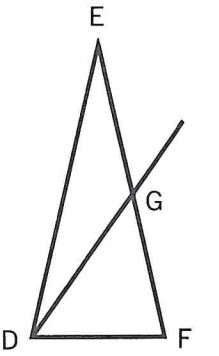
1. Sam draws an isosceles triangle ABC with base 2 in. long and sides 4 in. long. He draws a second isosceles triangle XYZ with base 1 in. long and sides 3 in. long. He says that angles BAC and YXZ are equal in size because the two triangles have the same shape.
Is Sam right?

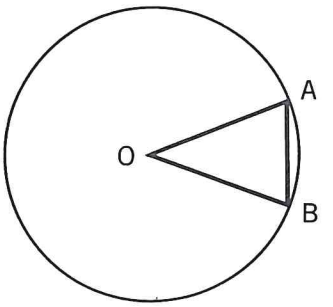




2. Laura bisects angle MOP by using the construction shown in the figure.
What is her mistake?

3. Bob says that it is easy to bisect a base angle of isosceles triangle DEF. He says to find the midpoint G of $\overline{EF}$ and draw the line through D and G.
How would you show Bob that this method is incorrect?





4. Gene says it is easy to construct an equilateral triangle with 2-inch sides. He says to draw a circle with a 2-inch radius OA, draw another radius OB, and then draw $\overline{AB}$. He says that triangle OAB is equilateral.
How would you show Gene that his method is incorrect?

5. Vernon says that he can construct isosceles triangle ABC with base AB 4 in. long and sides AC and BC $1\frac{1}{2}$ in. long. Can he?

Checkout

Find each product.

1. 256×8
2. 4590×9
3. 76×45
4. 9325×38

Find each quotient.

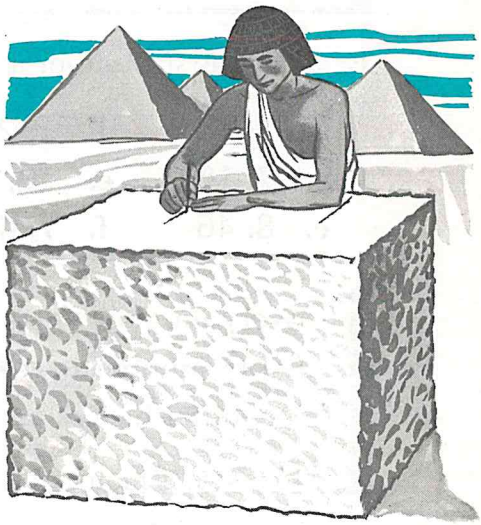
5. $4\overline{)768}$
6. $7\overline{)931}$
7. $9\overline{)5076}$
8. $6\overline{)3972}$

Find each difference.

9. $846 - 39$
10. $687 - 494$
11. $8402 - 3967$
12. $5000 - 2986$

Choose $=$ or $\neq$.

13. $64 + 38 \bigcirc 38 + 64$
14. $87 \times 6 \bigcirc 6 \times 87$
15. $19 - 0 \bigcirc 0 - 19$
16. $10 \div 1 \bigcirc 1 \div 10$
17. $(8 \times 2) \times 4 \bigcirc 8 \times (2 \times 4)$



Complete each statement.

18. A non-zero number divided by itself is ■.
19. Zero divided by a non-zero number is ■.
20. ■ times any number is zero.
21. ■ times any number is the number.
22. ■ plus any number is the number.
23. Any number divided by ■ is the number.
24. Any number minus ■ is the number.
25. Fred has 13 bags of marbles, with 28 marbles in each bag.
    a. How many marbles does he have?
    b. Joe has half as many marbles as Fred. How many marbles does he have?

Part 5 Factors and Multiples

Factors

7 is a **factor** of 28 because 28 can be divided by 7 with no remainder. We say that 28 is **divisible** by 7.

$$28 \div 7 = 4$$

5 is not a factor of 28 because 28 can not be divided by 5 with no remainder. We say that 28 is not divisible by 5.

$$28 \div 5 = 5\frac{3}{5}$$

1. Is the first number a factor of the second?

a. 3; 12	**d.** 1; 27	**g.** 6; 38	**j.** 12; 96	**m.** 17; 17
b. 7; 15	**e.** 10; 5	**h.** 8; 64	**k.** 30; 10	**n.** 6; 1
c. 8; 46	**f.** 7; 49	**i.** 9; 84	**l.** 12; 72	**o.** 6; 42

Every number n has the factor 1.
Every number n, $n \neq 0$, is a factor of itself.
Any other factors are between 1 and the number itself.
The set of factors of 24 is $\{1, 2, 3, 4, 6, 8, 12, 24\}$.

2. Find the set of all factors of each number.

a. 4	**f.** 6	**k.** 7	**p.** 2	**u.** 27
b. 9	**g.** 1	**l.** 16	**q.** 30	**v.** 18
c. 5	**h.** 12	**m.** 20	**r.** 15	**w.** 36
d. 10	**i.** 14	**n.** 42	**s.** 26	**x.** 28
e. 45	**j.** 25	**o.** 17	**t.** 19	**y.** 40

Common Factors

A **common factor** of two numbers is a factor of both of them.

8 is a factor of 32. 8 is a factor of 56.

8 is a common factor of 32 and 56.

1. Is the first number a common factor of the pair of numbers?

a. 1; 43 and 27
b. 5; 10 and 45
c. 4; 32 and 42
d. 3; 16 and 23
e. 6; 12 and 114
f. 11; 7 and 121
g. 12; 72 and 156
h. 7; 21 and 91
i. 10; 100 and 16
j. 9; 90 and 333
k. 8; 16 and 64
l. 20; 80 and 70

We can find all the common factors of two or more numbers.

What are the common factors of 42, 56, and 98?

Let A be the set of all factors of 42.

$$A = \{1, 2, 3, 6, 7, 14, 21, 42\}$$

Let B be the set of all factors of 56.

$$B = \{1, 2, 4, 7, 8, 14, 28, 56\}$$

Let C be the set of all factors of 98.

$$C = \{1, 2, 7, 14, 49, 98\}$$

The intersection of the three sets gives the set of all common factors.

$$A \cap B \cap C = \{1, 2, 7, 14\}$$

2. Use the above method to find all the common factors of each list of numbers.

a. 4 and 12
b. 36 and 16
c. 6 and 5
d. 18 and 9
e. 24 and 16
f. 25, 15, and 70
g. 32, 48, and 16
h. 27, 18, and 45
i. 12, 27, and 20

Greatest Common Factor

The **greatest common factor (G.C.F.)** of a set of numbers is the greatest number in the set of common factors.

The set of common factors of 42, 56, and 98 is $\{1, 2, 7, 14\}$.
The G.C.F. of 42, 56, and 98 is 14.

1. Find the G.C.F. for each list of numbers.

   a. 8 and 12
   b. 9 and 27
   c. 24 and 16
   d. 30, 40, and 50
   e. 17, 22, and 31
   f. 32, 16, and 48
   g. 15, 20, and 45
   h. 8, 12, and 24
   i. 9, 6, and 10

2. Jane has a cake measuring 15 in. by 21 in. She wants to cut it into square pieces all the same size. What is the largest size she can make the square pieces without having any left?

3. Mark has a block of wood measuring 8 in. by 12 in. by 20 in. He wants to cut this into cubes all the same size. What is the largest size cube he can cut so that no wood will be left over?

4. Jim has a block of wood measuring 6 in. by 8 in. by 12 in. He wants to cut it into cubes all the same size. What is the largest size cube he can cut so that no wood will be left over?

5. Mrs. Cole's rectangular coffee table measures 52 inches by 44 inches. She is covering it with square tiles that are all the same size. What is the largest size of square tile she can use without having to cut any tiles?

Prime Numbers

1. Find all the factors for each number.

a. 8	e. 2	i. 6	m. 50
b. 5	f. 18	j. 11	n. 36
c. 22	g. 21	k. 24	o. 27
d. 14	h. 3	l. 90	p. 28

A number with exactly two factors is a **prime number.**
These factors are 1 and the number itself.

The factors of 13 are 1 and 13.	13 is prime.
The factors of 12 are 1, 2, 3, 4, 6, and 12.	12 is not prime.
The only factor of 1 is 1.	1 is not prime.

2. Which numbers in Exercise 1 are prime numbers?

3. Find all the factors for each number and state whether each is prime.

a. 15	e. 31	i. 16	m. 19	q. 41
b. 37	f. 48	j. 44	n. 33	r. 38
c. 29	g. 30	k. 35	o. 23	s. 34
d. 20	h. 25	l. 10	p. 42	t. 26

The first four prime numbers are 2, 3, 5, and 7.
If a number less than 100 is not divisible by 2, 3, 5, or 7, then that number is prime.

51 is divisible by 3.	51 is not prime.
47 is not divisible by 2, 3, 5, or 7.	47 is prime.

4. Which of these numbers are prime?

a. 53	c. 45	e. 68	g. 58	i. 87	k. 43
b. 49	d. 59	f. 84	h. 77	j. 39	l. 56

Multiples

36 is a **multiple** of 9 because 36 is divisible by 9.
36 is not a multiple of 7 because 36 is not divisible by 7.

1. Is 33 a multiple of 3?
2. Is 33 a multiple of 2?
3. Is 33 a multiple of 11?
4. Is 33 a multiple of 7?
5. Is 17 a multiple of 1?
6. Is 1 a multiple of 17?
7. Is 12 a multiple of 5?
8. Is 15 a multiple of 5?
9. Is 43 a multiple of 3?
10. Is 32 a multiple of 2?
11. Is 57 a multiple of 9?
12. Is 72 a multiple of 8?

13. Give the set of the first ten multiples of each number.

The set of the first ten multiples of 3 is
$\{0, 3, 6, 9, 12, 15, 18, 21, 24, 27\}$.

a. 1
b. 4
c. 8
d. 10
e. 7
f. 6
g. 5
h. 9
i. 12
j. 11

Zero is a multiple of every positive whole number n.

$0 \div n = 0$ if $n \neq 0$.

We cannot divide by zero.

14. Which of these numbers are multiples of 7?

a. 147
b. 81
c. 274
d. 66
e. 97
f. 322
g. 17
h. 777
i. 914
j. 826
k. 21
l. 217
m. 483
n. 522
o. 63
p. 87

Least Common Multiple

A **common multiple** of a set of numbers is a multiple of each number in the set.

60 is a multiple of 5. 60 is a multiple of 2. 60 is a multiple of 3.
60 is a common multiple of 2, 3, and 5.

1. Is the first number a common multiple of the list of numbers?

a. 8; 2 and 4	**d.** 12; 2 and 3	**g.** 72; 36, 18, and 25
b. 3; 9 and 6	**e.** 30; 6 and 5	**h.** 81; 3, 9, and 27
c. 7; 1 and 10	**f.** 4; 12 and 2	**i.** 91; 8, 7, and 4

We can find the **least common multiple** (L.C.M.) of 5, 10, and 6.

Let A = the set of positive multiples of 5.

$A = \{5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, \ldots\}$

Let B = the set of positive multiples of 10.

$B = \{10, 20, 30, 40, 50, 60, 70, \ldots\}$

Let C = the set of positive multiples of 6.

$C = \{6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, \ldots\}$

$A \cap B \cap C$ gives the set of common positive multiples.

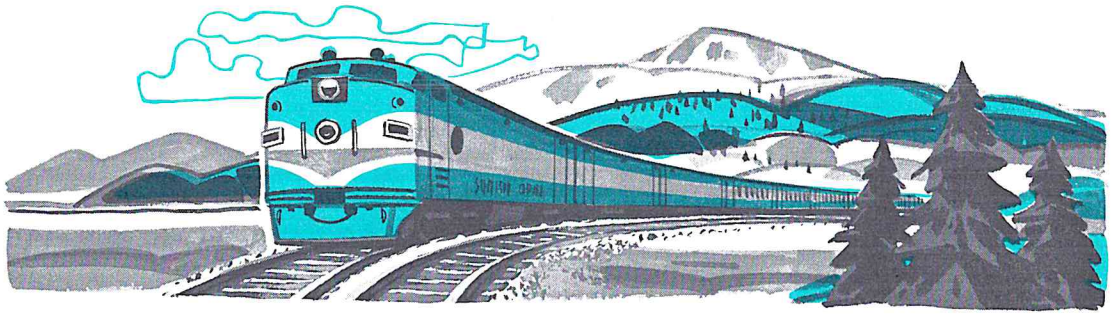
$A \cap B \cap C = \{30, 60, \ldots\}$

The least common multiple of 5, 10, and 6 is 30.

We consider only positive multiples in finding the least common multiple.

2. Find the L.C.M. of each list of numbers.

a. 2 and 5	**d.** 4 and 7	**g.** 12, 6, and 5
b. 3 and 7	**e.** 8 and 12	**h.** 9, 2, and 6
c. 4 and 10	**f.** 9 and 6	**i.** 2, 5, and 10



1. A train travels from Allentown to Billings every 24 min.; a train travels from Citadel to Billings every 40 min. The trains from Allentown and Citadel just arrived at the same time. How long will it be before this happens again?

2. At 2 P.M. a flight from Miami arrived in New York City just as a flight was leaving for Portland. Flights leave for Portland every 3 hours. If Mr. Lombardi arrives in New York City from Miami at 8:45 P.M., how long will he have to wait for a flight to Portland?

3. It takes 3 hours to travel by train from Durham to Edgarton. At 8 A.M. a train leaves Durham for Edgarton, and a train leaves Edgarton for Burnie. Trains leave Edgarton for Burnie every 5 hours. If Miss Greenberg leaves Durham by train at 2 P.M., how long will she have to wait in Edgarton for a train to Burnie?

4. At exactly 1 A.M. flights from Seattle, Los Angeles, and Las Vegas arrive in San Francisco. Every hour a plane arrives in San Francisco from Los Angeles. Every 2 hours a plane arrives in San Francisco from Las Vegas. Every 5 hours a plane arrives in San Francisco from Seattle. What is the next time at which these flights will arrive together?

5. A train arrives in Dover from Trenton every 40 min. and in Dover from Baltimore every 50 min. If both trains arrive at 1 P.M., what is the next time they will both arrive together?

1. Give the first twenty multiples of 2.

2. Which of these numbers are multiples of 2?

a. 90	c. 43	e. 35	g. 128	i. 94	k. 100
b. 21	d. 1000	f. 29	h. 76	j. 117	l. 85

3. Give the first twenty multiples of 5.

4. Which of the numbers in Exercise 2 are multiples of 5?

5. Which of the numbers in Exercise 2 are multiples of 10?

6. Can you find a multiple of 10 which is not a multiple of 5?

7. Can you find a multiple of 10 which is not a multiple of 2?

Any number whose last digit is 0, 2, 4, 6, or 8 is divisible by 2.
Any number whose last digit is 0 or 5 is divisible by 5.
Any number whose last digit is 0 is divisible by 10.

8. Which of these numbers are divisible by 2?

a. 55	d. 97	g. 32	j. 40	m. 73	p. 95
b. 30	e. 68	h. 41	k. 1010	n. 10	q. 52
c. 28	f. 42	i. 87	l. 965	o. 91	r. 110

9. Which of the numbers in Exercise 8 are divisible by 5?

10. Which of the numbers in Exercise 8 are divisible by both 2 and 5?

11. Which of the numbers in Exercise 8 are multiples of 10?

12. Busses leave Milton for Burton every 30 min. The trips take 45 min. Busses leave Burton for Hillside every 75 min. At 10 A.M. a bus left Milton for Burton and a bus left Burton for Hillside. If Joe leaves Milton by bus at 12:30 P.M., how long must he wait in Burton for a bus to Hillside?

A number is divisible by 3 if the sum of its digits is a multiple of 3.

192 is divisible by 3 because $1 + 9 + 2 = 12$.

A number is divisible by 9 if the sum of its digits is a multiple of 9.

495 is divisible by 9 because $4 + 9 + 5 = 18$.

1. Which of these numbers is a multiple of 3?

a.	9	d.	100	g.	47	j.	111	m.	597	p.	90,124
b.	53	e.	333	h.	13	k.	401	n.	674	q.	10,287
c.	36	f.	479	i.	45	l.	386	o.	8473	r.	90,123

2. Which of the numbers in Exercise 1 are divisible by 9?

3. Can you find a number that is divisible by 9 that is not divisible by 3?

Any number is divisible by 4 if the number given by the last two digits is a multiple of 4.

2936 is divisible by 4 because 36 is a multiple of 4.

Any number is divisible by 8 if the number given by the last three digits is a multiple of 8.

50,488 is divisible by 8 because 488 is a multiple of 8.

4. Which of these numbers are divisible by 4?

a.	24	d.	91	g.	216	j.	60,159	m.	8487
b.	307	e.	4834	h.	9470	k.	55,096	n.	2758
c.	4932	f.	1800	i.	8015	l.	58,937	o.	15,374

5. Which numbers in Exercise 4 are divisible by:

a. 8 b. 2 c. 3 d. 2 and 3?

6. Are all the numbers that are divisible by both 2 and 3 also divisible by 6?

6 is a factor of any common multiple of 2 and 3.

Powers

Multiplying is adding the same number many times.

$$5 \times 4 = 4 + 4 + 4 + 4 + 4$$

Using **powers** is multiplying the same number many times.

$$3^4 = 3 \times 3 \times 3 \times 3 \qquad 6^2 = 6 \times 6$$

1. Express each product as a power.

a. 4×4 **c.** $7 \times 7 \times 7$ **e.** $8 \times 8 \times 8 \times 8$

b. $2 \times 2 \times 2 \times 2$ **d.** $9 \times 9 \times 9 \times 9$ **f.** $3 \times 3 \times 3 \times 3 \times 3$

2. Express each power as a product.

$$8^5 = 8 \times 8 \times 8 \times 8 \times 8$$

a. 2^3 **c.** 7^4 **e.** 6^5 **g.** 3^2 **i.** 2^5

b. 5^6 **d.** 9^2 **f.** 8^3 **h.** 4^4 **j.** 9^3

In the power p^m, the number p is the **base**, the number m is the **exponent**.
The exponent tells how many times to multiply p by itself.
Usually we do not write the exponent 1: $3^1 = 3$

3. For each problem in Exercise 2, which number is the exponent?

Sometimes we can express a product as a product of powers.

$$3 \times 5 \times 5 \times 5 \times 7 \times 7 = 3 \times 5^3 \times 7^2$$

4. Express each product as a product of powers.

a. $2 \times 2 \times 3 \times 3 \times 3$ **d.** $2 \times 2 \times 2 \times 2 \times 7 \times 7$

b. $5 \times 5 \times 5 \times 7 \times 11$ **e.** $5 \times 5 \times 11 \times 11 \times 11 \times 11$

c. $11 \times 11 \times 13 \times 13 \times 13$ **f.** $3 \times 3 \times 3 \times 3 \times 17 \times 19$

Products of Prime Factors

1. Give all the factors of each number.

a. 9	d. 7	g. 14	j. 13	m. 16	p. 41
b. 11	e. 4	h. 3	k. 23	n. 20	q. 83
c. 2	f. 10	i. 12	l. 15	o. 53	r. 74

2. Which numbers in Exercise 1 are prime?

$2 \times 3 \times 5$ is a product of **prime factors** because 2, 5, and 3 are all prime numbers.
$2 \times 7 \times 9$ is not a product of prime factors because 9 is not a prime number.

3. Which are products of prime factors?

a. $3 \times 3 \times 8$	c. $4 \times 7 \times 5$	e. $5 \times 5 \times 5 \times 5$
b. $7 \times 9 \times 1$	d. $3 \times 5 \times 7 \times 7$	f. $3 \times 7 \times 2 \times 9$

Any number that is not prime can be expressed as a product of prime factors in terms of powers of primes. We list the factors in order starting with the least factor greater than 1.

90 is not prime
9 and 10 are not prime

$$\begin{aligned} 90 &= 9 \times 10 \\ &= 3 \times 3 \times 2 \times 5 \\ &= 2 \times 3^2 \times 5 \end{aligned}$$

There is only one such expression for each number.

4. Express each number as a product of prime factors in terms of powers of primes. List the factors in order starting with the least factor.

a. 12	d. 25	g. 28	j. 42	m. 24	p. 49
b. 35	e. 33	h. 18	k. 38	n. 55	q. 16
c. 26	f. 45	i. 32	l. 21	o. 40	r. 14

We can also find the L.C.M. of a list of numbers by looking at their prime factors.

What is the L.C.M. of 24, 90, and 42?

$24 = 2^3 \times 3 \qquad 90 = 2 \times 3^2 \times 5 \qquad 42 = 2 \times 3 \times 7$

The prime factors are 2, 3, 5, and 7.

The highest power of 2 shown is 2^3. The highest power of 5 shown is 5.

The highest power of 3 shown is 3^2. The highest power of 7 shown is 7.

The least common multiple is the product of the highest powers of each prime factor.

The L.C.M. of 24, 90, and 42 is $2^3 \times 3^2 \times 5 \times 7$.

1. Express the L.C.M. as a product of prime factors.

$27 = 3^3 \qquad 12 = 2^2 \times 3 \qquad 7$ is prime

The L.C.M. of 27, 12, and 7 is $2^2 \times 3^3 \times 7$.

a. 18 and 6
b. 12 and 32
c. 10 and 15
d. 24 and 36
e. 48 and 16
f. 27 and 20
g. 21 and 12
h. 16 and 10
i. 5, 8, and 12
j. 6, 10 and 14
k. 9, 12, and 14
l. 8, 11, and 20

2. Ann has enough eggs in this box to divide them into 2, 3, 4, or 5 groups of the same size.

a. What is the least number of eggs she could have?
b. Could Ann divide the eggs into 6 groups of the same size?
c. Could she divide the eggs into 12 groups of the same size?

3. Mrs. Nerloff drives Gail to Brownies every 7 days, Joe to Red Cross meetings every 21 days, and Patty to her music class every 2 weeks. Today, she drove to all 3 places. How many days will it be before she must drive to all 3 places on the same day again?

Checkout

1. Give all the factors of these numbers.
   a. 18 b. 19 c. 20 d. 24 e. 60

2. Find the greatest common factor for each set of numbers.
   a. $\{6, 15\}$ b. $\{7, 12\}$ c. $\{9, 21, 63\}$ d. $\{18, 54, 6\}$

3. Find the least common multiple for each set of numbers.
   a. $\{8, 9\}$ b. $\{13, 52\}$ c. $\{12, 20, 40\}$ d. $\{32, 24, 8\}$

4. Give each number as a product of prime factors.
   a. 35 b. 42 c. 58 d. 80 e. 24

5. Which of these numbers are prime?

89	27	85	2	70	88	59
38	53	57	1	45	23	97
10	3	76	83	35	16	86

6. Give the twelve prime numbers between 4 and 44.

7. Give the prime numbers between 65 and 80.

8. True or false?
   a. 4 is a prime number.
   b. 17 is a factor of 153.
   c. 152 is divisible by 4.
   d. 41 is a prime number.
   e. No prime number has a last digit 8.
   f. The G.C.F. of two primes is their product.
   g. The L.C.M. of two primes is 1.
   h. 9 is a factor of 1,843,701.

Checkout

Solve each equation.

1. $\frac{4}{7} + \frac{3}{14} = n$
2. $\frac{3}{4} + \frac{3}{5} = n$
3. $\frac{4}{7} + \frac{3}{4} = n$
4. $\frac{2}{5} + \frac{3}{10} = n$
5. $\frac{1}{6} + \frac{2}{3} = n$
6. $\frac{2}{3} + \frac{7}{12} = n$
7. $\frac{5}{8} - \frac{1}{4} = n$
8. $\frac{3}{5} - \frac{3}{10} = n$
9. $\frac{2}{3} - \frac{1}{4} = n$

Choose $\cup$ or $\cap$ or $-$.

10. $\{1, 4, 7\} \bullet \{2, 4\} = \{1, 7\}$
11. $\{1, 4, 7\} \bullet \{2, 4\} = \{4\}$
12. $\{8, 10\} \bullet \{8, 9\} = \{8, 9, 10\}$
13. $\{1, 3\} \bullet \{2, 4\} = \{\ \}$
14. $\{\text{Ann}\} \bullet \{\text{Sue}\} = \{\text{Sue, Ann}\}$
15. $\{0, 2, 4\} \bullet \{1, 3\} = \{0, 1, 2, 3, 4\}$

Match each law of arithmetic with a mathematical sentence.

16. Associative law for addition
17. Associative law for multiplication
18. Commutative law for addition
19. Commutative law for multiplication
20. Distributive law for multiplication

A. $(2 \times 3) \times 6 = 2 \times (3 \times 6)$
B. $(a + 3) + 4 = a + (3 + 4)$
C. $4 \times (1 + n) = (4 \times 1) + (4 \times n)$
D. $3 + n = n + 3$
E. $5 \times 80 = 80 \times 5$

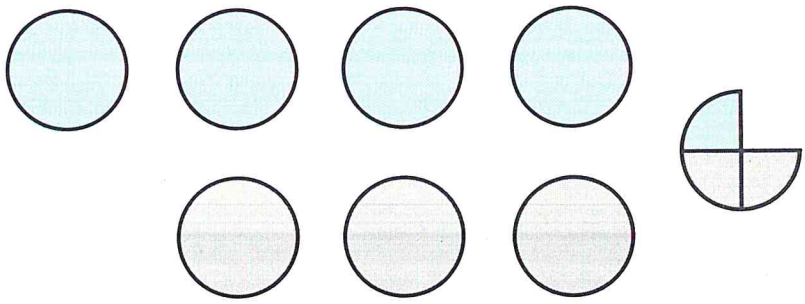
21. Clara made 4 bows of equal length from a strip of ribbon 48 inches long. How much ribbon did she use for each bow?

22. Helen sold 45 tickets for a school play and Brenda sold 38 tickets. How many tickets did both girls sell?

Part 6 More About Fractions

$4\frac{1}{4}$

$3\frac{2}{4}$



In adding or subtracting mixed numbers, first add or subtract the fractions.

Addition:

$$\begin{array}{r} 4\frac{1}{4} \\ +3\frac{2}{4} \\ \hline \end{array}$$

First step:

$$\begin{array}{r} 4\frac{1}{4} \\ +3\frac{2}{4} \\ \hline \frac{3}{4} \end{array}$$

Second step:

$$\begin{array}{r} 4\frac{1}{4} \\ +3\frac{2}{4} \\ \hline 7\frac{3}{4} \end{array}$$

Subtraction:

$$\begin{array}{r} 7\frac{3}{4} \\ -4\frac{1}{4} \\ \hline \end{array}$$

First step:

$$\begin{array}{r} 7\frac{3}{4} \\ -4\frac{1}{4} \\ \hline \frac{2}{4} \end{array}$$

Second step:

$$\begin{array}{r} 7\frac{3}{4} \\ -4\frac{1}{4} \\ \hline 3\frac{2}{4} \end{array}$$

Find each sum or difference.

1. $\begin{array}{r} 2\frac{1}{3} \\ +3\frac{1}{3} \\ \hline \end{array}$

2. $\begin{array}{r} 4\frac{2}{4} \\ -2\frac{1}{4} \\ \hline \end{array}$

3. $\begin{array}{r} 7\frac{3}{10} \\ +1\frac{4}{10} \\ \hline \end{array}$

4. $\begin{array}{r} 9\frac{15}{25} \\ -1\frac{6}{25} \\ \hline \end{array}$

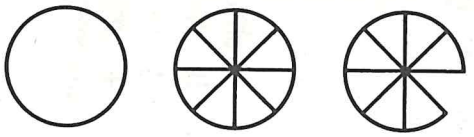
5. $\begin{array}{r} 1\frac{2}{4} \\ +4\frac{1}{4} \\ \hline \end{array}$

6. $\begin{array}{r} 7\frac{4}{5} \\ -1\frac{3}{5} \\ \hline \end{array}$

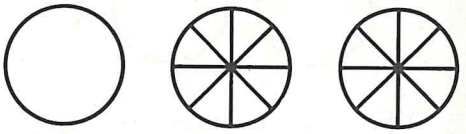
7. $\begin{array}{r} 8\frac{2}{7} \\ +6\frac{4}{7} \\ \hline \end{array}$

8. $\begin{array}{r} 7\frac{3}{8} \\ +5\frac{4}{8} \\ \hline \end{array}$

In a mixed number the fraction should be a proper fraction.



$1\frac{15}{8} = 1 + 1\frac{7}{8} = 2\frac{7}{8}$



$1\frac{16}{8} = 1 + 2 = 3$

Find an equal whole number or mixed number.

1. $1\frac{3}{3} =$ ■
2. $1\frac{5}{3} =$ ■
3. $2\frac{9}{5} =$ ■
4. $8\frac{7}{4} =$ ■
5. $3\frac{11}{11} =$ ■
6. $5\frac{14}{6} =$ ■
7. $17\frac{15}{13} =$ ■
8. $23\frac{8}{8} =$ ■
9. $9\frac{23}{5} =$ ■

Find each sum.

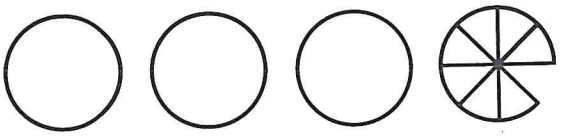
10. $3\frac{1}{4}$ + $1\frac{3}{4}$
11. $9\frac{3}{5}$ + $4\frac{1}{5}$
12. $5\frac{4}{7}$ + $3\frac{3}{7}$
13. $5\frac{5}{6}$ + $2\frac{4}{6}$
14. $3\frac{7}{12}$ + $7\frac{6}{12}$
15. $9\frac{1}{2}$ + $7\frac{1}{2}$
16. $35\frac{1}{4}$ + $78\frac{2}{4}$
17. $16\frac{2}{3}$ + $33\frac{1}{3}$

18. Bob ran $3\frac{4}{10}$ miles on Monday, $1\frac{7}{10}$ miles on Tuesday, and $2\frac{3}{10}$ miles on Wednesday. How far did he run all together?

19. Mrs. Adams bought $5\frac{3}{4}$ pounds of potatoes, $4\frac{1}{4}$ pounds of meat, 2 pounds of butter, and $3\frac{3}{4}$ pounds of carrots. How much did her groceries weigh?

20. Each day Jon sleeps for $9\frac{1}{4}$ hours and spends $1\frac{1}{4}$ hours eating. How much time does he spend eating and sleeping?

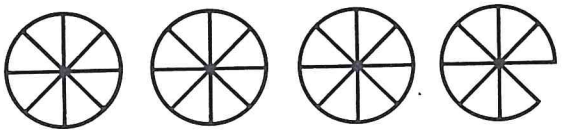
21. Ray can throw a ball $30\frac{3}{4}$ yards, and Jim can throw it $29\frac{1}{4}$ yards. Who can throw it farther and how much farther?



To find a fraction equal to a mixed number, we first multiply to find the parts in the wholes. Then we add the fractions.

$$3\tfrac{7}{8} = 3 + \tfrac{7}{8}$$
$$= \tfrac{24}{8} + \tfrac{7}{8}$$
$$= \tfrac{31}{8}$$

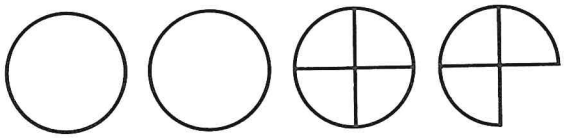
$$3 = \tfrac{3 \times 8}{1 \times 8} = \tfrac{24}{8}$$



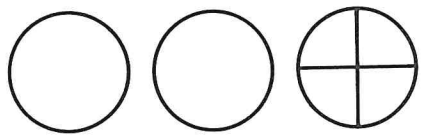
Express as a fraction.

1. $2\frac{3}{4} =$ ▢
2. $3\frac{2}{5} =$ ▢
3. $1\frac{1}{2} =$ ▢
4. $9\frac{7}{10} =$ ▢
5. $5\frac{3}{6} =$ ▢
6. $2\frac{4}{7} =$ ▢
7. $5\frac{5}{9} =$ ▢
8. $7\frac{1}{2} =$ ▢
9. $3\frac{7}{12} =$ ▢
10. $1\frac{5}{24} =$ ▢
11. $20\frac{2}{3} =$ ▢
12. $4\frac{6}{11} =$ ▢

Sometimes we need to express a mixed number as a whole number and an improper fraction.



$$3\tfrac{3}{4} = 2 + 1\tfrac{3}{4} = 2\tfrac{7}{4}$$



$$3 = 2 + 1 = 2\tfrac{4}{4}$$

Each time the improper fraction is less than 2.

Complete each problem.

13. $2\frac{1}{3} = 1\frac{▢}{3}$
14. $5\frac{6}{8} = 4\frac{▢}{8}$
15. $8\frac{7}{12} = 7\frac{▢}{12}$
16. $4 = 3\frac{▢}{8}$
17. $9\frac{4}{7} = 8\frac{▢}{7}$
18. $6 = 5\frac{▢}{3}$
19. $6\frac{1}{12} = 5\frac{▢}{12}$
20. $8\frac{7}{11} = 7\frac{▢}{11}$
21. $9\frac{3}{7} = 8\frac{▢}{7}$
22. $13 = 12\frac{▢}{6}$
23. $29\frac{3}{5} = 28\frac{▢}{5}$
24. $45\frac{2}{9} = 44\frac{▢}{9}$

Sometimes in subtracting mixed numbers, the fraction to be subtracted is the greater.

$$\begin{array}{r} 6\frac{1}{8} \\ -3\frac{7}{8} \\ \hline \end{array}$$

We find another name for $6\frac{1}{8}$.

$$\begin{array}{rr} 6\frac{1}{8} = & 5\frac{9}{8} \\ -3\frac{7}{8} = & -3\frac{7}{8} \\ \hline & 2\frac{2}{8} \end{array}$$

Find each difference.

1. $3\frac{1}{4} - 1\frac{3}{4}$
2. $8\frac{2}{6} - 5\frac{5}{6}$
3. $7\frac{4}{9} - 4\frac{2}{9}$
4. $34\frac{3}{12} - 8\frac{8}{12}$
5. $6\frac{2}{5} - 2\frac{4}{5}$
6. $9\frac{3}{12} - 4\frac{4}{12}$
7. $5\frac{4}{10} - 3\frac{8}{10}$
8. $57\frac{4}{10} - 21\frac{9}{10}$
9. $11\frac{1}{7} - 9\frac{2}{7}$
10. $16\frac{14}{24} - 9\frac{9}{24}$
11. $22\frac{4}{13} - 17\frac{9}{13}$
12. $73\frac{3}{24} - 28\frac{19}{24}$

13. Mike has $4\frac{5}{6}$ candy bars. He gives $2\frac{1}{6}$ candy bars to Liz. How much is left?

14. Dave buys $4\frac{3}{4}$ pounds of hamburger. He fries $1\frac{1}{4}$ pounds one day and $1\frac{1}{4}$ pounds the next day. How much is left?

15. Rosa has $7\frac{1}{6}$ yards of tape. She needs $4\frac{2}{6}$ yards for her dress and $6\frac{3}{6}$ yards for her coat. How much more tape must she buy?

16. Gary rides $2\frac{3}{10}$ miles in the school bus going to school and $2\frac{4}{10}$ miles returning from school. How far does he ride in the school bus each day?

We can add or subtract mixed numbers, whole numbers, and fractions.

$$\begin{array}{r} 8 \\ +4\frac{1}{3} \\ \hline 12\frac{1}{3} \end{array} \qquad \begin{array}{r} 3\frac{1}{5} \\ +\ \frac{3}{5} \\ \hline 3\frac{4}{5} \end{array} \qquad \begin{array}{r} 7 = 6\frac{3}{3} \\ -2\frac{1}{3} = -2\frac{1}{3} \\ \hline 4\frac{2}{3} \end{array} \qquad \begin{array}{r} 9\frac{1}{5} = 8\frac{6}{5} \\ -7\frac{3}{5} = -7\frac{3}{5} \\ \hline 1\frac{3}{5} \end{array}$$

Find each sum or difference.

1. $\begin{array}{r} 4 \\ +4\frac{4}{7} \\ \hline \end{array}$

2. $\begin{array}{r} 23\frac{1}{3} \\ -9 \\ \hline \end{array}$

3. $\begin{array}{r} \frac{1}{4} \\ +19\frac{3}{4} \\ \hline \end{array}$

4. $\begin{array}{r} 13\frac{11}{12} \\ -7\frac{7}{12} \\ \hline \end{array}$

5. $\begin{array}{r} 11 \\ -4\frac{3}{8} \\ \hline \end{array}$

6. $\begin{array}{r} 17\frac{1}{5} \\ -6\frac{2}{5} \\ \hline \end{array}$

7. $\begin{array}{r} 31 \\ -1\frac{1}{7} \\ \hline \end{array}$

8. $\begin{array}{r} 1\frac{4}{100} \\ +\ \frac{51}{100} \\ \hline \end{array}$

9. $\begin{array}{r} 8\frac{1}{24} \\ \frac{7}{24} \\ +1\frac{6}{24} \\ \hline \end{array}$

10. $\begin{array}{r} 4\frac{5}{15} \\ 1\frac{3}{15} \\ +7 \\ \hline \end{array}$

11. $\begin{array}{r} \frac{7}{21} \\ 4 \\ +\ \frac{18}{21} \\ \hline \end{array}$

12. $\begin{array}{r} \frac{1}{6} \\ 7\frac{5}{6} \\ +15\frac{1}{6} \\ \hline \end{array}$

13. James has 5 cakes. He gives $3\frac{1}{4}$ cakes away. How many cakes are left?

14. Mrs. Jones uses $\frac{3}{8}$ pound of butter. She had 2 pounds. How many pounds are left?

15. Melissa has $4\frac{5}{8}$ yards of ribbon. She gives $\frac{3}{8}$ yard of ribbon to each of two friends. How much ribbon does Melissa have left?

16. Karen and Peggy sold pies at the school carnival. They started with 18 pies. They sold $5\frac{5}{8}$ pies. Then one of the teachers bought another pie. Later Karen sold $5\frac{3}{8}$ pies and Peggy sold $5\frac{7}{8}$ pies. How many pies were left at the end of the day?

17. Rachel rode her bicycle $4\frac{3}{10}$ miles to the beach and $3\frac{4}{10}$ miles back by a different route. How far did she ride?

The least common denominator (L.C.D.) for $\frac{1}{4}$ and $\frac{5}{6}$ is 12 because the least common multiple (L.C.M.) of 4 and 6 is 12. To find the least common denominator, find the least common multiple of the denominators.

1. What is the L.C.M. of each set of numbers?

a. 2 and 4	d. 2 and 6	g. 6 and 9	j. 2, 3, and 6
b. 4 and 3	e. 4 and 6	h. 3 and 5	k. 3, 8, and 6
c. 5 and 2	f. 9 and 2	i. 8 and 2	l. 3, 9, and 2

2. Find the L.C.D.

a. $\frac{1}{2}, \frac{1}{4}$	d. $\frac{1}{6}, \frac{1}{9}$	g. $\frac{3}{4}, \frac{1}{5}, \frac{3}{10}$
b. $\frac{3}{4}, \frac{1}{2}$	e. $\frac{1}{2}, \frac{4}{5}$	h. $\frac{1}{8}, \frac{1}{2}, \frac{3}{4}$
c. $\frac{1}{4}, \frac{2}{3}$	f. $\frac{4}{5}, \frac{2}{3}$	i. $\frac{2}{3}, \frac{1}{9}, \frac{1}{2}$

3. Rename the fractions in each part using the L.C.D.

a. $\frac{3}{8}, \frac{1}{2}$	d. $\frac{3}{5}, \frac{1}{3}$	g. $\frac{1}{2}, \frac{2}{3}, \frac{5}{6}$
b. $\frac{3}{4}, \frac{1}{6}$	e. $\frac{2}{3}, \frac{2}{5}$	h. $\frac{2}{3}, \frac{1}{8}, \frac{1}{6}$
c. $\frac{1}{5}, \frac{1}{3}$	f. $\frac{4}{9}, \frac{1}{2}$	i. $\frac{5}{8}, \frac{5}{6}, \frac{1}{3}$

4. Choose $<$ or $=$ or $>$.

a. $\frac{1}{2} \bigcirc \frac{3}{8}$	f. $\frac{3}{5} \bigcirc \frac{2}{3}$	k. $\frac{1}{2} \bigcirc \frac{7}{14}$
b. $\frac{5}{6} \bigcirc \frac{2}{3}$	g. $\frac{3}{4} \bigcirc \frac{2}{3}$	l. $\frac{4}{7} \bigcirc \frac{3}{5}$
c. $\frac{5}{8} \bigcirc \frac{1}{2}$	h. $\frac{2}{5} \bigcirc \frac{6}{15}$	m. $\frac{4}{9} \bigcirc \frac{5}{12}$
d. $\frac{3}{4} \bigcirc \frac{5}{6}$	i. $\frac{4}{5} \bigcirc \frac{1}{2}$	n. $\frac{5}{7} \bigcirc \frac{3}{4}$
e. $\frac{5}{6} \bigcirc \frac{15}{18}$	j. $\frac{2}{3} \bigcirc \frac{5}{7}$	o. $\frac{7}{8} \bigcirc \frac{5}{6}$

To add or subtract fractions:

1. Find the L.C.M. of the denominators.
2. Express both fractions with this common denominator.
3. Add or subtract.

$$\begin{array}{r} \frac{3}{4} = \frac{9}{12} \\ +\frac{2}{3} = \frac{8}{12} \\ \hline \frac{17}{12} = 1\frac{5}{12} \end{array} \qquad \begin{array}{r} \frac{3}{4} = \frac{9}{12} \\ -\frac{2}{3} = -\frac{8}{12} \\ \hline \frac{1}{12} \end{array}$$

Add or subtract. If possible, express each answer either as a whole number or as a mixed number.

1. $\begin{array}{r} \frac{1}{2} \\ +\frac{1}{3} \\ \hline \end{array}$
2. $\begin{array}{r} \frac{2}{3} \\ -\frac{1}{9} \\ \hline \end{array}$
3. $\begin{array}{r} \frac{3}{7} \\ -\frac{1}{3} \\ \hline \end{array}$
4. $\begin{array}{r} \frac{3}{7} \\ +\frac{3}{4} \\ \hline \end{array}$
5. $\begin{array}{r} \frac{11}{12} \\ -\frac{5}{8} \\ \hline \end{array}$
6. $\begin{array}{r} \frac{4}{7} \\ +\frac{1}{2} \\ \hline \end{array}$
7. $\begin{array}{r} \frac{7}{8} \\ -\frac{1}{4} \\ \hline \end{array}$
8. $\begin{array}{r} \frac{1}{2} \\ +\frac{7}{9} \\ \hline \end{array}$
9. $\begin{array}{r} \frac{5}{8} \\ -\frac{2}{5} \\ \hline \end{array}$
10. $\begin{array}{r} \frac{5}{12} \\ +\frac{3}{8} \\ \hline \end{array}$
11. $\begin{array}{r} \frac{1}{5} \\ +\frac{2}{3} \\ \hline \end{array}$
12. $\begin{array}{r} \frac{4}{5} \\ -\frac{2}{3} \\ \hline \end{array}$
13. $\begin{array}{r} \frac{5}{6} \\ +\frac{1}{4} \\ \hline \end{array}$
14. $\begin{array}{r} \frac{7}{8} \\ -\frac{5}{6} \\ \hline \end{array}$
15. $\begin{array}{r} \frac{1}{3} \\ +\frac{6}{7} \\ \hline \end{array}$
16. $\begin{array}{r} \frac{1}{2} \\ +\frac{3}{8} \\ \hline \end{array}$
17. $\begin{array}{r} \frac{2}{3} \\ -\frac{1}{4} \\ \hline \end{array}$
18. $\begin{array}{r} \frac{5}{8} \\ -\frac{1}{3} \\ \hline \end{array}$
19. $\begin{array}{r} \frac{1}{6} \\ +\frac{2}{7} \\ \hline \end{array}$
20. $\begin{array}{r} \frac{4}{9} \\ +\frac{5}{12} \\ \hline \end{array}$

21. Jenny bought $\frac{3}{4}$ lb. of stick candy, $\frac{1}{2}$ lb. of licorice, and $\frac{1}{4}$ lb. of chocolate bars. How many pounds of candy was this?

22. Mrs. Grayko's diet permits her only $\frac{7}{16}$ lb. of butter each week. She eats $\frac{3}{16}$ lb. on Monday and $\frac{1}{8}$ lb. on Tuesday. How much butter is left?

We often say that a fraction is expressed in lowest terms when the greatest common factor (G.C.F.) of the numerator and denominator is 1.

$\frac{8}{15}$ is expressed in lowest terms because the G.C.F. of 8 and 15 is 1.

$\frac{9}{15}$ is not expressed in lowest terms because the G.C.F. of 9 and 15 is 3.

1. Find the G.C.F. of each pair of numbers.

a. 2 and 6	d. 12 and 34	g. 4 and 8	j. 24 and 42
b. 7 and 12	e. 9 and 21	h. 13 and 24	k. 1 and 17
c. 10 and 15	f. 16 and 24	i. 17 and 17	l. 9 and 36

2. Which of these fractions are expressed in lowest terms?

a. $\frac{1}{2}$	d. $\frac{6}{8}$	g. $\frac{1}{24}$	j. $\frac{3}{9}$	m. $\frac{3}{10}$
b. $\frac{2}{8}$	e. $\frac{9}{24}$	h. $\frac{1}{5}$	k. $\frac{15}{25}$	n. $\frac{14}{21}$
c. $\frac{5}{6}$	f. $\frac{4}{6}$	i. $\frac{21}{24}$	l. $\frac{4}{15}$	o. $\frac{12}{17}$

To express a fraction in lowest terms, divide both the numerator and denominator by their G.C.F.

When we divide both the numerator and denominator by the same number, we are dividing the fraction by 1.

The G.C.F. of 12 and 20 is 4.

$$\frac{12}{20} = \frac{12 \div 4}{20 \div 4} = \frac{3}{5}$$

We have divided $\frac{12}{20}$ by 1 because we divided by $\frac{4}{4}$.

3. Express each fraction in lowest terms.

a. $\frac{9}{15}$	d. $\frac{12}{34}$	g. $\frac{18}{24}$	j. $\frac{9}{36}$	m. $\frac{21}{56}$
b. $\frac{2}{6}$	e. $\frac{9}{21}$	h. $\frac{17}{17}$	k. $\frac{15}{75}$	n. $\frac{60}{100}$
c. $\frac{9}{12}$	f. $\frac{16}{24}$	i. $\frac{24}{42}$	l. $\frac{10}{35}$	o. $\frac{70}{100}$

To add fractions we may need all these steps:

1. Express the fractions using the L.C.D.
2. Add these fractions.
3. If possible, express the sum as a whole number or as a mixed number.
4. Express the fraction in lowest terms.

$$\begin{aligned} \tfrac{5}{6} &= \tfrac{10}{12} \\ +\tfrac{5}{12} &= \tfrac{5}{12} \\ \hline \tfrac{15}{12} &= 1\tfrac{3}{12} \\ &= 1\tfrac{1}{4} \end{aligned}$$

Find each sum.

1. $\begin{array}{r} \frac{1}{4} \\ +\frac{1}{2} \\ \hline \end{array}$
2. $\begin{array}{r} \frac{1}{2} \\ +\frac{2}{5} \\ \hline \end{array}$
3. $\begin{array}{r} \frac{2}{7} \\ +\frac{1}{3} \\ \hline \end{array}$
4. $\begin{array}{r} \frac{5}{12} \\ +\frac{1}{3} \\ \hline \end{array}$
5. $\begin{array}{r} \frac{2}{3} \\ +\frac{8}{15} \\ \hline \end{array}$
6. $\begin{array}{r} \frac{2}{3} \\ +\frac{1}{4} \\ \hline \end{array}$
7. $\begin{array}{r} \frac{2}{3} \\ +\frac{3}{8} \\ \hline \end{array}$
8. $\begin{array}{r} \frac{5}{8} \\ +\frac{3}{4} \\ \hline \end{array}$
9. $\begin{array}{r} \frac{7}{8} \\ +\frac{1}{6} \\ \hline \end{array}$
10. $\begin{array}{r} \frac{5}{6} \\ +\frac{3}{10} \\ \hline \end{array}$

To subtract fractions we may need all these steps:

1. Express the fractions using the L.C.D.
2. Subtract these fractions.
3. Express the fraction in lowest terms.

$$\begin{aligned} \tfrac{5}{6} &= \tfrac{10}{12} \\ -\tfrac{1}{12} &= -\tfrac{1}{12} \\ \hline \tfrac{9}{12} &= \tfrac{3}{4} \end{aligned}$$

Find each difference.

11. $\begin{array}{r} \frac{1}{2} \\ -\frac{1}{4} \\ \hline \end{array}$
12. $\begin{array}{r} \frac{4}{7} \\ -\frac{1}{3} \\ \hline \end{array}$
13. $\begin{array}{r} \frac{2}{3} \\ -\frac{2}{5} \\ \hline \end{array}$
14. $\begin{array}{r} \frac{17}{24} \\ -\frac{3}{8} \\ \hline \end{array}$
15. $\begin{array}{r} \frac{5}{14} \\ -\frac{1}{6} \\ \hline \end{array}$
16. $\begin{array}{r} \frac{4}{5} \\ -\frac{3}{4} \\ \hline \end{array}$
17. $\begin{array}{r} \frac{5}{6} \\ -\frac{2}{9} \\ \hline \end{array}$
18. $\begin{array}{r} \frac{3}{4} \\ -\frac{2}{3} \\ \hline \end{array}$
19. $\begin{array}{r} \frac{9}{10} \\ -\frac{1}{15} \\ \hline \end{array}$
20. $\begin{array}{r} \frac{7}{12} \\ -\frac{1}{4} \\ \hline \end{array}$
21. $\begin{array}{r} \frac{7}{16} \\ -\frac{3}{8} \\ \hline \end{array}$
22. $\begin{array}{r} \frac{3}{5} \\ -\frac{1}{6} \\ \hline \end{array}$
23. $\begin{array}{r} \frac{4}{7} \\ -\frac{5}{14} \\ \hline \end{array}$
24. $\begin{array}{r} \frac{9}{11} \\ -\frac{2}{3} \\ \hline \end{array}$
25. $\begin{array}{r} \frac{3}{4} \\ -\frac{1}{5} \\ \hline \end{array}$

To add mixed numbers:

1. Express the fractions using the L.C.D.
2. Add the fractions and express the sum in lowest terms.
3. Add this result to the sum of the whole numbers.

To find $2\frac{2}{3} + 5\frac{7}{12}$:

Step 1: $\frac{2}{3} = \frac{8}{12}$, $\frac{7}{12} = \frac{7}{12}$

Step 2: $\frac{8}{12} + \frac{7}{12} = \frac{15}{12} = 1\frac{3}{12} = 1\frac{1}{4}$

Step 3: $2 + 5 + 1\frac{1}{4} = 8\frac{1}{4}$

$$2\frac{2}{3} + 5\frac{7}{12} = 8\frac{1}{4}$$

Find each sum.

1. $2\frac{1}{2} + 3\frac{1}{4}$
2. $4\frac{3}{4} + 1\frac{1}{2}$
3. $5\frac{3}{4} + 2\frac{3}{8}$
4. $3\frac{1}{5} + 1\frac{7}{10}$
5. $3\frac{1}{3} + 6\frac{2}{3}$
6. $4\frac{2}{3} + 2\frac{1}{4}$
7. $1\frac{2}{3} + \frac{5}{6}$
8. $7\frac{1}{2} + 6\frac{2}{3}$
9. $19\frac{12}{17} + 2\frac{1}{2}$
10. $4\frac{4}{5} + 1\frac{1}{2}$
11. $\frac{5}{6} + 11\frac{3}{10}$
12. $10\frac{1}{6} + 10\frac{2}{3}$
13. $4\frac{1}{4} + 7\frac{1}{6}$
14. $2\frac{5}{6} + 5\frac{1}{4}$
15. $6\frac{3}{10} + 9\frac{1}{6}$
16. $2\frac{4}{5} + \frac{2}{3}$
17. $8\frac{5}{8} + 7\frac{1}{3}$
18. $4\frac{5}{8} + 31\frac{1}{6}$
19. $16\frac{2}{3} + 16\frac{2}{3}$
20. $53\frac{7}{10} + 56\frac{51}{100}$
21. $241\frac{3}{4} + 84\frac{9}{16}$

22. Mrs. Hyland buys $2\frac{3}{4}$ lb. hamburger, $\frac{3}{4}$ lb. cheese, and a jar of relish that weighs $1\frac{1}{16}$ lb. How much do her groceries weigh all together?

23. Mrs. Swanson buys 2 lb. stew meat, $1\frac{1}{8}$ lb. onions, and $1\frac{3}{4}$ lb. cheese. Do her groceries weigh more than Mrs. Hyland's?

24. The desk is $4\frac{11}{12}$ ft. long, the bookcase is $3\frac{1}{2}$ ft. long, and this chair is $1\frac{5}{6}$ ft. wide. Can I place them all along a wall $10\frac{1}{6}$ ft. long?

To subtract mixed numbers:

1. Express the fractions using the L.C.D.
2. If the fraction to be subtracted is the greater, regroup the first mixed number.
3. Subtract the fractions.
4. Subtract the whole numbers.
5. Express the fraction in lowest terms.

To find $5\frac{5}{12} - 2\frac{2}{3}$:

$$\begin{array}{rrr} 5\frac{5}{12} = & 5\frac{5}{12} = & 4\frac{17}{12} \\ \underline{-2\frac{2}{3}} = & \underline{-2\frac{8}{12}} = & \underline{-2\frac{8}{12}} \\ & & 2\frac{9}{12} = 2\frac{3}{4} \end{array}$$

Find each difference.

1. $4\frac{2}{3} - 2\frac{1}{6}$
2. $7\frac{3}{4} - 1\frac{5}{8}$
3. $13\frac{4}{7} - 7$
4. $5\frac{1}{4} - \frac{2}{5}$
5. $1\frac{3}{7} - \frac{1}{2}$
6. $12 - 5\frac{1}{4}$
7. $10\frac{1}{2} - 6\frac{3}{8}$
8. $8\frac{3}{5} - 3\frac{3}{10}$
9. $9\frac{7}{10} - 3\frac{2}{5}$
10. $11\frac{2}{3} - 5\frac{1}{6}$
11. $16 - 9\frac{12}{17}$
12. $4\frac{1}{8} - 2\frac{2}{3}$
13. $5\frac{1}{3} - 4\frac{1}{12}$
14. $41\frac{3}{4} - 7\frac{1}{6}$
15. $10\frac{1}{6} - \frac{5}{6}$
16. $4\frac{8}{9} - 1\frac{5}{18}$
17. $29\frac{1}{9} - 1\frac{1}{3}$
18. $51\frac{1}{5} - 2\frac{5}{6}$
19. $66\frac{2}{3} - 33\frac{1}{3}$
20. $115\frac{1}{7} - 21\frac{3}{7}$
21. $101\frac{1}{4} - 2\frac{1}{3}$

22. Gary is $5\frac{3}{4}$ ft. high and David is $5\frac{7}{12}$ ft. How much taller is Gary?

23. Terry collects $52\frac{1}{8}$ lb. of old newspapers for the paper drive. Scott collects $39\frac{3}{8}$ lb. How many more pounds does Terry collect?

24. Henry weighs $215\frac{1}{2}$ lb. Jack weighs $123\frac{5}{8}$ lb. How much heavier is Henry?

1. Three boys went on a hiking trip. Billy carried a canteen that weighed $3\frac{1}{2}$ lb. and a first aid kit that weighed $1\frac{1}{4}$ lb. How much weight was this?

2. Ralph carried a skillet, some bacon, some cheese, and some fruit in his pack. The skillet weighed $2\frac{1}{4}$ lb., the bacon 1 lb., the cheese $\frac{7}{8}$ lb., and the fruit $2\frac{1}{4}$ lb. How much weight was this?

3. Sam carried some canned goods that weighed $5\frac{1}{4}$ lb. and some potatoes that weighed $2\frac{1}{2}$ lb. How much weight was this?

4. What was the total weight of the food and supplies that the boys took with them?

5. How much heavier was the weight that Ralph carried than the weight that Billy carried?

6. How much heavier was the weight that Sam carried than the weight that Ralph carried?

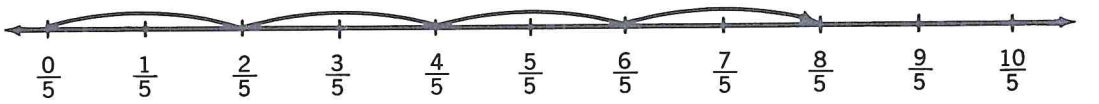
7. How much heavier was the weight that Sam carried than the weight that Billy carried?

8. The boys hiked $4\frac{1}{2}$ miles in the morning and $3\frac{3}{4}$ miles in the afternoon. How far did they hike?

We can multiply a fraction by a whole number.

$$4 \times \frac{2}{5} = \frac{2}{5} + \frac{2}{5} + \frac{2}{5} + \frac{2}{5} = \frac{8}{5} = 1\frac{3}{5}$$

We can show this on a number line.



$$\frac{2}{5} + \frac{2}{5} + \frac{2}{5} + \frac{2}{5} = 4 \times \frac{2}{5} = \frac{8}{5} = 1\frac{3}{5}$$

Show each problem on a number line. Write a multiplication equation for each problem and find the product.

1. $\frac{5}{7} + \frac{5}{7}$
2. $\frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3}$
3. $\frac{3}{2} + \frac{3}{2} + \frac{3}{2} + \frac{3}{2} + \frac{3}{2}$
4. $\frac{7}{9} + \frac{7}{9}$
5. $\frac{4}{5} + \frac{4}{5} + \frac{4}{5}$
6. $\frac{2}{13} + \frac{2}{13} + \frac{2}{13} + \frac{2}{13}$

To multiply a fraction by a whole number:

1. Multiply the numerator by the whole number.
2. Express the product as a mixed number or as a whole number if possible.
3. Express the fraction in lowest terms.

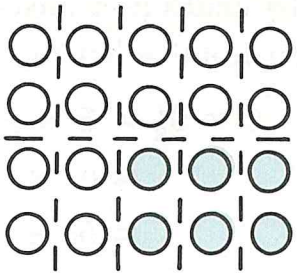
$$4 \times \frac{3}{8} = \frac{4 \times 3}{8} = \frac{12}{8} = 1\frac{4}{8} = 1\frac{1}{2}$$

Find each product.

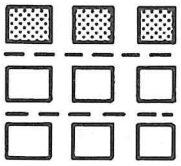
7. $2 \times \frac{3}{7}$
8. $3 \times \frac{3}{4}$
9. $4 \times \frac{1}{3}$
10. $7 \times \frac{1}{2}$
11. $5 \times \frac{2}{3}$
12. $3 \times \frac{1}{21}$
13. $2 \times \frac{5}{6}$
14. $11 \times \frac{7}{11}$
15. $20 \times \frac{2}{7}$
16. $24 \times \frac{1}{8}$
17. $17 \times \frac{2}{3}$
18. $3 \times \frac{4}{9}$
19. $12 \times \frac{5}{6}$
20. $22 \times \frac{3}{4}$
21. $25 \times \frac{1}{2}$
22. $100 \times \frac{2}{3}$

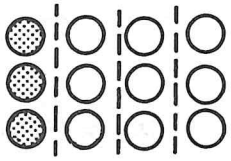
There are 20 balls divided into 10 groups.
3 of the small groups are colored.
$\frac{3}{10}$ of the whole group is colored.
6 balls are colored.

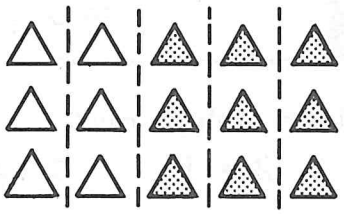
$$\frac{3}{10} \times 20 = 6$$

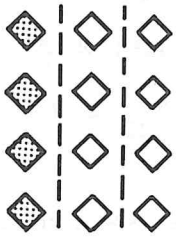


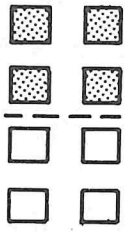
Give the multiplication equation for each problem.

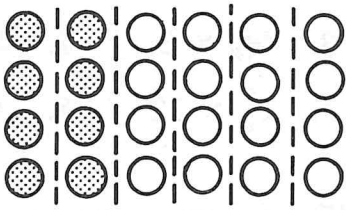
1. 

2. 

3. 

4. 

5. 

6. 

Find each product.

7. a. $\frac{1}{2} \times 8$ b. $8 \times \frac{1}{2}$

8. a. $\frac{3}{7} \times 21$ b. $21 \times \frac{3}{7}$

9. a. $\frac{5}{6} \times 12$ b. $12 \times \frac{5}{6}$

10. a. $\frac{2}{3} \times 18$ b. $18 \times \frac{2}{3}$

11. In multiplying does it matter whether the fraction or the whole number comes first?

Find each product.

12. $16 \times \frac{1}{8}$
13. $20 \times \frac{2}{5}$
14. $\frac{1}{6} \times 30$
15. $\frac{1}{3} \times 8$
16. $\frac{5}{6} \times 9$
17. $10 \times \frac{3}{8}$
18. $\frac{3}{8} \times 16$
19. $16 \times \frac{3}{8}$
20. $\frac{1}{4} \times 20$
21. $9 \times \frac{1}{4}$
22. $\frac{5}{8} \times 40$
23. $18 \times \frac{2}{5}$

To multiply a mixed number by a whole number, we may apply the distributive law for multiplication over addition.

$$
\begin{aligned}
6 \times 9\tfrac{7}{8} &= 6 \times (9 + \tfrac{7}{8}) && \text{because } 9\tfrac{7}{8} = 9 + \tfrac{7}{8} \\
&= (6 \times 9) + (6 \times \tfrac{7}{8}) && \text{because of the distributive law} \\
&= 54 + \tfrac{42}{8} && \text{because } 6 \times \tfrac{7}{8} = \tfrac{42}{8} \\
&= 54 + 5\tfrac{2}{8} && \text{because } \tfrac{42}{8} = 5\tfrac{2}{8} \\
&= 54 + 5\tfrac{1}{4} && \text{because } \tfrac{2}{8} = \tfrac{1}{4} \\
&= 59\tfrac{1}{4} && \text{because } 54 + 5\tfrac{1}{4} = 59\tfrac{1}{4}
\end{aligned}
$$

Find each product.

1. $2 \times 1\frac{1}{5}$
2. $3 \times 2\frac{1}{6}$
3. $4\frac{5}{18} \times 3$
4. $1\frac{1}{2} \times 12$
5. $\frac{3}{2} \times 12$
6. $4 \times 10\frac{2}{3}$
7. $4 \times \frac{3}{7}$
8. $1\frac{3}{7} \times 4$
9. $\frac{1}{2} \times 100$
10. $2\frac{3}{10} \times 15$
11. $3 \times 66\frac{2}{3}$
12. $6\frac{7}{16} \times 4$
13. $\frac{3}{8} \times 100$
14. $4\frac{3}{8} \times 100$
15. $103 \times \frac{4}{5}$
16. $5\frac{1}{12} \times 18$

17. If 1760 yards are equal in length to 1 mile, how many yards are equal in length to $\frac{3}{4}$ mile?

18. Dave's school is $2\frac{1}{4}$ miles away. He cycles there and back each school day. From Monday through Friday, how many miles does he cycle going back and forth to school?

19. During 1 hour Bob saw 9 commercials on television. Each one lasted $1\frac{2}{3}$ minutes. How long did he watch commercials?

20. Teresa swims from one end of the pool to the other in $2\frac{3}{10}$ minutes. How long does she take to swim 5 lengths?

21. The world is about 25,000 miles around. How far away is a place $\frac{2}{5}$ of the way around the world from here?

Complete each problem.

1. $4\frac{3}{7} + \frac{1}{3} =$ ■
2. $17 \times \frac{2}{9} =$ ■
3. $17 - \frac{2}{9} =$ ■
4. $\frac{5}{6} + \frac{2}{3} =$ ■
5. $3 \times 4\frac{2}{5} =$ ■
6. $\frac{5}{12} + \frac{7}{15} =$ ■
7. $\frac{7}{15} - \frac{5}{12} =$ ■
8. $\frac{3}{4} \times 20 =$ ■
9. $3\frac{1}{8} + 6\frac{3}{8} =$ ■
10. $7\frac{2}{5} - \frac{3}{4} =$ ■
11. $3\frac{1}{5} \times 9 =$ ■
12. $4\frac{5}{6} - 1\frac{1}{3} =$ ■
13. $6 \times 16\frac{2}{3} =$ ■
14. $\frac{27}{100} \times 50 =$ ■
15. $1\frac{4}{7} - \frac{6}{7} =$ ■
16. $\frac{5}{6} + \frac{7}{15} =$ ■
17. $11\frac{3}{4} + 11\frac{3}{4} =$ ■
18. $\frac{4}{5} \times 100 =$ ■
19. $5 \times 3\frac{7}{10} =$ ■
20. $2\frac{1}{3} \times 7 =$ ■
21. $11\frac{1}{6} - 5\frac{2}{3} =$ ■
22. $\frac{13}{18} + 39\frac{5}{6} =$ ■
23. $30\frac{7}{24} - 16\frac{1}{8} =$ ■
24. $11 \times 2\frac{3}{8} =$ ■
25. $2\frac{19}{24} + 27\frac{3}{8} =$ ■
26. $7 \times 1\frac{1}{2} =$ ■
27. $131\frac{1}{2} - 42\frac{6}{7} =$ ■
28. $\frac{3}{7} \times 30 =$ ■
29. $100 - 33\frac{1}{3} =$ ■
30. $3\frac{1}{3} + 3\frac{1}{3} =$ ■

31. Each can of peaches weighs $2\frac{1}{8}$ lb. What do 4 cans weigh?

32. Each sack of apples weighs $5\frac{3}{8}$ lb. What do 5 sacks weigh?

33. Ellen's desk is 9 pencil-lengths wide. The pencil is $5\frac{3}{8}$ in. long. How wide is the desk?

34. In a story a sultan was weighed in bags of gold and jewels. His weight was that of 100 bags of gold, 9 bags of diamonds, and 4 bags of assorted jewels. Each bag of gold weighed $1\frac{3}{8}$ lb. Each bag of diamonds weighed $2\frac{1}{12}$ lb., and each bag of jewels weighed $1\frac{5}{12}$ lb. How much did he weigh?

35. In the same story the sultan's wife was also weighed. Her weight was that of 80 bags of gold, 11 bags of diamonds, and 3 bags of jewels. How much more did the sultan weigh than his wife?

36. Norman swims from one end of the pool to the other in $\frac{7}{10}$ minute. How long does he take to swim 8 lengths?

Checkout

Find each sum or difference.

1. $8\frac{1}{3} - 2\frac{1}{4}$
2. $6\frac{3}{4} + 8\frac{2}{3}$
3. $14\frac{2}{5} + 8\frac{1}{4}$
4. $72\frac{3}{8} - 29\frac{1}{2}$

Express as a fraction.

5. $4\frac{3}{2} = \square$
6. $6\frac{2}{5} = \square$
7. $7\frac{5}{4} = \square$
8. $8\frac{2}{3} = \square$
9. $4\frac{1}{8} = \square$
10. $5\frac{2}{9} = \square$

Find the L.C.M. of each set of numbers.

11. $\{2, 5\}$
12. $\{6, 8\}$
13. $\{4, 20\}$
14. $\{7,5\}$

Choose $<$ or $=$ or $>$.

15. $\frac{2}{3} \bigcirc \frac{2}{5}$
16. $\frac{1}{2} \bigcirc \frac{8}{16}$
17. $\frac{4}{8} \bigcirc \frac{5}{8}$
18. $\frac{5}{8} \bigcirc \frac{4}{5}$
19. $\frac{3}{5} \bigcirc \frac{4}{6}$
20. $\frac{5}{9} \bigcirc \frac{6}{10}$

Solve for n.

21. $8 \times \frac{2}{5} = n$
22. $24 \times \frac{1}{2} = n$
23. $18 \times \frac{5}{6} = n$
24. $50 \times 1\frac{2}{3} = n$
25. $\frac{4}{5} \times 25 = n$
26. $\frac{3}{8} \times 20 = n$

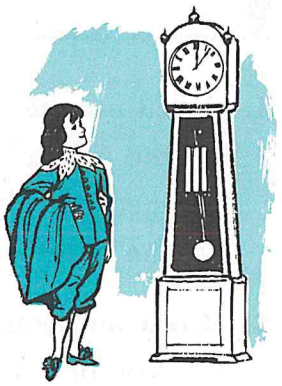
27. Helen made 4 belts that each required $\frac{7}{8}$ yd. of ribbon. How many yards of ribbon did she need?

28. List all the prime numbers between 40 and 60.

29. Ruth's school is $\frac{3}{4}$ mile from her home. She cycled $\frac{3}{10}$ mile going home before the chain on her bicycle wheel broke. How far did she have to push the bicycle before reaching her home?

Checkout

1. $\{\text{Ellen, Sue, Betty, Carol}\} \cap \{\text{Betty, Carol}\} = \blacksquare$
2. $\{\text{Mars, Venus, Mercury}\} - \{\text{Mercury, Earth}\} = \blacksquare$
3. $\{\text{Iowa, Texas, Utah, Maine}\} \cap \{\text{Utah, Ohio}\} = \blacksquare$
4. $\{a, b, c\} \cup \{d, e\} = \blacksquare$
5. $\{\} - \{3, 2, 1, 0\} = \blacksquare$



Find each sum.

6.	7.	8.	9.
14	18	503	421
11	19	702	932
30	36	988	547
15	41	776	673

Complete each problem.

10. $\frac{1}{8} + \blacksquare = \frac{5}{8}$
11. $\blacksquare + \frac{2}{4} = \frac{5}{8}$
12. $\blacksquare + \frac{1}{2} = \frac{5}{8}$
13. $\frac{1}{12} + \blacksquare = \frac{5}{12}$
14. $\blacksquare + \frac{2}{6} = \frac{5}{12}$
15. $\blacksquare + \frac{1}{3} = \frac{5}{12}$
16. $\blacksquare + \frac{3}{6} = \frac{7}{12}$
17. $\blacksquare + \frac{2}{4} = \frac{7}{12}$
18. $\frac{1}{2} + \blacksquare = \frac{7}{12}$

Find each difference.

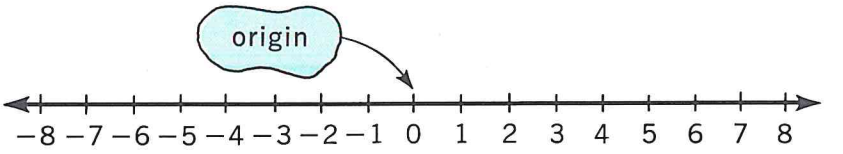
19.	20.	21.	22.	23.
27	38	376	491	600
−18	−29	−308	−279	−249

Find each product.

24.	25.	26.	27.	28.
7306	5477	73	42	759
×8	×6	×81	×63	×45

Part 7 Negative Numbers

The set of integers $= \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$



1. What number corresponds to
   a. the origin?
   b. the point three units to the right of the origin?
   c. the point five units to the left of the origin?

2. Copy and complete each sentence.
   a. Numbers corresponding to points to the ▢ of the origin are positive numbers.
   b. Numbers corresponding to points to the ▢ of the origin are negative numbers.
   c. ▢ is neither a positive nor a negative number.

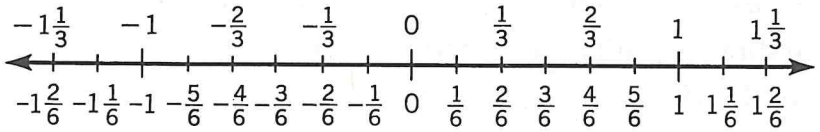
3. Choose $<$ or $=$ or $>$.

a. $2 \bigcirc -5$	c. $0 \bigcirc -4$	e. $-1 \bigcirc -4$	g. $3 \bigcirc -4$
b. $-1 \bigcirc 1$	d. $6 \bigcirc -6$	f. $-8 \bigcirc 0$	h. $-2 \bigcirc -1$

4. Make a line, mark off units on it, and label the origin. Label points corresponding to the numbers -10 through 10. Look at this number line to answer the following questions. What numbers correspond to the points that are
   a. 7 units from the origin?
   b. 2 units from the origin?
   c. 1 unit from the origin?
   d. 6 units from the origin?

Some numbers such as $-\frac{1}{2}$ and $1\frac{1}{6}$ are not integers.

We can use more than one scale for a number line to show fractions.


$-1\frac{1}{3}$ -1 $-\frac{2}{3}$ $-\frac{1}{3}$ 0 $\frac{1}{3}$ $\frac{2}{3}$ 1 $1\frac{1}{3}$
$-1\frac{2}{6}$ $-1\frac{1}{6}$ -1 $-\frac{5}{6}$ $-\frac{4}{6}$ $-\frac{3}{6}$ $-\frac{2}{6}$ $-\frac{1}{6}$ 0 $\frac{1}{6}$ $\frac{2}{6}$ $\frac{3}{6}$ $\frac{4}{6}$ $\frac{5}{6}$ 1 $1\frac{1}{6}$ $1\frac{2}{6}$

The scale above the number line shows the number line with a scale of $\frac{1}{3}$.
The scale below the number line shows the number line with a scale of $\frac{1}{6}$.

1. Make a number line with units of $\frac{1}{8}$. Label points corresponding to the numbers $-2\frac{1}{8}$ through $2\frac{3}{8}$. Below the number line, put a scale showing fourths. Then complete these problems.

   a. $\frac{2}{4} = \frac{\square}{8}$ b. $-\frac{3}{4} = -\frac{\square}{8}$ c. $\frac{2}{8} = \frac{\square}{4}$ d. $-1\frac{1}{4} = -1\frac{\square}{8}$

2. Make a line and mark off units of two-tenths on it. Label the origin, then label points corresponding to the numbers from $-3\frac{2}{10}$ to $2\frac{8}{10}$. Then complete these problems. Choose $<$ or $=$ or $>$.

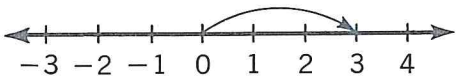
   a. $\frac{4}{10} \bigcirc \frac{2}{10}$ b. $\frac{6}{10} \bigcirc -\frac{8}{10}$ c. $-\frac{24}{10} \bigcirc -\frac{16}{10}$ d. $0 \bigcirc \frac{2}{10}$

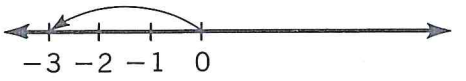
3. Copy each sentence and complete it with **left** or **right.**
   a. To go in a **positive** direction on the number line, we go to the ■.
   b. To go in a **negative** direction, we go to the ■.

4. Copy each sentence and complete it with **less than** or **greater than.**
   a. If a number corresponds to a point to the left of the origin, that number is ■ zero.
   b. If a number corresponds to a point to the right of the origin, that number is ■ zero.
   c. Any negative number is ■ any positive number.

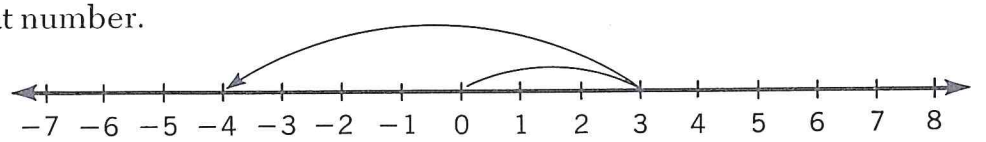
Addition of Negative Numbers

We can use a number line to show addition. Start at the origin. Move the number of units indicated by the first number, in the direction indicated by the sign of that number.





A plus sign tells us to make our second move the number of units indicated by the second number, in the direction indicated by the sign of that number.



$3 + (-7) = -4$

Make a number line for each problem. Follow the rules above. Then complete each equation.

1. $4 + (-2) =$ ▢
2. $(-5) + 6 =$ ▢
3. $(-3) + 3 =$ ▢
4. $(-4) + (-5) =$ ▢
5. $5 + (-6) =$ ▢
6. $(-2) + (-7) =$ ▢

Find each sum. If you need help on any problem, make a number line and show the addition on it.

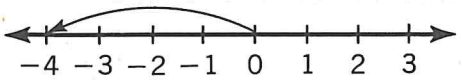
7. $\begin{array}{r} -4 \\ 2 \\ \hline \end{array}$
8. $\begin{array}{r} 8 \\ -5 \\ \hline \end{array}$
9. $\begin{array}{r} -7 \\ -4 \\ \hline \end{array}$
10. $\begin{array}{r} -9 \\ 8 \\ \hline \end{array}$
11. $\begin{array}{r} -6 \\ -3 \\ \hline \end{array}$

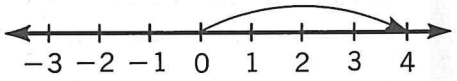
12. $\begin{array}{r} 4\frac{4}{5} \\ -2\frac{3}{5} \\ \hline \end{array}$
13. $\begin{array}{r} 6\frac{3}{4} \\ -4\frac{1}{4} \\ \hline \end{array}$
14. $\begin{array}{r} -5\frac{2}{3} \\ -2\frac{1}{3} \\ \hline \end{array}$
15. $\begin{array}{r} -6\frac{4}{6} \\ 3\frac{1}{6} \\ \hline \end{array}$
16. $\begin{array}{r} -6\frac{5}{8} \\ -9\frac{7}{8} \\ \hline \end{array}$

17. $\begin{array}{r} -8\frac{4}{10} \\ 5\frac{6}{10} \\ \hline \end{array}$
18. $\begin{array}{r} -6\frac{9}{10} \\ -4\frac{8}{10} \\ \hline \end{array}$
19. $\begin{array}{r} -9\frac{4}{10} \\ 6\frac{9}{10} \\ \hline \end{array}$
20. $\begin{array}{r} -7\frac{8}{10} \\ -9\frac{4}{10} \\ \hline \end{array}$
21. $\begin{array}{r} -8\frac{1}{10} \\ -6\frac{5}{10} \\ \hline \end{array}$

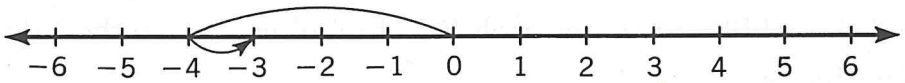
Subtraction of Negative Numbers

We can use a number line to show subtraction. Start at the origin. Move the number of units indicated by the first number, in the direction indicated by the sign of that number.





A minus sign tells us to make our second move the number of units indicated by the second number, but in the opposite direction from the sign of that number.



$(-4) - (-1) = -3$

Make a number line to show each problem. Follow the rules above. Then complete each equation.

1. $6 - (-3) =$ ■
2. $(-4) - (-4) =$ ■
3. $(-4) - 4 =$ ■
4. $4 - (-4) =$ ■
5. $(-2) - (-6) =$ ■
6. $5 - (-2) =$ ■

Find each difference. If you need help on any problem, make a number line and show the subtraction on it.

7. $\begin{array}{r} 9 \\ 4 \\ \hline \end{array}$
8. $\begin{array}{r} -8 \\ -3 \\ \hline \end{array}$
9. $\begin{array}{r} -6 \\ 4 \\ \hline \end{array}$
10. $\begin{array}{r} 7 \\ -4 \\ \hline \end{array}$
11. $\begin{array}{r} 9 \\ -7 \\ \hline \end{array}$
12. $\begin{array}{r} 4\frac{3}{8} \\ 9\frac{2}{8} \\ \hline \end{array}$
13. $\begin{array}{r} 5\frac{1}{2} \\ -6\frac{1}{2} \\ \hline \end{array}$
14. $\begin{array}{r} -3\frac{4}{9} \\ -1\frac{1}{9} \\ \hline \end{array}$
15. $\begin{array}{r} 9\frac{2}{3} \\ -4\frac{1}{3} \\ \hline \end{array}$
16. $\begin{array}{r} -8\frac{6}{7} \\ 3\frac{2}{7} \\ \hline \end{array}$
17. $\begin{array}{r} 3\frac{5}{10} \\ -4\frac{7}{10} \\ \hline \end{array}$
18. $\begin{array}{r} -6\frac{6}{10} \\ -9\frac{4}{10} \\ \hline \end{array}$
19. $\begin{array}{r} -4\frac{9}{10} \\ 7\frac{4}{10} \\ \hline \end{array}$
20. $\begin{array}{r} 6\frac{9}{10} \\ -6\frac{9}{10} \\ \hline \end{array}$
21. $\begin{array}{r} 9\frac{2}{10} \\ 4\frac{8}{10} \\ \hline \end{array}$

In each list of numbers, we follow a rule to find the next number. In this list find the missing numbers

$$4, 3, 2, \square, \square, \square, -2$$

by using an addition rule or a subtraction rule.

Addition Rule: Add -1 to the preceding number. 4, 3, 2, 1, 0, $-1, -2$.

Subtraction Rule: Subtract 1 from the preceding number. 4, 3, 2, 1, 0, $-1, -2$.

1. Find the addition rule for each list of numbers. Give the rule and complete the list.

a. $-10, -8, -6, \square, \square, \square, \square, \square, \square, 8$

b. $13, 4, -5, \square, \square, \square, \square, \square, \square, -68$

c. $14, 11\frac{1}{2}, 9, \square, \square, \square, \square, \square, \square, -8\frac{1}{2}$

d. $2, 3\frac{1}{4}, 4\frac{1}{2}, \square, \square, \square, \square, \square, 12, \square$

2. Find the subtraction rule for each list of numbers. Give the rule and complete the list.

a. $21, 13, 5, \square, \square, \square, \square, \square, \square, -51$

b. $-11, -9, -7, \square, \square, \square, \square, \square, \square, 7$

c. $40, 29\frac{1}{2}, 19, \square, \square, \square, \square, \square, \square, -54\frac{1}{2}$

d. $9, 6\frac{1}{2}, 4, \square, \square, \square, \square, \square, -11, \square$

3. Find an addition rule and a subtraction rule for each list of numbers. Give each rule and complete the list.

a. $6, 12, 18, \square, \square, \square, \square, \square, \square, 60$

b. $10, 5, 0, \square, \square, \square, \square, \square, \square, -35$

c. $17, 15, 13, \square, \square, \square, \square, \square, \square, -1$

d. $4, 0, -4, \square, \square, \square, \square, \square, \square, -32$

e. $-1, -4, -7, \square, \square, \square, \square, \square, \square, -28$

1. Mr. Morgan runs a small grocery store that is not very successful. Sometimes he has a little money ahead, and at other times he is in debt. On September 1, he had $225.00 in his account. On September 2, he received $55 from Mr. Smithson, who paid his August bill. What was the new balance in Mr. Morgan's account?
2. On September 3, Mr. Morgan paid one bill of $60 and another bill of $125. What was the new balance in his account?
3. On September 4, he received $43 from Mr. Johnson. What was the new balance in his account?
4. The next ten days are represented in the following table. What was the balance in his account at the end of September 5?

Date	Paid Out	Received
Sept. 5	$150	
Sept. 6		$34
Sept. 7		$57
Sept. 8	$290	
Sept. 9		$72

Date	Paid Out	Received
Sept. 10		$19
Sept. 11		$92
Sept. 12	$133	
Sept. 13		$110
Sept. 14		$185

5. What was the balance in his account at the end of September 8?
6. What was the balance in his account at the end of September 11?
7. What was the balance in his account at the end of September 14?

Multiplication of Negative Numbers.

Look at this pattern.

Notice that the product of two positive factors is positive. The product of a positive factor and a negative factor is negative.

$2 \times 3 = 6$
$2 \times 2 = 4$
$2 \times 1 = 2$
$2 \times 0 = 0$
$2 \times (-1) = -2$
$2 \times (-2) = -4$
$2 \times (-3) = -6$
$2 \times (-4) = -8$
$2 \times (-5) = -10$

Each number given in this column is two less than the number given above it.

Each number given in this column is one less than the number given above it.

Copy and complete each pattern.

1. $3 \times 3 =$ ■
   $3 \times 2 =$ ■
   $3 \times 1 =$ ■
   $3 \times 0 =$ ■
   $3 \times (-1) =$ ■
   $3 \times (-2) =$ ■
   $3 \times (-3) =$ ■

2. $4 \times 3 =$ ■
   $4 \times 2 =$ ■
   $4 \times 1 =$ ■
   $4 \times 0 =$ ■
   $4 \times (-1) =$ ■
   $4 \times (-2) =$ ■
   $4 \times (-3) =$ ■

3. $5 \times 3 =$ ■
   $5 \times 2 =$ ■
   $5 \times 1 =$ ■
   $5 \times 0 =$ ■
   $5 \times -1 =$ ■
   $5 \times -2 =$ ■
   $5 \times -3 =$ ■

4. Each number given in the answer column in Exercise 1 is ■ less than the number given just above it.

5. Each number given in the answer column in Exercise 2 is ■ less than the number given just above it.

6. Each number given in the answer column in Exercise 3 is ■ less than the number given just above it.

7. Complete each sentence with **positive** or **negative**.
   a. A positive number times a positive number is a ■ number.
   b. A positive number times a negative number is a ■ number.

Look at this pattern.

$(-2) \times 3 = -6$
$(-2) \times 2 = -4$
$(-2) \times 1 = -2$
$(-2) \times 0 = 0$
$(-2) \times (-1) = 2$
$(-2) \times (-2) = 4$
$(-2) \times (-3) = 6$
$(-2) \times (-4) = 8$
$(-2) \times (-5) = 10$

Each number given in this column is two more than the number given above it.

Notice that the product of two negative factors is positive.

Copy and complete each pattern.

1. $(-3) \times 3 = -9$
$(-3) \times 2 =$ ■
$(-3) \times 1 =$ ■
$(-3) \times 0 =$ ■
$(-3) \times (-1) =$ ■
$(-3) \times (-2) =$ ■
$(-3) \times (-3) =$ ■
$(-3) \times (-4) =$ ■
$(-3) \times (-5) =$ ■

2. $(-5) \times 3 =$ ■
$(-5) \times 2 =$ ■
$(-5) \times 1 =$ ■
$(-5) \times 0 =$ ■
$(-5) \times (-1) =$ ■
$(-5) \times (-2) =$ ■
$(-5) \times (-3) =$ ■
$(-5) \times (-4) =$ ■
$(-5) \times (-5) =$ ■

3. $(-8) \times 3 =$ ■
$(-8) \times 2 =$ ■
$(-8) \times 1 =$ ■
$(-8) \times 0 =$ ■
$(-8) \times (-1) =$ ■
$(-8) \times (-2) =$ ■
$(-8) \times (-3) =$ ■
$(-8) \times (-4) =$ ■
$(-8) \times (-5) =$ ■

4. Each number given in the answer column in Exercise 1 is ■ more than the number given just above it.

5. Each number given in the answer column in Exercise 2 is ■ more than the number given just above it.

6. Each number given in the answer column in Exercise 3 is ■ more than the number given just above it.

7. Complete each problem with **positive** or **negative.**

a. negative × positive = ■
b. positive × positive = ■
c. negative × negative = ■
d. positive × negative = ■

Find each product.

1. a. $8 \times (-7) =$ ▢
   b. $3 \times (-4) =$ ▢
   c. $2 \times (-7) =$ ▢
   d. $9 \times (-8) =$ ▢
   e. $4 \times (-7) =$ ▢
   f. $6 \times (-9) =$ ▢
   g. $(-5) \times 7 =$ ▢
   h. $(-9) \times 9 =$ ▢
   i. $(-7) \times 6 =$ ▢
   j. $(-3) \times 8 =$ ▢
   k. $(-6) \times 3 =$ ▢
   l. $(-4) \times 6 =$ ▢

2. Copy and complete this multiplication table.

×	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6
6							-36						0						
7		-77																	
8								-40								24			
9															18				
10	-120				-80														
11										-33									
12														12					

3. Find each product.

   a. $(-4) \times (-7) =$ ▢
   b. $(-3) \times (-9) =$ ▢
   c. $(-6) \times (-8) =$ ▢
   d. $(-1) \times (-9) =$ ▢
   e. $(-1) \times (-1) =$ ▢
   f. $(-5) \times 7 =$ ▢
   g. $(-9) \times 6 =$ ▢
   h. $(-7) \times 3 =$ ▢
   i. $(-6) \times 4 =$ ▢
   j. $(-1) \times 1 =$ ▢
   k. $2 \times (-4) =$ ▢
   l. $5 \times (-9) =$ ▢
   m. $8 \times (-5) =$ ▢
   n. $9 \times (-7) =$ ▢
   o. $1 \times (-1) =$ ▢

4. Copy each problem and find the product.

   a. $\begin{array}{r} -14 \\ \times\ \ 8 \\ \hline \end{array}$
   b. $\begin{array}{r} -25 \\ \times -9 \\ \hline \end{array}$
   c. $\begin{array}{r} 39 \\ \times -7 \\ \hline \end{array}$
   d. $\begin{array}{r} -43 \\ \times -6 \\ \hline \end{array}$
   e. $\begin{array}{r} 84 \\ \times -4 \\ \hline \end{array}$

   f. $\begin{array}{r} -34 \\ \times -9 \\ \hline \end{array}$
   g. $\begin{array}{r} 42 \\ \times -5 \\ \hline \end{array}$
   h. $\begin{array}{r} -57 \\ \times\ \ 6 \\ \hline \end{array}$
   i. $\begin{array}{r} 47 \\ \times -7 \\ \hline \end{array}$
   j. $\begin{array}{r} -84 \\ \times -8 \\ \hline \end{array}$

Division of Negative Numbers.

Find each missing factor.

1. a. $6 \times \square = 30$
   b. $\square \times 7 = 56$
   c. $(-6) \times \square = 30$
   d. $\square \times (-7) = 56$

2. a. $8 \times \square = 48$
   b. $8 \times \square = -48$
   c. $\square \times 6 = -36$
   d. $\square \times 6 = 36$

3. a. $(-7) \times \square = 42$
   b. $5 \times \square = 25$
   c. $(-8) \times \square = 24$
   d. $\square \times (-9) = 45$

To find the missing factor is to find the quotient.

$9 \times \square = 72$ — missing factor

$9 \times 8 = 72$

$72 \div 9 = \square$ — quotient

$72 \div 9 = 8$

Complete each problem.

4. a. $4 \times \square = 32$
   b. $32 \div 4 = \square$

5. a. $(-7) \times \square = -63$
   b. $(-63) \div (-7) = \square$

6. a. $(-8) \times \square = -56$
   b. $(-56) \div (-8) = \square$

7. a. $(-4) \times \square = -48$
   b. $(-48) \div (-4) = \square$

8. a. $\square \times (-9) = 81$
   b. $81 \div (-9) = \square$

9. a. $6 \times \square = -72$
   b. $(-72) \div 6 = \square$

10. a. $\square \times 11 = -55$
    b. $(-55) \div 11 = \square$

11. a. $\square \times 8 = -96$
    b. $(-96) \div 8 = \square$

Choose **negative** or **positive**.

12. a. negative $\times \square =$ positive
    b. positive $\div$ negative $= \square$

13. a. negative $\times \square =$ negative
    b. negative $\div$ negative $= \square$

Find each quotient.

1. $(-44) \div (-11) =$ ▢
2. $(-36) \div 12 =$ ▢
3. $50 \div (-10) =$ ▢
4. $28 \div 2 =$ ▢
5. $32 \div (-4) =$ ▢
6. $20 \div (-5) =$ ▢
7. $30 \div 6 =$ ▢
8. $(-36) \div 3 =$ ▢
9. $(-40) \div (-8) =$ ▢
10. $27 \div (-3) =$ ▢
11. $28 \div (-7) =$ ▢
12. $(-40) \div 10 =$ ▢
13. $(-36) \div (-4) =$ ▢
14. $56 \div 8 =$ ▢
15. $63 \div (-7) =$ ▢

Find the multiplication rule for each list of numbers. Give the rule and complete the list.

16. $1, -2, 4,$ ▢, ▢, ▢, ▢, ▢, ▢, -512
17. $-16, 8, -4,$ ▢, ▢, ▢, ▢, ▢, ▢, $\frac{1}{32}$
18. $27, 9, 3,$ ▢, ▢, ▢, ▢, ▢, ▢, $\frac{1}{729}$

Find the division rule for each list of numbers. Give the rule and complete the list.

19. $1200, 600, 300,$ ▢, ▢, ▢
20. $48, 24,$ ▢, ▢, ▢, $\frac{3}{2}$
21. $81, 27,$ ▢, ▢, $1,$ ▢

Find the rule used in each list of numbers. Give this rule and complete the list. Use the operation given before the list of numbers in stating the rule.

22. addition $25, 21, 17,$ ▢, ▢, ▢, ▢, -3
23. division $64, 32, 16,$ ▢, ▢, ▢, ▢, $\frac{1}{2}$
24. multiplication $-2, 6, -18,$ ▢, ▢, ▢, ▢, 4374
25. subtraction $8, 9, 10,$ ▢, ▢, ▢, ▢, 15
26. division $-64, 32, -16,$ ▢, ▢, ▢, ▢, $\frac{1}{2}$
27. addition $-10, -6, -2,$ ▢, ▢, ▢, ▢, 18
28. multiplication $-1, -2, -4,$ ▢, ▢, ▢, ▢, -128

Checkout

Find each sum.

1. $(-5) + 4 =$ ■
2. $(-4) + (-3) =$ ■
3. $8 + (-12) =$ ■
4. $(-4) + 6 =$ ■

Complete each problem.

5. $(-3) \times (-8) =$ ■
6. $8 \times (-4) =$ ■
7. $(-6) \times 9 =$ ■
8. $8 \times 7 =$ ■
9. $35 \div (-7) =$ ■
10. $(-40) \div (-10) =$ ■
11. $(-36) \div 6 =$ ■
12. $0 \div 5 =$ ■

Find each sum or difference.

13. $\frac{2}{3} + \frac{1}{4}$
14. $\frac{5}{6} + \frac{1}{5}$
15. $\frac{1}{2} + \frac{3}{7}$
16. $\frac{7}{8} + \frac{1}{3}$
17. $\frac{7}{12} - \frac{1}{3}$
18. $\frac{9}{10} - \frac{3}{4}$
19. $\frac{4}{5} - \frac{3}{4}$
20. $\frac{2}{3} - \frac{3}{5}$

21. Marian and Penny together weigh 152 pounds. If Penny weighs 80 pounds, how much does Marian weigh?

22. Ken and Eric together collected 45 stamps. Phil collected 25 stamps and Bruce collected 28 stamps. How many stamps did the 4 boys collect all together?

23. One day Jane baked cupcakes. If each pan held 8 cupcakes and she baked 3 pans of cupcakes, how many cupcakes did she bake that day?

24. Phil has a piece of cord that is 42 feet long. How many five-foot pieces can be cut from the cord?

Part 8 Measuring

Linear and Area Measure

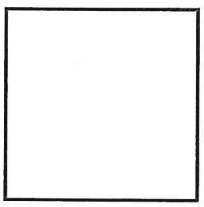
Distance may be measured in units of inches, feet, yards, rods, or miles.

12 inches (in.) = 1 foot (ft.)
36 inches = 1 yard (yd.)
3 feet = 1 yard
$16\frac{1}{2}$ feet = 1 rod (rd.)
5280 feet = 1 mile (mi.)
320 rods = 1 mile

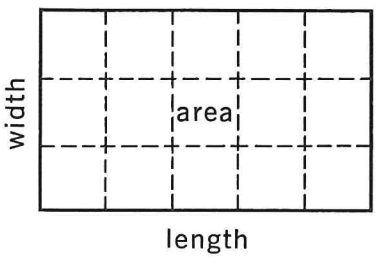
Complete each problem.

1. 7 ft. = ▢ in.
2. 7 yd. = ▢ in.
3. 42 yd. = ▢ ft.
4. 960 rd. = ▢ mi.
5. 6 mi. = ▢ ft.
6. 2 rd. = ▢ ft.
7. 8442 ft. = ▢ yd.
8. 8 mi. = ▢ rd.
9. 4 ft. 3 in. = ▢ in.
10. 7 yd. 2 ft. = ▢ ft.
11. 6 yd. 8 in. = ▢ in.
12. 2 yd. 2 ft. 4 in. = ▢ in.

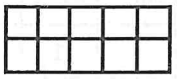
A square with 1-inch sides has an area of one **square inch.** (1 sq. in.)
The square inch is a standard unit of measure for area.

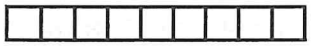


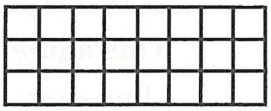
13. Each square represents a 1-inch square.
    a. What is the length of the rectangle shown in the figure?
    b. What is its width?
    c. Length times width = area. What is the area of the rectangle?



1. Find the area of each rectangle by using a square inch (sq. in.) as your unit of measure. Each square represents a 1-inch square.

a. 

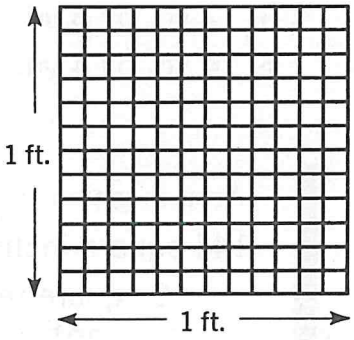
b. 

c. 

2. The length and the width of a rectangle are its **dimensions.** Find the area of rectangles with these dimensions.

a. 4 in. by 2 in.
b. 7 in. by 3 in.
c. 6 in. by 1 in.
d. 2 in. by 5 in.
e. 15 in. by 12 in.
f. 17 in. by 13 in.
g. 22 in. by 10 in.
h. 50 in. by 11 in.
i. 73 in. by 15 in.

3. A square with 1-foot sides has an area of one **square foot** (1 sq. ft.).
This figure represents a square with an area of one square foot. It is divided into 1-inch squares.



a. What is the length in inches of one side of the large square?
b. What is the area in sq. in. of the large square?

To find the area of a rectangle, the length and width must be given in the same units. We give the dimensions of a rectangle that measures 6 ft. by 24 in. as 6 ft. by 2 ft. or 72 in. by 24 in.

4. Find the area of rectangles with these dimensions.

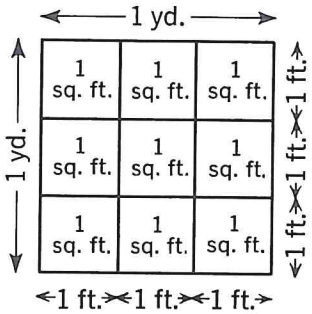
a. 3 ft. by 3 ft.
b. 4 in. by 8 in.
c. 9 ft. by 12 ft.
d. 6 ft. by 12 in.
e. 8 ft. by 48 in.
f. 24 in. by 8 ft.
g. 12 in. by 12 in.
h. 4 in. by 4 in.
i. 16 ft. by 36 in.

1. A square with sides that are one yard in length has an area of one **square yard.** (1 sq. yd.)

   This figure represents a square with an area of one square yard (1 sq. yd.). It is divided into 1-foot squares.

   a. What is the length in feet of one side of the large square?

   b. What is the area in sq. ft. of the large square?



2. Find the area of rectangles with these dimensions.

   a. 8 yd. by 7 yd.
   b. 2 yd. by 6 yd.
   c. 4 yd. by 4 yd.
   d. 8 yd. by 3 ft.
   e. 9 ft. by 12 ft.
   f. 2 yd. by 15 ft.
   g. 6 yd. by 36 in.
   h. 3 yd. by 6 ft.
   i. 48 in. by 2 ft.

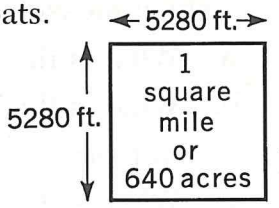
Area is given in square units.

144 square inches (sq. in.) = 1 square foot (sq. ft.)
9 square feet (sq. ft.) = 1 square yard (sq. yd.)
$30\frac{1}{4}$ square yards = 1 square rod (sq. rd.)
160 square rods = 1 acre (A.)
640 acres = 1 square mile (sq. mi.)

Complete each problem.

3. 2 sq. yd. = ▢ sq. ft.
4. 3 sq. ft. = ▢ sq. in.
5. 5 A. = ▢ sq. rd.
6. 4 sq. mi. = ▢ A.
7. 27 sq. ft. = ▢ sq. yd.
8. 576 sq. in. = ▢ sq. ft.
9. 90 sq. yd. = ▢ sq. ft.
10. 1 sq. yd. = ▢ sq. in.
11. $\frac{1}{2}$ A. = ▢ sq. rd.
12. $\frac{1}{2}$ sq. mi. = ▢ A.

1. Mr. Moore owns 640 acres (1 sq. mi.) of farmland. He plants 180 A. of wheat, 160 A. of corn, 200 A. of alfalfa, and 90 A. of oats.



   a. How many acres are left for planting vegetables?
   b. He wants to grow 4 kinds of vegetables. How many acres of each kind can he plant if he assigns the same size plot to each vegetable?

2. Mr. Gordon is painting a wall measuring 15 ft. by 8 ft. One quart of paint covers 100 square feet. Will he need more or less than one quart of paint?

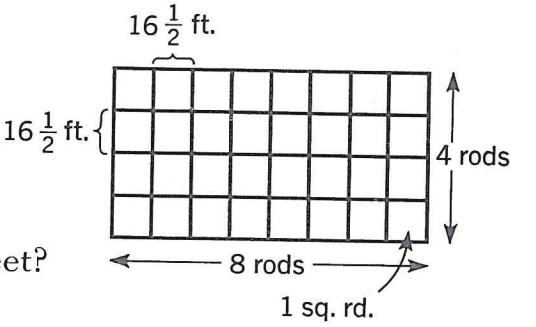
3. Mrs. O'Neil buys a rug measuring 9 ft. by 12 ft. The clerk figures the price by the square yard. What is the area of the rug in square yards?

4. Some boys got permission to use a vacant lot for a baseball diamond. They made a square measuring 75 feet on each side. What is the area of the square?

5. Greg makes a doghouse for his puppy. The floor has the shape of a square with sides measuring 36 inches each.
   a. What is the area of the floor in square inches?
   b. What is the area in square feet?
   c. What is the area in square yards?

6. Lisa is making a blanket. She uses 6-inch squares of cloth and sews them together in rows of 8 for the width and 12 for the length.
   a. What is the width in feet?
   b. What is the length in feet?
   c. What is the area of the blanket in square feet?

1. Mr. Scott has a vegetable garden which measures 8 rods by 4 rods.
   a. What is the area in square rods?
   b. What is the length of the garden in feet?
   c. What is the width of the garden in feet?



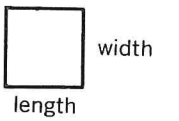
2. Susan plants a flower garden measuring 7 ft. by 3 ft. Connie plants a vegetable garden measuring 6 ft. by 4 ft. Whose garden is larger?

3. Eric has a square chess board measuring 16 in. on each side. Each small square is 2 in. long.
   a. How many small squares are there along one side of the board?
   b. How many small squares are there in all?
   c. What is the area in square inches of the board?

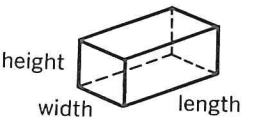
4. What is the area in sq. yd. of a lawn measuring 20 yd. by 25 yd.?

5. Find the area of the following:
   a. a rug measuring 12 ft. by 15 ft.
   b. a floor measuring 5 yd. by 13 yd.
   c. a tennis court measuring 80 ft. by 40 ft.

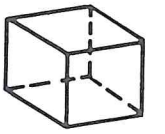
The area of a rectangular region is given in square units. You need to know 2 dimensions (length and width) to find the area.



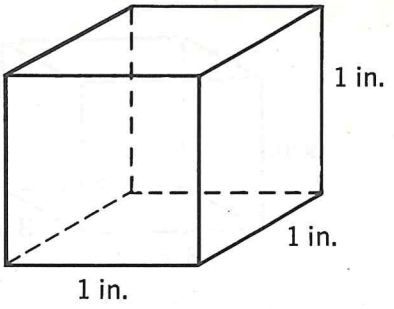
The volume of a rectangular box is given in **cubic** units. You need to know 3 dimensions (length, width, and height) to find the volume.



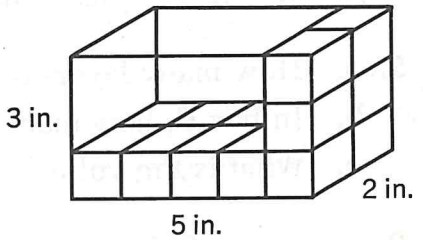
A **cube** is a rectangular solid. It has 6 square faces.



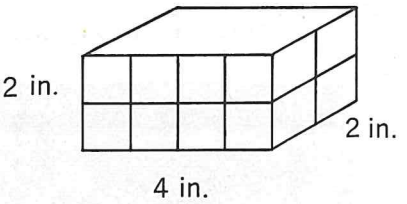
A cubic inch is the volume of a cube that is 1 inch long, 1 inch wide, and 1 inch high.



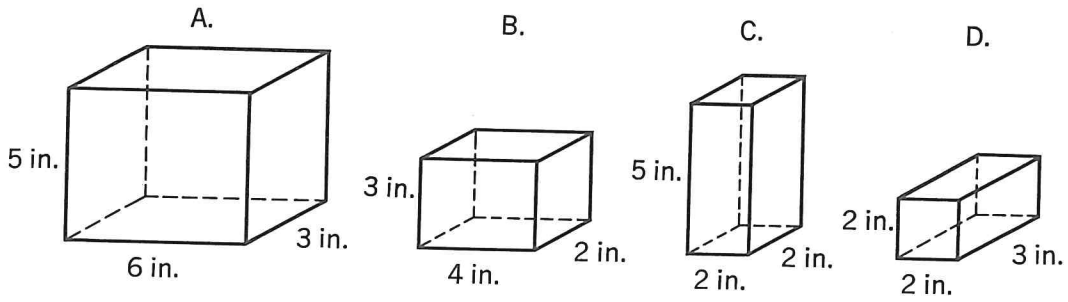
1. a. If you fill the box with 1-inch cubes, how many cubes will be in the bottom layer?
   b. How many cubes will be in the next layer?
   c. How many cubes will be in the top layer?
   d. Is the number of cubes in 1 layer the same as the number of square inches in the area of the base of the box?
   e. How many layers are there?
   f. If you multiply the area times the height, you get the ■.
   g. Length times width times height equals ■.



2. a. If you fill the box with 1-in. cubes, how many cubes will be in the bottom layer?
   b. How many cubes will be in the top layer?
   c. What is the volume of the box?



3. David's tool box is 26 in. long, 22 in. wide, and 24 in. high. Mike's tool box is 36 in. long, 20 in. wide, and 22 in. high.
   a. Which box has the greater volume?
   b. How many more cubic inches of space does the larger box contain?



Write the answer for each question.

1. a. In box A, how many 1-inch cubes are in each layer?
   b. How many layers of cubes will fit in box A?
   c. In cubic inches, what is the volume of box A?

2. a. How many layers of 1-inch cubes will fit in box B?
   b. In box B, how many cubes are in each layer?
   c. What is the volume of box B in cubic inches?

3. a. In box C, how many 1-inch cubes are in each layer?
   b. How many layers of cubes will fit in box C?
   c. What is the volume of box C?

4. a. How many 1-inch cubes are in each layer in box D?
   b. What is the volume of box D?

5. An aquarium is 40 in. wide, 20 in. deep, and 28 in. high. What is the volume of the aquarium in cubic inches?

1. a. How many faces does the cube pictured in Figure A have?
   b. What are the dimensions of each face?
   c. What is the shape of each face?
   d. Since ■ inches equal 1 foot in linear measure, the volume of this cube is 1 cubic foot.
   e. What is the volume of Figure A in cubic inches?
   f. The area of each face is ■ sq. ft.
   g. The surface area of the cube is ■ sq. ft.

Figure A

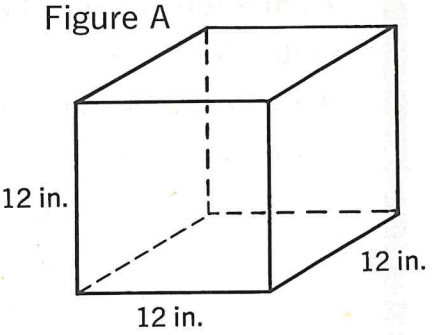
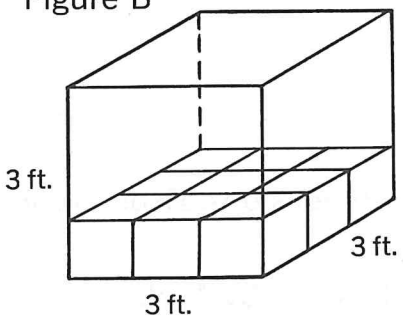


Figure B



2. a. What are the dimensions of the cube pictured in Figure B?
   b. In each layer of 1-foot cubes, how many cubes are there?
   c. How many layers will fit inside the box?
   d. What is the volume of Figure B in cubic feet?
   e. Since ■ feet equal 1 yard, the volume of the box is 1 cubic yard.

3. The dimensions of a living room measure 15 ft. by 12 ft. by 9 ft.
   a. How many cubic feet of space does the room contain?
   b. How many cubic yards of space does this room contain?

4. The dimensions of a storage bin are 8 yd. by 5 yd. by 3 yd.
   a. What is the volume of the storage bin in cubic yards?
   b. What is the volume of the storage bin in cubic feet?

1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.)
27 cubic feet (cu. ft.) = 1 cubic yard (cu. yd.)

To find the volume of a box, the length, width, and height must be given in the same units.

We give the dimensions of a box that measures 2 yd. by 9 ft. by 12 in. any of these ways:

2 yd. by 3 yd. by $\frac{1}{3}$ yd.
6 ft. by 9 ft. by 1 ft.
72 in. by 108 in. by 12 in.

Find the volume of the boxes having the following dimensions. Express each answer in cubic inches or cubic feet.

	Length	Width	Height
1.	7 in.	4 in.	5 in.
2.	10 in.	8 in.	$\frac{1}{2}$ ft.
3.	13 in.	1 ft.	3 in.
4.	2 ft.	16 in.	12 in.

Find the volume of the boxes having the following dimensions. Express each answer in cubic inches, cubic feet, or cubic yards.

5. 1 in. by 1 in. by 1 in.
6. 2 ft. by 2 ft. by 2 ft.
7. 1 ft. by 1 ft. by 12 in.
8. 4 yd. by 9 ft. by 5 yd.
9. 7 ft. by 9 ft. by 48 in.
10. 16 yd. by 8 yd. by 4 yd.
11. 4 yd. by 10 ft. by 6 ft.
12. 1 ft. by 14 in. by 9 in.
13. 10 yd. by 10 yd. by 10 yd.
14. 25 ft. by 20 ft. by 5 yd.

Complete each problem.

15. 3 cu. ft. = ▢ cu. in.
16. 3 cu. yd. = ▢ cu. ft.
17. 6912 cu. in. = ▢ cu. ft.
18. ▢ cu. in. = 1 cu. ft.
19. ▢ cu. ft. = 8640 cu. in.
20. ▢ cu. yd. = 108 cu. ft.
21. 6 cu. ft. = ▢ cu. in.
22. 216 cu. ft. = ▢ cu. yd.

23. Which is the greater volume, 8640 cu. in. or 4 cu. ft.?

For many purposes weight is measured in units of ounces, pounds, or tons.

16 ounces (oz.) = 1 pound (lb.)
100 lb. = 1 hundredweight (cwt.)
2000 lb. = 1 ton (T.)

Complete each problem.

1. 20 oz. = 1 lb. ■ oz.
2. 54 oz. = 3 lb. ■ oz.
3. 79 oz. = 4 lb. ■ oz.
4. 400 lb. = ■ cwt.
5. 10 cwt. = ■ T.
6. 1500 lb. = ■ cwt.
7. $2\frac{1}{2}$ T. = ■ lb.
8. 30 T. = ■ cwt.
9. 6000 lb. = ■ T.

Find each weight.

$$\begin{array}{r} 2 \text{ lb. } 9 \text{ oz.} \\ +3 \text{ lb. } 8 \text{ oz.} \\ \hline 5 \text{ lb. } 17 \text{ oz.} \end{array} = 6 \text{ lb. } 1 \text{ oz.} \qquad \begin{array}{r} 7 \text{ lb. } 3 \text{ oz.} \\ -2 \text{ lb. } 7 \text{ oz.} \\ \hline 4 \text{ lb. } 12 \text{ oz.} \end{array} = \begin{array}{r} 6 \text{ lb. } 19 \text{ oz.} \\ -2 \text{ lb. } 7 \text{ oz.} \\ \hline \end{array}$$

10. $\begin{array}{r} 6 \text{ lb. } 7 \text{ oz.} \\ +6 \text{ lb. } 7 \text{ oz.} \\ \hline \end{array}$
11. $\begin{array}{r} 9 \text{ lb. } 6 \text{ oz.} \\ -2 \text{ lb. } 11 \text{ oz.} \\ \hline \end{array}$
12. $\begin{array}{r} 1 \text{ lb. } 11 \text{ oz.} \\ + \quad 5 \text{ oz.} \\ \hline \end{array}$

The volume of liquids is usually measured in units of teaspoons, tablespoons, fluid ounces, cups, pints, quarts, or gallons.

3 teaspoons (t.) = 1 tablespoon (tbsp.)
2 tablespoons = 1 fluid ounce. (fl. oz.)
8 fluid ounces = 1 cup (c.)
2 cups = 1 pint (pt.)
2 pt. = 1 quart (qt.)
4 qt. = 1 gallon (gal.)

Complete each problem.

13. 4 c. = ■ pt.
14. 12 gal. = ■ qt.
15. 18 pt. = ■ qt.
16. 2 tbsp. = ■ t.
17. 8 c. = ■ fl. oz.
18. 48 fl. oz. = ■ c.
19. 10 pt. = ■ c.
20. 22 qt. = ■ gal.
21. 5 pt. = ■ fl. oz.

22. $\begin{array}{r} 5 \text{ qt. } 2 \text{ pt.} \\ -1 \text{ qt. } 1 \text{ pt.} \\ \hline \end{array}$
23. $\begin{array}{r} 7 \text{ gal. } 3 \text{ qt.} \\ -4 \text{ gal. } 6 \text{ qt.} \\ \hline \end{array}$
24. $\begin{array}{r} 3 \text{ qt. } 1 \text{ pt.} \\ + \quad 3 \text{ pt.} \\ \hline \end{array}$
25. $\begin{array}{r} 7 \text{ pt. } 3 \text{ c.} \\ -2 \text{ pt. } 7 \text{ c.} \\ \hline \end{array}$

Measuring Time

Periods of time are measured in units of seconds, minutes, hours, days, weeks, months, and years. Long periods of time are measured in decades, and centuries.

60 seconds (sec.) = 1 minute (min.)
60 minutes = 1 hour (hr.)
24 hours = 1 day (da.)
7 days = 1 week (wk.)
52 weeks = 1 year (yr.)
12 months = 1 year
10 years = 1 decade
100 years = 1 century

1. Mrs. Larson baked her cake from 4:15 P.M. until 5:05 P.M. How many minutes did her cake bake?

2. Mr. Cooper's plane was due to arrive at 11:35 A.M. but it was 45 minutes late. What time did his plane arrive?

3. Mr. Lawrence drove for 4 hours and averaged 50 miles per hour. How many miles did he drive?

4. A millennium is 100 decades. How many years is this?

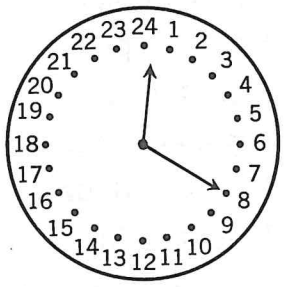
5. In 1519 Magellan sailed around the world in 3 years. How many days was this?

6. The *Graf Zeppelin* was the first airship to cross the Pacific Ocean from west to east. It traveled from Japan to Los Angeles, a distance of 5800 miles, in $3\frac{1}{3}$ days. How many hours was this?

7. It took a Ranger spacecraft about 66 hours to travel from the Earth to the moon.
   a. How many minutes was this?
   b. How many seconds was this?

The 24-hour Clock

The military services, airlines, and other groups who work at all times of the day and night use a 24-hour clock, which measures the time from midnight to midnight.



Four digits are used to express the time. The first two digits tell the hour; the two digits at the right indicate minutes. For example 0530 (read "zero five thirty") is 5:30 A.M. on a twelve-hour clock. 2100 is 21 hours after midnight, or 9 P.M. on a watch. "2100" is read "twenty-one hundred".

1. Change to 12-hour time:
   a. 1545 c. 1650
   b. 2140 d. 0134

2. Change to 24-hour time:
   a. 9:15 A.M. c. 7:19 P.M.
   b. 11:30 P.M. d. 9:15 A.M.

3. Which is earlier in the day?
   a. 1100 or 11 P.M. c. 2100 or 9:15 P.M. e. 1500 or 3:00 A.M.
   b. 1720 or 5:30 P.M. d. 0115 or 1:00 P.M. f. 2310 or 11:00 A.M.

4. The stewardess on Flight #162 will go on duty at 1930. What time will her watch say?

5. If the computer operates from 0300 to 1500, how many hours is this? At what time would it start on a 12-hour clock? At what time would it finish?

6. When it is 8 A.M. in California, it is 3 hours later in New York. If a plane leaves California at 1350 what time will the New York airport clock say when the plane arrives in New York if the flight time is 5 hours?

Checkout

Copy and complete each problem.

1. 1 c. = ▢ pt.
2. 1 qt. = ▢ pt.
3. 1 gal. = ▢ qt.
4. 1 c. = ▢ fl. oz.
5. 1 ft. = ▢ in.
6. 1 yd. = ▢ ft.
7. 1 yd. = ▢ in.
8. 1 mi. = ▢ ft.
9. 1 min. = ▢ sec.
10. 1 hr. = ▢ min.
11. 1 da. = ▢ hr.
12. 1 wk. = ▢ da.
13. 1 yr. = ▢ wk.
14. 1 yr. = ▢ mo.
15. 1 lb. = ▢ oz.
16. 1 T. = ▢ lb.
17. $2\frac{1}{2}$ lb. = ▢ oz.
18. $3\frac{1}{2}$ T. = ▢ lb.

19. It takes Ann half an hour and Betty 40 minutes to walk to school. What time must each leave in order to arrive at 8:30?

20. Judy bought 4 quarts of lemonade, 3 quarts of grape juice, and 5 quarts of ginger ale to make a punch. If she serves 1 pint of punch to each guest, how many people can she serve?

21. Dave and Al kept records of how much lemonade each sold. At the end of a week, Dave's record showed he had sold 3 quarts, 7 pints, and 3 cups. Al had sold 12 pints and 6 cups. Which had sold more?

22. Pete bought 12 oz. of gumdrops, 14 oz. of mints, 11 oz. of jellybeans, and 6 oz. of licorice. The candy cost 3¢ an oz. How much did he spend? How many lb. of candy did he buy?

23. Kevin has 3 rectangular gardens. One plot measures 8 ft. by 23 ft., another 2 yd. by 21 ft., and the third $7\frac{1}{2}$ ft. by 23 ft. He has 19 yd. of fence. Which plot can he fence in? How much fence is left?

24. Joan has 3 pt. of paint. Each pint will cover 46 sq. ft. She wants to paint 2 walls without doors or windows, each measuring 12 ft. by 8 ft. Does she have enough?

Checkout

List the members of each set.

1. $\{n: 3 < n < 7\}$
2. $\{n: n - 1 < 4\}$
3. $\{n: n + 1 < 5\}$
4. $\{n: n < 19 \text{ and is divisible by } 3\}$
5. $\{n: 5 < n < 13 \text{ and } n \text{ is even}\}$
6. $\{n: 11 < n < 23 \text{ and } n \text{ is odd}\}$

Find each sum.

7. $8\frac{8}{10}$ / $8\frac{8}{10}$ / $+8\frac{8}{10}$

8. $4\frac{2}{3}$ / $9\frac{1}{3}$ / $+3\frac{1}{3}$

9. $13\frac{9}{10}$ / $29\frac{4}{10}$ / $+74\frac{8}{10}$

10. $2\frac{3}{4}$ / $3\frac{3}{4}$ / $+1\frac{2}{4}$

Find each product.

11. 4089×9

12. 1678×5

13. 58×35

14. 142×93

15. Find all the factors of each number.
    a. 42 b. 75 c. 31 d. 24

16. Find the L.C.M. of each pair of numbers.
    a. 21 and 35 b. 18 and 12 c. 5 and 4

17. Find the G.C.F. of each pair of numbers.
    a. 32 and 24 b. 63 and 18 c. 25 and 40.

18. List all the prime numbers between 10 and 40.

Part 9 Decimals

We can count by tenths: $\frac{1}{10}, \frac{2}{10}, \frac{3}{10}, \ldots$.
We can write these fractions as **decimals:** .1, .2, .3, ...
We read ".1, .2, .3" as "one tenth, two tenths, three tenths".

1. Copy and complete each pattern.
   a. $\frac{1}{10}, \frac{2}{10}, \frac{3}{10}, \square, \square, \square, \square, \square, \square$
   b. .1, .2, .3, $\square, \square, \square, \square, \square, \square$

We can count by hundredths: $\frac{1}{100}, \frac{2}{100}, \frac{3}{100}, \ldots$.
We can use decimals to count by hundredths: .01, .02, .03, ...
We read ".01, .02, .03" as
"one hundredth, two hundredths, three hundredths".

2. The table shows how we count from .01 $\left(\text{or } \frac{1}{100}\right)$ to .99 $\left(\text{or } \frac{99}{100}\right)$.
Copy the table and complete it.

	.01	.02	.03						
.10	.11								.19
.20									
			.33						
								.48	
					.55				
		.62							
.70									
									.89
.90								.98	

3. Read these numerals aloud. Tell what fraction each is equal to.

a.	.3	c.	.31	e.	.7	g.	.89	i.	.72	k.	.81
b.	.9	d.	.44	f.	.6	h.	.50	j.	.2	l.	.02

1. Express each fraction as a decimal.

a. $\frac{3}{10} =$ ■ c. $\frac{18}{100} =$ ■ e. $\frac{4}{10} =$ ■ g. $\frac{40}{100} =$ ■

b. $\frac{5}{100} =$ ■ d. $\frac{7}{10} =$ ■ f. $\frac{4}{100} =$ ■ h. $\frac{92}{100} =$ ■

Find each sum or difference.

2. a. $\frac{1}{10} + \frac{2}{10}$
   b. $.1 + .2$

3. a. $\frac{2}{10} + \frac{4}{10}$
   b. $.2 + .4$

4. a. $\frac{24}{100} + \frac{16}{100}$
   b. $.24 + .16$

5. a. $\frac{5}{10} - \frac{4}{10}$
   b. $.5 - .4$

6. a. $\frac{13}{100} - \frac{5}{100}$
   b. $.13 - .05$

7. a. $\frac{42}{100} - \frac{31}{100}$
   b. $.42 - .31$

Find each sum or difference. Then give the equation using decimals.

$$\frac{48}{100} - \frac{13}{100} = \frac{35}{100}$$
$$.48 - .13 = .35$$

8. a. $\frac{6}{10} + \frac{3}{10}$
   b. $\frac{1}{10} + \frac{6}{10}$
   c. $\frac{32}{100} + \frac{41}{100}$
   d. $\frac{10}{100} - \frac{6}{100}$

9. a. $\frac{8}{10} - \frac{1}{10}$
   b. $\frac{4}{100} - \frac{2}{100}$
   c. $\frac{19}{100} + \frac{28}{100}$
   d. $\frac{51}{100} - \frac{44}{100}$

10. a. $\frac{7}{10} + \frac{2}{10}$
    b. $\frac{3}{10} + \frac{5}{10}$
    c. $\frac{9}{100} - \frac{3}{100}$
    d. $\frac{3}{100} - \frac{2}{100}$

Choose $<$ or $=$ or $>$.

11. a. .3 ● .8
    b. .7 ● .5
    c. .39 ● .23
    d. .40 ● .50

12. a. .37 ● .45
    b. .07 ● .09
    c. .03 ● .20
    d. .17 ● .08

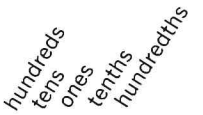
13. a. .01 ● .10
    b. .40 ● .4
    c. .9 ● .95
    d. .89 ● .98

14. a. .81 ● .8
    b. .3 ● .30
    c. .4 ● .26
    d. .5 ● .43

15. a. .42 ● .70
    b. .42 ● .7
    c. .36 ● .30
    d. .36 ● .3

16. a. .6 ● .54
    b. .3 ● .44
    c. .9 ● .81
    d. .1 ● .09

Decimals show place value.



$916.14 = 900 + 10 + 6 + \frac{1}{10} + \frac{4}{100}$

We read "916.14" as "nine hundred sixteen and fourteen hundredths".

1. Express each numeral as shown above.

a. 2.58	d. .36	g. 142.8	j. 4.23
b. 13.06	e. .04	h. 24.39	k. 28.02
c. 9.15	f. .58	i. 16.07	l. 9.73

2. Write in decimal notation.

a. $\frac{6}{100}$	d. $2\frac{3}{10}$	g. $9\frac{83}{100}$	j. $\frac{1}{10}$
b. $\frac{31}{100}$	e. $\frac{4}{100}$	h. $\frac{90}{100}$	k. $\frac{1}{100}$
c. $4\frac{71}{100}$	f. $8\frac{4}{100}$	i. $6\frac{18}{100}$	l. $4\frac{11}{100}$

3. Express each sum in decimal notation.

a. $6 + \frac{4}{10} + \frac{5}{100} =$ ▢	e. $30 + \frac{6}{10} + \frac{9}{100} =$ ▢
b. $8 + \frac{3}{10} + \frac{4}{100} =$ ▢	f. $200 + 4 + \frac{5}{10} =$ ▢
c. $20 + 3 + \frac{8}{10} =$ ▢	g. $50 + 2 + \frac{7}{100} =$ ▢
d. $40 + 2 + \frac{6}{100} =$ ▢	h. $300 + \frac{3}{100} =$ ▢

4. Find each sum or difference.

a. .7 +.2	b. .5 +.4	c. .3 −.2	d. .6 −.3	e. .9 −.4
f. .74 +.25	g. .85 +.14	h. .96 −.03	i. .47 −.32	j. .88 −.71

We regroup when adding decimals as when adding whole numbers.

10 hundredths = 1 tenth

$$\begin{array}{r} {\scriptstyle 1} \\ .73 \\ +.18 \\ \hline .91 \end{array}$$

10 tenths = 1 whole

$$\begin{array}{r} {\scriptstyle 6\ 13} \\ .7\cancel{3} \\ -.18 \\ \hline .55 \end{array}$$

Write each problem in column form, keeping the decimal points in a straight line, so that you can add or subtract the tenths and hundredths. Find each sum or difference.

1. .32 + .41
2. .76 + .14
3. .83 + .16
4. .54 + .39
5. .88 + .09
6. .46 − .05
7. .52 − .25
8. .73 − .28
9. .39 − .18
10. .29 − .10
11. .56 + .26
12. .93 + .05
13. .27 + .27
14. .34 + .49
15. .63 + .08
16. .32 − .16
17. .72 − .63
18. .47 − .26
19. .98 − .39
20. .72 − .64
21. .08 + .07
22. .35 + .56
23. .44 + .38
24. .09 + .03
25. .76 + .15
26. .52 − .27
27. .29 − .14
28. .55 − .37
29. .28 − .19
30. .90 − .45

Divide to express each fraction as a mixed number or as a whole number. Then write each as a decimal.

$\frac{117}{10} = 11\frac{7}{10} = 11.7$ $\qquad$ $\frac{300}{100} = 3 = 3.00$

31. a. $\frac{31}{10}$ b. $\frac{42}{10}$ c. $\frac{19}{10}$ d. $\frac{200}{10}$
32. a. $\frac{186}{10}$ b. $\frac{214}{10}$ c. $\frac{508}{10}$ d. $\frac{191}{10}$
33. a. $\frac{128}{100}$ b. $\frac{327}{100}$ c. $\frac{900}{100}$ d. $\frac{608}{100}$
34. a. $\frac{409}{100}$ b. $\frac{547}{100}$ c. $\frac{222}{100}$ d. $\frac{307}{100}$

Write each problem in column form and find each sum or difference. Then solve each problem using decimals.

$$\begin{array}{r} \frac{17}{10} \\ +\frac{4}{10} \\ \hline \frac{21}{10} \end{array} \qquad \begin{array}{r} 1.7 \\ +\ .4 \\ \hline 2.1 \end{array} \qquad \begin{array}{r} \frac{113}{100} \\ -\frac{108}{100} \\ \hline \frac{5}{100} \end{array} \qquad \begin{array}{r} 1.13 \\ -1.08 \\ \hline .05 \end{array}$$

1. $\frac{9}{10} + \frac{8}{10}$
2. $\frac{24}{10} + \frac{19}{10}$
3. $\frac{37}{10} + \frac{25}{10}$
4. $\frac{196}{100} + \frac{273}{100}$
5. $\frac{23}{100} + \frac{76}{100}$
6. $\frac{177}{100} + \frac{238}{100}$
7. $\frac{405}{100} + \frac{7}{100}$
8. $\frac{930}{100} + \frac{81}{100}$
9. $\frac{9}{10} - \frac{7}{10}$
10. $\frac{35}{10} - \frac{21}{10}$
11. $\frac{42}{10} - \frac{8}{10}$
12. $\frac{103}{100} - \frac{27}{100}$
13. $\frac{59}{100} - \frac{38}{100}$
14. $\frac{938}{100} - \frac{72}{100}$
15. $\frac{735}{100} - \frac{567}{100}$

Write each problem in column form and find each sum or difference.

16. $3.7 + 2.1$
17. $1.2 + 4.3$
18. $2.0 + 1.8$
19. $5.67 + 9.32$
20. $1.08 + 6.44$
21. $9.67 + 23.16$
22. $17.53 + 6.84$
23. $6.38 + 19.45$
24. $14.21 + 7.69$
25. $3.1 - 2.0$
26. $6.3 - 3.4$
27. $18.0 - 6.2$
28. $19.35 - 13.84$
29. $16.29 - 10.88$
30. $7.32 - 1.45$
31. $21.55 - 17.67$
32. $34.51 - 26.73$
33. $14.44 - 11.56$

34. A rectangle is 1.52 in. long and .47 in. wide. What is its perimeter?

35. Penny has \$2.35, Joyce has \$1.77, Pam has \$2.85, and Ginny has \$3.65. They want to buy an anniversary present for their parents that costs \$10.98. Do they have enough money?

36. From Earth, about .59 of the moon's surface is visible. How much more than half of the moon's surface is visible from Earth? (Hint: One half equals .50.)

Choose $<$ or $=$ or $>$.

1. $1.3 \bigcirc 2.0$
2. $1.33 \bigcirc 1.21$
3. $3.81 \bigcirc 3.90$
4. $4.10 \bigcirc 4.08$
5. $.17 \bigcirc .34$
6. $.1 \bigcirc .2$
7. $.31 \bigcirc .29$
8. $.65 \bigcirc .7$
9. $3.2 \bigcirc 3.20$
10. $2.84 \bigcirc 2.75$
11. $2.13 \bigcirc 2.10$
12. $1.08 \bigcirc 1.1$

Sometimes we need to rename decimals.

$$.41 + .5 = .41 + .50 \quad \text{(because } .5 = .50\text{)}$$
$$= .91$$
$$2 - 1.17 = 2.00 - 1.17 \quad \text{(because } 2 = 2.00\text{)}$$
$$= .83$$

Write each problem in column form and find each sum or difference.

13. $1.7 + 2.34$
14. $.6 + .98$
15. $1.3 + 1.49$
16. $.47 + .2$
17. $.39 + .7$
18. $3.61 + 2.9$
19. $.9 - .77$
20. $2.8 - 1.65$
21. $4 - 3.23$
22. $3.17 - 2$
23. $2.45 - 1.1$
24. $7.63 - .8$

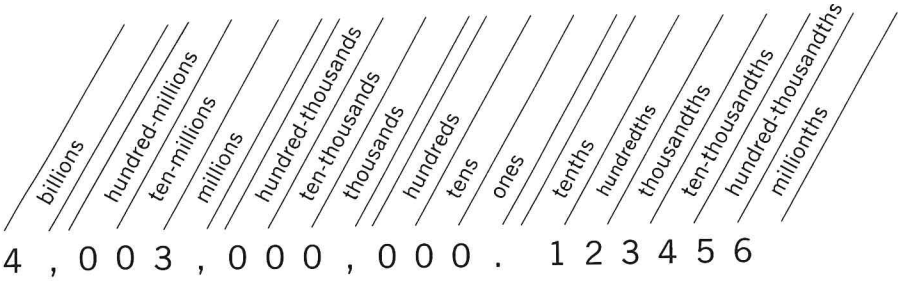
25. The perimeter of a rectangle is 3.36 in. Its width is .68 in. What is its length?

26. Mrs. Gomez gives a clerk a twenty-dollar bill to pay for a dress that costs \$12.98, gloves that cost \$1.75, stockings that cost \$1.25, and a scarf that costs \$1.98. How much change does she get?

27. The perimeter of a quadrilateral is 12.29 ft. Two sides measure 3.47 ft. each, and a third side measures 4.09 ft. How long is the fourth side?

28. The area of a rectangular field is 27 square yards. Its width is 3 yards. Jim has 24.5 yards of fence.
    a. Is this enough to surround the field?
    b. How much more will Jim need, or how much will he have left?

We use decimals to name more place values.



We read "4,003,000,000.123456" as "four billion three million and one hundred twenty-three thousand four hundred fifty-six millionths".
We read the numerals to the right of the decimal point as if we were reading a whole number. Then we give the place value of the last digit.

We read: ".12" as "twelve hundredths".
".078" as "seventy-eight thousandths".
".3012" as "three thousand twelve ten-thousandths".

Read each numeral aloud and give place values as in the example above.

1.	6,143,859,146	5.	349.16375	9.	736,039.72108
2.	1,489,700,000	6.	5,921.3334	10.	92,100.493217
3.	2,000,000,001.4	7.	16,728.00498	11.	297,445.000024
4.	7,000,146,024.26	8.	389,000.3002	12.	3,030,303.030303

Express each decimal as a fraction.

$.2314 = \frac{2314}{10{,}000}$ $.751347 = \frac{751347}{1{,}000{,}000}$

13.	.9	17.	.1237	21.	.00008	25.	.31768
14.	.176	18.	.8205	22.	.000213	26.	.44501
15.	.008	19.	.93167	23.	.006412	27.	.307009
16.	.93	20.	.03071	24.	.017003	28.	.9595

Often we must regroup when adding or subtracting decimals.

10 millionths = 1 hundred-thousandth
10 hundred-thousandths = 1 ten-thousandth
10 ten-thousandths = 1 thousandth
10 thousandths = 1 hundredth
10 hundredths = 1 tenth
10 tenths = 1 whole

1 1 1 1 3.3864 + .6798 4.0662	7 14 .3984 −.1647 .2337	5 9 12 .602 −.179 .423

Find each sum or difference.

1. 4.6089 +3.7218	**2.** .00918 +.06723	**3.** .35912 −.17608
4. 75.902 + 3.146	**5.** .001967 −.000188	**6.** 39.7014 −17.0824
7. .03068 +.10712	**8.** 9.0013 −1.6204	**9.** .34198 −.12634
10. 16.082 +39.198	**11.** .72163 +.11417	**12.** .09643 −.08999

13. Choose < or = or >.

a. .303 ● .33	**f.** .3000 ● .30	**k.** .000020 ● .00002
b. .1717 ● .01717	**g.** .7081 ● .69999	**l.** .12345 ● .123456
c. .6245 ● .6254	**h.** .5115 ● .5151	**m.** .03030 ● 0.303
d. .602 ● .6002	**i.** .3846 ● .3	**n.** .4 ● .2807
e. .58457 ● .6	**j.** .8999 ● .9	**o.** .9999 ● .99997

If a fraction has 10, 100, 1000, 10,000, 100,000, or 1,000,000 as a denominator, we can express the fraction as a decimal.

Sometimes we must first find equal fractions before we can express a fraction as a decimal.

$$\frac{2}{5} = \frac{2 \times 2}{5 \times 2} = \frac{4}{10} = .4 \qquad \frac{3}{25} = \frac{3 \times 4}{25 \times 4} = \frac{12}{100} = .12 \qquad \frac{1}{16} = \frac{1 \times 625}{16 \times 625} = \frac{625}{10{,}000} = .0625$$

Find the missing numerator, and then express the fraction as a decimal.

1. $\frac{1}{2} = \frac{\square}{10}$
2. $\frac{1}{4} = \frac{\square}{100}$
3. $\frac{5}{4} = \frac{\square}{100}$
4. $\frac{3}{4} = \frac{\square}{100}$
5. $\frac{1}{5} = \frac{\square}{10}$
6. $\frac{6}{5} = \frac{\square}{10}$
7. $\frac{3}{5} = \frac{\square}{100}$
8. $\frac{4}{5} = \frac{\square}{100}$
9. $\frac{1}{8} = \frac{\square}{1000}$
10. $\frac{3}{8} = \frac{\square}{1000}$
11. $\frac{5}{8} = \frac{\square}{1000}$
12. $\frac{7}{8} = \frac{\square}{1000}$
13. $\frac{1}{25} = \frac{\square}{100}$
14. $\frac{6}{25} = \frac{\square}{100}$
15. $\frac{9}{25} = \frac{\square}{100}$
16. $\frac{13}{25} = \frac{\square}{100}$
17. $\frac{5}{16} = \frac{\square}{10{,}000}$
18. $\frac{3}{16} = \frac{\square}{10{,}000}$

We can write:

17 of 100 parts as $\frac{17}{100}$ or .17

3 of 50 parts as $\frac{3}{50}$ or $\frac{6}{100}$ or .06

4 of 25 parts as $\frac{4}{25}$ or $\frac{16}{100}$ or .16

Express each as a decimal.

19. 2 of 100 parts
20. 37 of 100 parts
21. 7 of 50 parts
22. 2 of 25 parts
23. 3 of 4 parts
24. 15 of 20 parts
25. 7 of 25 parts
26. 58 of 100 parts
27. 7 of 20 parts
28. 24 of 25 parts
29. 13 of 20 parts
30. 19 of 50 parts

Per Cent

We can express decimals as per cents.

$\frac{1}{100} = .01 = 1\%$ We read "1%" as "one per cent".

$\frac{89}{100} = .89 = 89\%$ We read "89%" as "eighty-nine per cent".

$\frac{4}{25} = \frac{16}{100} = .16 = 16\%$ We read "16%" as "sixteen per cent".

1. Express each decimal as a per cent.

a. .04	c. .68	e. .02	g. .74	i. .98
b. .92	d. .17	f. .15	h. .09	j. .14

2. Tell what each per cent means. 89% means 89 of 100 parts.

a. 73%	c. 13%	e. 41%	g. 65%	i. 8%
b. 28%	d. 2%	f. 50%	h. 20%	j. 100%

3. Choose $<$ or $=$ or $>$.

$\frac{3}{5}$ ● 58% $\frac{3}{5} = \frac{3 \times 20}{5 \times 20} = \frac{60}{100} = 60\%$ so $\frac{3}{5} > 58\%$

a. $\frac{16}{25}$ ● 63%	c. $\frac{1}{50}$ ● 2%	e. $\frac{3}{10}$ ● 29%
b. $\frac{3}{20}$ ● 15%	d. $\frac{9}{10}$ ● 87%	f. $\frac{49}{50}$ ● 99%

4. Express each fraction as a decimal and then as a per cent.

$\frac{375}{1000} = .375 = 37.5\%$ $\frac{625}{10,000} = .0625 = 6.25\%$ $\frac{1}{10,000} = .0001 = .01\%$

a. $\frac{495}{1000}$	c. $\frac{508}{1000}$	e. $\frac{16}{1000}$	g. $\frac{408}{10,000}$
b. $\frac{683}{1000}$	d. $\frac{7}{1000}$	f. $\frac{3426}{10,000}$	h. $\frac{11}{10,000}$

1. There were 100 problems on a test. Mary had 85% of the problems correct. Jack had 94% correct, Fred had 91% correct, and Joyce had 87% correct. How many problems did each have correct? How many problems did each have incorrect?

2. There were 20 problems on a quiz. Anne missed 3, Pete missed 2, Polly missed 1, and Don missed none. What per cent did each miss? What per cent did each have correct?

If Penny missed 4, she missed 4 of 20.

$\frac{4}{20} = \frac{20}{100} = 20\%$ She missed 20%.

$20 - 4 = 16$ She had 16 correct.

$\frac{16}{20} = \frac{80}{100} = 80\%$ She had 80% correct.

3. There were 25 problems on a quiz. Paula missed 1, Jeff missed 2, Al missed 4, and Betty missed 5. What per cent did each have correct? What per cent did each miss?

4. Kent has read 30 pages of a 50-page book. Mike has read 16 pages of a 20-page book. Who has read the greater per cent of his book?

Checkout

Complete each problem.

1. $.63 + .09 + .17 =$ ■
2. $.25 + .36 + .31 =$ ■
3. $.73 + .09 + .11 =$ ■
4. $.33 + .30 + .42 =$ ■
5. $.75 - .45 =$ ■
6. $.89 - .76 =$ ■
7. $.47 - .08 =$ ■
8. $.36 - .19 =$ ■

List the members of each set.

9. $\{n: 109 < n < 111\} =$ ■
10. $\{n: 379 < n < 382\} =$ ■
11. $\{n: 13 < n < 18\} =$ ■
12. $\{24 < n < 27\} =$ ■

Solve each equation.

13. $\blacksquare - \frac{2}{3} = \frac{1}{9}$
14. $\frac{3}{8} - \blacksquare = \frac{1}{8}$
15. $\blacksquare - \frac{1}{4} = \frac{1}{8}$
16. $\blacksquare - \frac{4}{6} = \frac{3}{12}$
17. $\blacksquare - \frac{2}{3} = \frac{3}{12}$
18. $\frac{7}{10} - \blacksquare = \frac{2}{10}$
19. $\blacksquare - \frac{2}{5} = \frac{7}{10}$
20. $\frac{9}{12} - \blacksquare = \frac{1}{12}$
21. $\blacksquare - \frac{4}{6} = \frac{1}{12}$

22. Draw a segment AB about 3 in. long. Construct equilateral triangle ACB. Bisect each angle of the triangle.

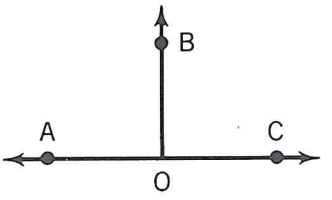
23. Patty made $12\frac{1}{2}$ dozen cookies for her party. She allowed 5 cookies for each guest. How many guests did she expect?

24. The volume of a rectangular box is 24 cu. in. The length of the box is $\frac{1}{3}$ ft., its width is 2 in. What is its height?

25. Each side of a pentagon measures 6.38 in. What is its perimeter?

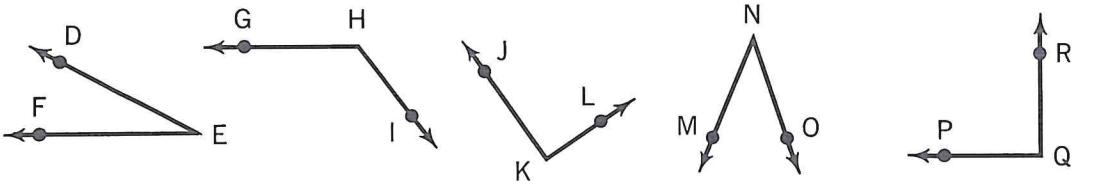
Part 10 Perpendiculars

A right angle has a particular size. Its measure is 90°.
The sum of the measures of two right angles is equal to the measure of a straight angle.



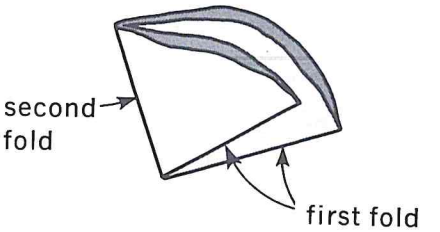
Angle AOB is a right angle.
Angle BOC is a right angle.
AC is a straight line.
Angle AOC is a straight angle.

1. Which are right angles?

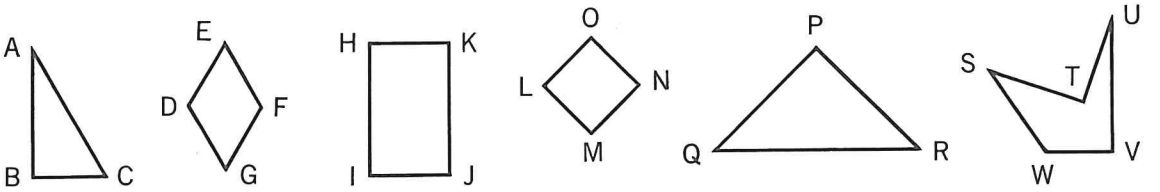


2. a. Which angles in Exercise 1 are obtuse angles?
b. Which angles in Exercise 1 are acute angles?

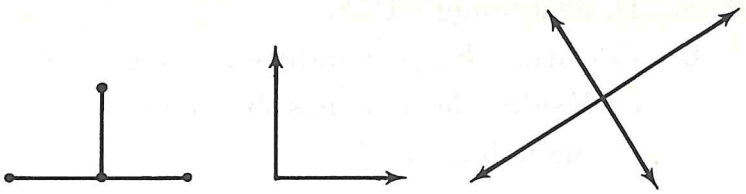
3. a. Fold a piece of paper along a line.
b. Fold it again making the parts of the first fold fit together.
c. These folds form a ▪ angle.



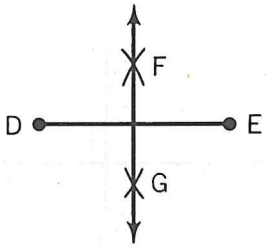
4. a. Name the right angles in the following figures.
b. How many right angles has each rectangle?



Perpendicular lines (or segments or rays) meet at right angles.



1. Problem: Construct the perpendicular bisector of a segment.
   - **a.** Draw segment DE.
   - **b.** Construct above and below $\overline{DE}$ two arcs of a circle with center D. This circle need not pass through E.
   - **c.** Construct two arcs of another circle of the same size with center E, intersecting the first two arcs in F and G. Draw $\overleftrightarrow{FG}$.

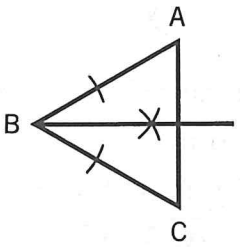


   - **d.** Label as "H" the point of intersection of $\overline{DE}$ and $\overleftrightarrow{FG}$.
   - **e.** Is $\overline{DH}$ congruent to $\overline{HE}$?
   - **f.** Is angle DHF congruent to angle EHF?
   - **g.** Is $\overleftrightarrow{FG}$ perpendicular to $\overline{DE}$?
   - **h.** Draw $\overline{DF}$ and $\overline{FE}$. Is $\overline{DF} \cong \overline{FE}$?

The perpendicular bisector of a segment forms right angles with the segment and passes through its midpoint.

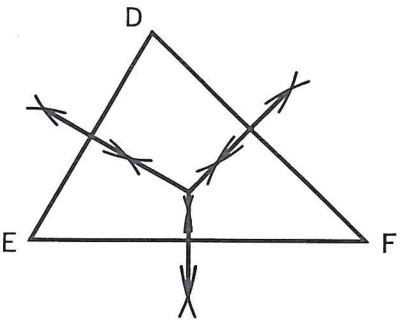
2. Draw three line segments and construct the perpendicular bisector of each one.

3. **a.** Construct equilateral triangle ABC.
   - **b.** Bisect angle ABC.
   - **c.** Is this angle bisector also the perpendicular bisector of side AC?



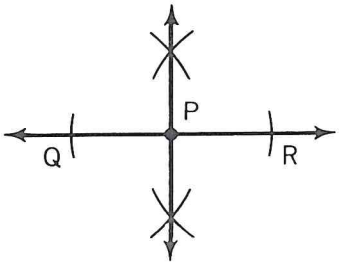
1. **a.** Draw a triangle DEF.

   b. Construct the perpendicular bisector of each side. These 3 lines should intersect in one point.



2. Draw a triangle FGH and construct the perpendicular bisector of each side.

3. Problem: Construct a perpendicular to a given line at a given point.



Method 1:

   a. Draw a line and mark point P on it.

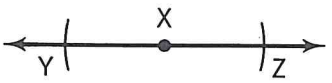
   b. Construct 2 arcs of a circle with center P, intersecting the line in Q and R.

   c. Construct the perpendicular bisector of $\overline{QR}$. The perpendicular passes through midpoint P.

Method 2:

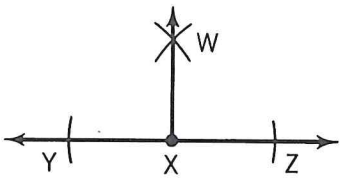
   a. Draw a line and mark point X on it.

   b. Construct 2 small arcs of a circle with center X, intersecting the 2 rays in Y and Z.



   c. Above X construct an arc with center Y.

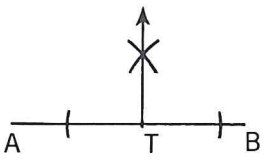
   d. Construct another arc with radius the same length and center Z, intersecting the first arc in W.



   e. Draw the ray from X through W. This is the perpendicular to the given line at given point X.

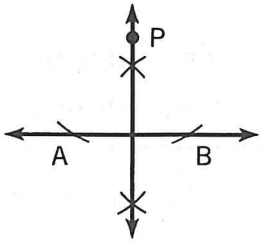
4. **a.** Draw a line segment AB and mark point T on it.

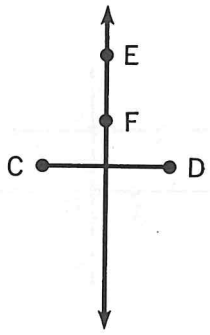
   b. Construct a perpendicular to $\overline{AB}$ at T.



   c. Is $\overline{AT} \cong \overline{BT}$?

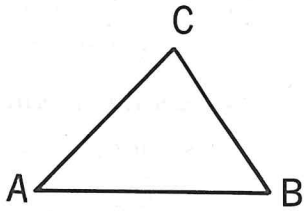
1. Problem: Construct a line perpendicular to a given line through a point not on the given line.
   a. With center P construct an arc that intersects the line at two points A and B.
   b. Construct the perpendicular bisector of $\overline{AB}$. The perpendicular bisector of $\overline{AB}$ is a perpendicular to $\overleftrightarrow{AB}$ which passes through point P.



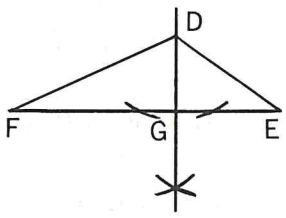


2. a. Draw a segment CD.
   b. Construct the perpendicular bisector of $\overline{CD}$.
   c. Label as "E" a point on the perpendicular bisector.
   d. Is the distance from E to C the same as the distance from E to D?
   e. Choose another point F on the perpendicular bisector.
   f. Is the distance from F to C the same as the distance from F to D?

3. a. Draw a triangle like the one at the right.
   b. Construct the perpendicular from C to $\overline{AB}$.
   c. Label as "D" the point of intersection of the perpendicular and $\overline{AB}$. $\overline{CD}$ is an **altitude** of triangle ABC.



4. a. Draw a triangle DEF.
   b. Construct the perpendicular from D to $\overline{EF}$.
   c. Label as "G" the point of intersection of the perpendicular and $\overline{EF}$. Is $\overline{DG}$ an altitude of triangle DEF?

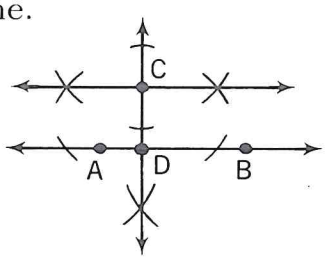


Parallels

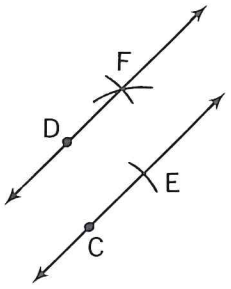
Two lines in the same plane are **parallel** if they never intersect. Two segments or rays are parallel if the lines of which they are a part are parallel.

1. Which pairs of lines, rays, or segments are parallel?

2. Problem: Construct a parallel to a given line through a given point.
   Method A
   a. Draw a line AB and mark point C not on the line.
   b. Construct the perpendicular to $\overleftrightarrow{AB}$ through point C. Label as "D" the point of intersection.
   c. Through point C construct a perpendicular to $\overleftrightarrow{CD}$. This line is parallel to $\overleftrightarrow{AB}$.



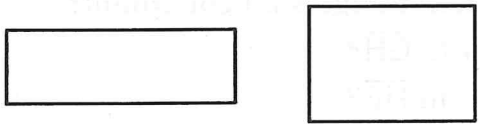
   Method B
   a. Draw a line. Mark a point C on the line and a point D not on the line.
   b. Construct an arc with center C and radius CD intersecting the line. Label the intersection "E".
   c. Using a congruent radius and center D, construct an arc above the line.
   d. Using a congruent radius and center E, construct another arc above the line. Label the intersection of the two arcs "F".
   e. Draw $\overleftrightarrow{DF}$. D is on $\overleftrightarrow{DF}$, and $\overleftrightarrow{DF}$ is parallel to $\overleftrightarrow{CE}$.



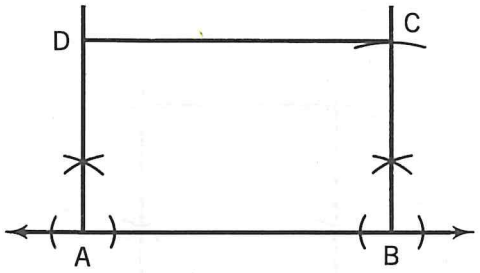
3. Are two sides of any triangle parallel?

4. If two lines are perpendicular to the same line, are they parallel?

A rectangle is a quadrilateral whose 4 interior angles are right angles.

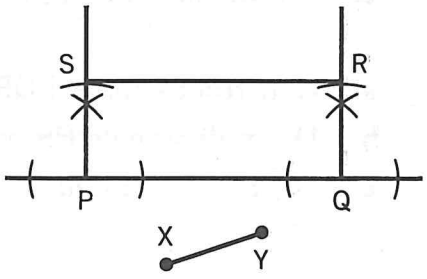


The opposite sides of a rectangle are the same length.



1. Problem: Construct a rectangle.
   - **a.** Draw a line AB.
   - **b.** Construct a perpendicular at A, and mark point D on it.
   - **c.** Construct a perpendicular at B.
   - **d.** Construct an arc with center B and radius the length of $\overline{AD}$, intersecting the perpendicular in C.
   - **e.** Draw $\overline{CD}$. ABCD is a rectangle.

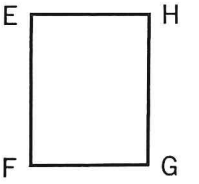
2. Problem: Construct a rectangle with the sides equal in length to given line segments.
   - **a.** Draw line segments PQ and XY.
   - **b.** Extend $\overline{PQ}$, and construct perpendiculars at P and Q.
   - **c.** On the two perpendiculars lay off $\overline{PS}$ and $\overline{QR}$ equal in length to $\overline{XY}$.
   - **d.** Draw segment RS.

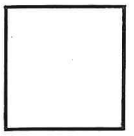


The sides PQ and RS of rectangle PQRS are the same length as $\overline{PQ}$. Sides QR and SP are the same length as $\overline{XY}$.

3. Are the opposite sides of a rectangle always parallel?
4. Are the opposite sides of a rectangle always equal in length?

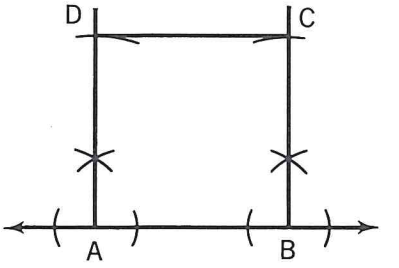
1. **a.** Construct a rectangle EFGH.
   b. Are the four interior angles all congruent?
   c. Is $\overline{EF}$ congruent to $\overline{GH}$?
   d. Is $\overline{FG}$ congruent to $\overline{HE}$?





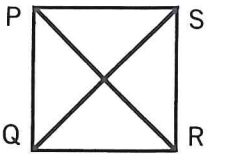
A square is a rectangle whose 4 sides are congruent.

2. Problem: Construct a square.
   a. Draw a line AB.
   b. At A and B construct perpendiculars to $\overleftrightarrow{AB}$.
   c. On these perpendiculars, lay off $\overline{AD}$ and $\overline{BC}$ congruent to $\overline{AB}$.
   d. Draw $\overline{CD}$. ABCD is a square.



3. **a.** Construct several squares.
   b. Are the angles congruent?
   c. Are the sides congruent?

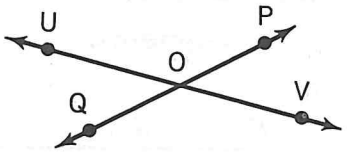
4. **a.** Construct square PQRS.
   b. Draw diagonals PR and QS.
   c. Is $\overline{PR}$ perpendicular to $\overline{QS}$?



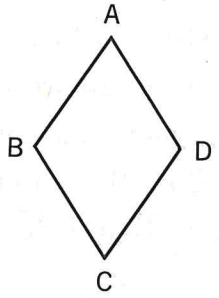
5. **a.** Construct a rectangle UVWX which is not a square.
   b. Draw diagonals UW and VX. Are they perpendicular?

6. **a.** Is a square a rectangle?
   b. Is a rectangle a square?

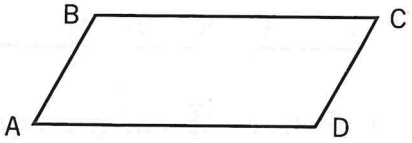
1. Len says that lines PQ and UV are perpendicular because angle POV is congruent to angle UOQ. What mistake has Len made?



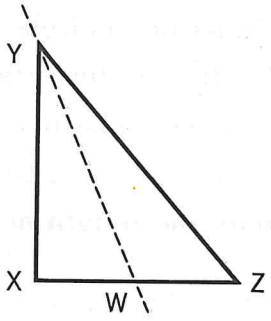
2. Karen says that quadrilateral ABCD is a square because all four sides are congruent. Why is she wrong?



3. Marilyn says that quadrilateral ABCD is a rectangle because $\overline{AB}$ is congruent to $\overline{CD}$, and $\overline{BC}$ is congruent to $\overline{AD}$. Why is she wrong?



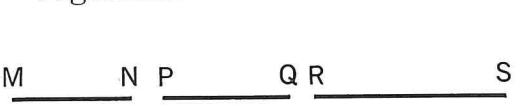
4. Garth says that $\overleftrightarrow{YW}$ is the perpendicular bisector of segment XZ, because W is the midpoint of $\overline{XZ}$, and Y is the vertex of the angle opposite to $\overline{XZ}$ in triangle XYZ. Why is he wrong?

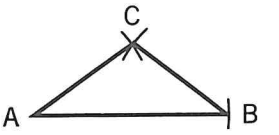


5. Bonnie says that if a quadrilateral has all 4 sides congruent and 1 right angle, the quadrilateral is a square. Is Bonnie right?

6. Steve says that if a quadrilateral has 1 right angle, it is a rectangle. Is Steve right?

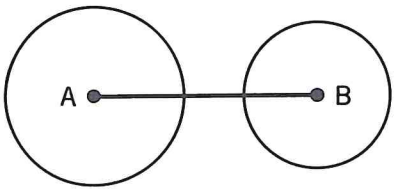
1. Problem: Construct a triangle ABC with sides congruent to three given segments.





   a. Construct $\overline{AB}$ congruent to $\overline{RS}$.
   b. Construct an arc with center A and radius the length of $\overline{PQ}$.
   c. Construct an arc with center B and radius the length of $\overline{MN}$.
   d. Let C be the point of intersection of the two arcs.
   e. Draw $\overline{AC}$ and $\overline{BC}$.

2. Problem: Construct a triangle ABC with sides congruent to the given segments.

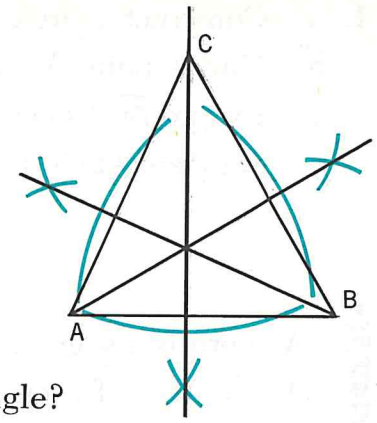


   a. Construct $\overline{AB}$ congruent to $\overline{MP}$.
   b. Construct a circle with center A and radius the length of $\overline{TV}$.
   c. Construct a circle with center B and radius the length of $\overline{KN}$.
   d. Do the circles intersect?
   e. Can we construct a triangle ABC if the circles do not intersect?
   f. Can we have a triangle if the sum of the lengths of two sides is less than the length of the third side?

3. a. Draw three line segments whose lengths are $1\frac{1}{2}$ inches, 2 inches, and 3 inches.
   b. Construct a triangle with sides congruent to these segments.

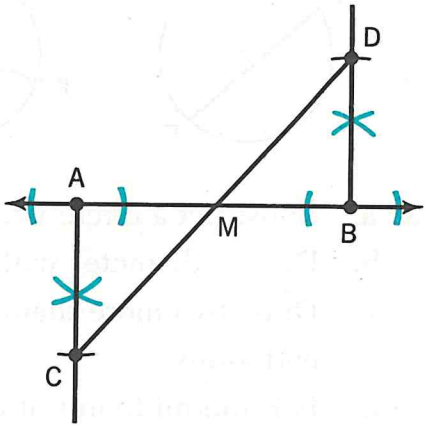
4. a. Draw three line segments whose lengths are 1 inch, 2 inches, and 4 inches.
   b. If you can, construct a triangle with sides congruent to the three given segments.

1. a. Construct triangle ABC having sides whose lengths are $3\frac{3}{4}$ in., $4\frac{1}{4}$ in., and 4 in.



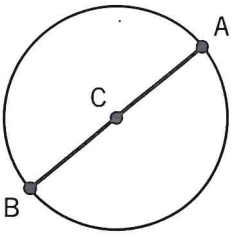
   b. Construct a line perpendicular to $\overline{AB}$ through C.
   c. Construct a line perpendicular to $\overline{AC}$ through B.
   d. Construct a line perpendicular to $\overline{BC}$ through A.
   e. Have you constructed 3 altitudes of the triangle?
   f. Do the three constructed lines intersect in a point?

2. a. Draw a line and mark points A and B on the line.
   b. Construct perpendiculars at points A and B as shown.
   c. Choose a point C on the perpendicular through A as shown.
   d. Lay off $\overline{BD}$ on the perpendicular through B so that $\overline{AC}$ and $\overline{BD}$ are congruent.



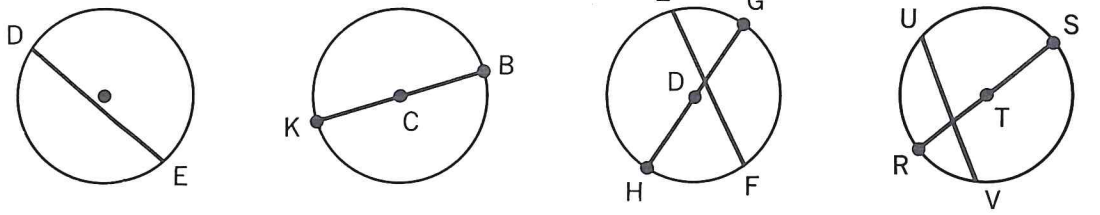
   e. Draw $\overline{CD}$ which intersects $\overline{AB}$ in M.
   f. Are angles ACM and BDM congruent?
   g. Are angles CAM and DBM congruent?
   h. Are angles AMC and BMD congruent?
   i. Do triangles AMC and BMD have the same shape?
   j. Are segments AM and BM congruent?
   k. Are segments CM and DM congruent?
   l. Are the triangles AMC and BMD the same size?
   m. Two triangles that have the same shape and size are **congruent** triangles. Is triangle AMC congruent to triangle BMD?
   n. Is $\overline{AC}$ parallel to $\overline{BD}$?
   o. Is $\overline{AC}$ parallel to $\overline{MB}$?

1. **a.** Construct a circle with center C.
   b. Mark a point A on the circle and draw $\overline{AC}$.
   c. Extend $\overline{AC}$ to point B on the circle.
   Segment AB is a diameter of the circle.

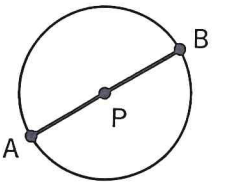


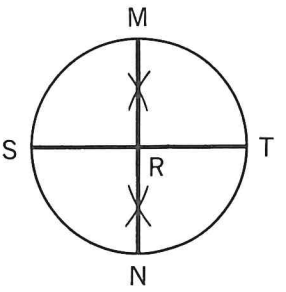
A **chord** is a segment connecting 2 points on a circle.
A **diameter** of a circle is a chord that passes through the center of the circle.

2. Which of the chords shown are diameters?



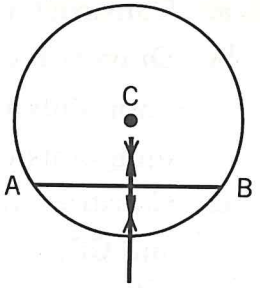
3. **a.** Construct a circle with center P.
   b. Draw a diameter with endpoints A and B.
   c. Draw two more diameters and label the endpoints.
   d. Is P the midpoint of each of these diameters?



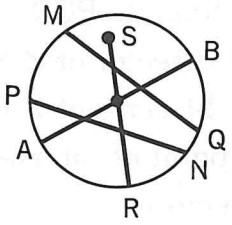


4. **a.** Construct a circle with center R.
   b. Draw diameter ST.
   c. Compare $\overline{SR}$ and $\overline{RT}$. Are they congruent?
   d. Construct diameter MN perpendicular to $\overline{ST}$.
   e. Compare $\overline{MR}$ and $\overline{RN}$. Are they congruent?
   f. $\overline{MN}$ is a ▢ of the circle.
   g. $\overline{MR}$ is a ▢ of the circle.
   h. $\overline{MN}$ is ▢ as long as $\overline{MR}$.

1. a. Construct a circle with center C.
   b. Draw a chord that is not a diameter, and label the endpoints "A" and "B".
   c. Construct the perpendicular bisector of $\overline{AB}$.
   d. Does the perpendicular bisector of $\overline{AB}$ pass through C?

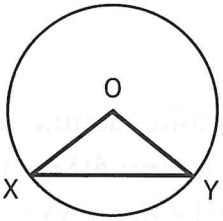


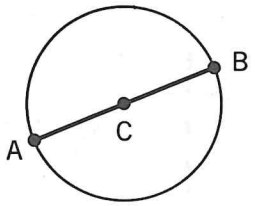
2. a. Repeat Exercise 1 with three different circles of different sizes.
   b. Does the perpendicular bisector of the chord always pass through the center of the circle?



3. a. Which of the line segments in the figure is a diameter of the circle?
   b. Which of the line segments in the figure are chords of the circle?

4. a. Construct a circle with center O.
   b. Draw a chord XY of the circle.
   c. Draw segments XO and YO.
   d. Is the triangle XOY isosceles? Why?

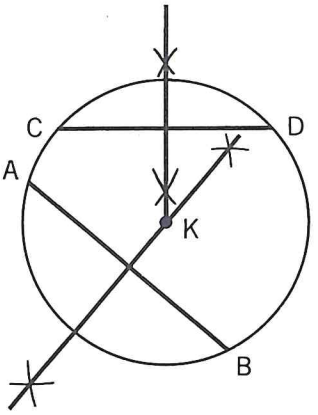


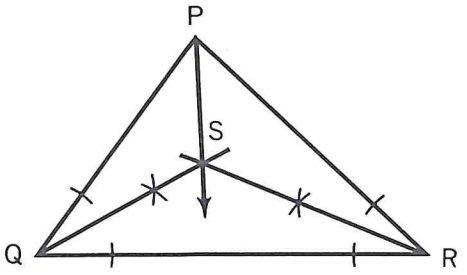


5. Problem: Given a line segment, construct a circle with the segment as a diameter.
   a. Draw $\overline{AB}$.
   b. Find the midpoint C of $\overline{AB}$.
   c. Construct a circle with center C and radius CA.

6. a. Can two radii of a circle be parallel?
   b. Can two diameters of a circle be parallel?
   c. Can two chords of a circle be parallel?

1. **a.** Construct a circle with center K.
   b. Draw two chords of the circle. Label the endpoints of one "A" and "B" and the endpoints of the other "C" and "D".
   c. Construct perpendicular bisectors of $\overline{AB}$ and $\overline{CD}$.
   d. What do you notice about the point of intersection of these two perpendicular bisectors?

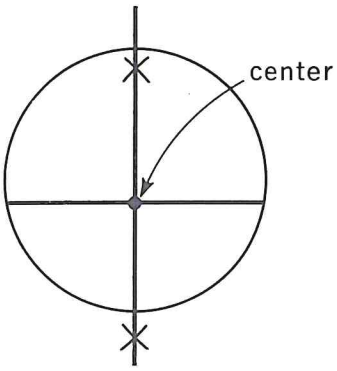




2. **a.** Draw a large triangle PQR.
   b. Construct the bisector of ∠ PQR.
   c. Construct the bisector of ∠ QRP.
   d. Let S be the point of intersection of these two bisectors.
   e. Draw the ray from P through S.
   f. Does $\overrightarrow{PS}$ bisect ∠ RPQ?

3. Don told Juanita that he constructed a triangle with sides that were 10 in., 20 in., and 40 in. in length. Why did Juanita know that Don had made a mistake at some point, either in drawing the triangle or in measuring the lengths of the sides?

4. Marvin says that he has found a simple way to find the center of a circle. This is his method.
   a. Draw a chord of the circle.
   b. Find the midpoint of the chord.
   c. This midpoint is the center of the circle. What is wrong with Marvin's method?



Checkout

Answer each question.

1. In arithmetic, which operations are commutative?
2. In arithmetic, which operations are associative?
3. Is multiplication distributive over addition?

List the whole numbers in each set.

4. $\{n: 117 < n < 123\} =$ ■
5. $\{n: 259 < n < 264\} =$ ■
6. $\{n: 1098 < n < 1101\} =$ ■
7. $\{n: 777 < n < 782\} =$ ■

Complete each problem.

8. $\frac{8}{12} = \frac{\square}{3}$
9. $1\frac{2}{5} = \frac{\square}{5}$
10. $\frac{19}{4} = 4\frac{\square}{4}$
11. $8\frac{1}{3} = \frac{\square}{3}$
12. $\frac{2}{9} = \frac{\square}{27}$
13. $\frac{5}{6} = \frac{\square}{30}$

14. Debbie has $2\frac{1}{4}$ yards of material. She needed 4 yards to make a dress How many more yards must she buy?

15. A rectangle measures 4.6 ft. by 3.2 ft. What is its perimeter?

16. On Monday the temperature was 72°, on Tuesday it was 78°, and on Wednesday it was 75°. What was the average temperature for the three-day period?

17. From the end of Tom's first finger to the first joint is $\frac{7}{8}$ in. long. A pencil is 5 times this length. How long is the pencil? How much shorter than 5 in. is this?

Part 11 Mathematical Sentences

English Sentence	Mathematical Sentence
Bert and Ken have 5 model planes.	$b + k = 5$
Ken has 2 model planes.	$k = 2$
Bert has more model planes than Ken.	$b > k$
Bert has 2 more model planes than Ken.	$b = k + 2$
Ken has fewer model planes than Bert.	$k < b$
Ken has 2 fewer model planes than Bert.	$k = b - 2$
Ken has twice as many planes as Bert.	$k = 2 \times b$

Choose the mathematical sentence that best matches each English sentence.

1. Bill and Tom together have 9 pets.
2. Bill has 2 fewer pets than Tom.
3. Tom has 2 more pets than Bill.
4. Betty has the same number of books as Therese.
5. Ted practices 2 hours less than Bob.
6. Terry has twice as many stamps as Bart.
7. Barbara has saved 9 more dimes than Toni.
8. Brett spent less time studying for the test than Tom.
9. Bonnie spent more time studying for the test than Tressie.
10. Beulah has gone to school 2 more years than Tammie.

A. $t = b - 2$
B. $b = t + 9$
C. $b = t$
D. $b = t + 2$
E. $t = 2 \times b$
F. $b < t$
G. $b + t = 9$
H. $t = b + 2$
I. $b > t$
J. $b = t - 2$

Write a mathematical sentence for each English sentence.

Marie has more books than Tressie.	$m > t$
Bob weighs 2 pounds less than Fred.	$b = f - 2$

1. Donna practices twice as long as Linda.
2. Barbara weighs 6 pounds more than Joanne.
3. Kate and Marilyn together have 60 cents to spend.
4. Dick has 5 more rocks in his collection than Paul.
5. Jerry has 4 fewer rocks in his collection than Paul.
6. Marla has 2 sisters.
7. Jill and Freda have saved the same amount of money as have Peggy and Toni.
8. Ralph, Charles, and Jim together own 9 model planes.
9. Jack has 3 times as many books as Mike.
10. Pete has twice as many books as Jack and Mike have together.
11. Phyllis bought 6 more oranges than Marilyn.
12. Sue weighs less than Ann.
13. Bill studied for 2 hours.
14. Pat has 4 fewer books than Ted.
15. Frances and Kathy together read 23 books.
16. Harry rakes more leaves than Mike and Stan together.

Variables

The letter that stands for an unknown number is called a **variable.**

In 'x + 4 = 6', 'x' is the variable.

In 'x + y < 9', 'x' and 'y' are variables.

What variable (or variables) is used in each of these mathematical sentences?

1. $14 \div m = 7$
2. $a < b + 3$
3. $9 \times (2 + t) = 45$
4. $f \neq g \times 3$
5. $m > 0$
6. $p = 16 \times (41 + 28)$

Write a mathematical sentence to match each English sentence. Tell what each variable stands for.

7. Joyce has 2 more apples than George.
8. Sue has 3 fewer brothers than Francie.
9. February had more rainy days than March and April together.
10. To convert feet to inches, multiply the number of feet by 12.
11. November has fewer days than August.
12. Jupiter is more than twice as large as Venus.
13. Al has 4 more peaches than Hank and Joe together.
14. Anne and Hal together have more than twice as many books as Betty.

When we have an addition problem in which we add the same number many times, we may change this to a multiplication problem.

$k + k + k = 3 \times k$ $\quad s + s + s + s + s = 5 \times s$

If possible, change each of these to a multiplication problem.

15. $7 + 7 + 7 + 7$
16. $r + r + r + r + r$
17. $5 + 6 + 7 + p$
18. $p + p + p$
19. $r + s + t$
20. $2 + 2 + 2 + 2$

Solving Equations

Solve each problem by using an equation. Check each problem, and then answer the question.

Jack and Jerry have the same number of sweaters. Together they have 6 sweaters. How many sweaters does each boy have?

$n + n = 6$ Check: $3 + 3 = 6$

$2 \times n = 6$

$n = 3$ Jack and Jerry each have 3 sweaters.

1. Pamela, Ted, Susan, and Larry each have the same number of books in their desks at school. Together, they have 24 books. How many books does each child have in his desk?

$n + n + n + n = 24$ Check: ■ + ■ + ■ + ■ = 24

$4 \times n = 24$

$n =$ ■ Each child has ■ books.

2. Mary, Steven, and Jennifer each ride their bikes the same number of miles from their houses to school. They ride a total of 9 miles. How many miles does each child ride to get to school?

3. Tim, Peter, Mark, Sam, and Don together bought 50¢ worth of licorice. Each boy put in the same amount of money. How much money did each boy spend?

4. Patty, Joanne, Sandra, Carolyn, Beth, and Julie went to the movies together. It cost $3.90 for all of their tickets. How much did one ticket cost?

5. Andy, Rhonda, Linda, and Sid went to a party. Each of the children had the same number of cookies. Together they had 20 cookies. How many cookies did each child have?

Sometimes we can change only part of an expression to a multiplication problem as in these examples.

$4 + 4 + 4 + 9 = (3 \times 4) + 9$
$8 + k + 8 + 8 = (3 \times 8) + k$
$3 + m + 3 + m + 7 = (2 \times 3) + (2 \times m) + 7$

If possible, change all or part of each problem to a multiplication problem.

1. $6 + 6 + 6$
2. $5 + 2 + 2 + 2 + 2$
3. $8 + r + r$
4. $7 + 7 + m + 7$
5. $5 + 5 + g + g + 5$
6. $6 + 4 + 6 + 4 + 6$
7. $9 + 9 + f + 9 + f$
8. $4 + 4 + 4 + 4$
9. $b + c + d + e$
10. $s + s + t + u + t$
11. $2 + 4 + m + p$
12. $6 + 6 + f + f + f$

In a multiplication problem, we can sometimes leave out the multiplication sign, if this does not change the meaning.

$f + f + f + f = 4 \times f$ $= 4f$	"$4 \times f$" and "$4f$" both mean 4 times f.
$6 + 6 + 6 + 6 = 4 \times 6$	"4×6" means 4 times 6 but "46" means forty-six. We can not leave out the multiplication sign in this case.
$m \times (n + p) = (m \times n) + (m \times p)$ $= (mn) + (mp)$	"$m \times n$" and "mn" both mean m times n, "$m \times p$" and "mp" both mean m times p.
$m \times (n + p) = m\ (n + p)$	"$m \times (n + p)$" and "$m\ (n + p)$" both mean m times $(n + p)$.

In which of these problems is it possible to leave out the multiplication sign without changing the meaning?

13. 3×4
14. $h \times m$
15. $7 \times r$
16. $s \times t$
17. $9 \times k$
18. 5×6
19. 25×43
20. $b \times (q + r)$

Solve each problem by using an equation. Solve the equation and answer each question.

Jean has twice as many headbands as Natalie. Jean has 8 headbands. How many headbands does Natalie have?

Instead of: $2 \times n = 8$ We write: $2n = 8$

$n = 4$ $n = 4$

Natalie has 4 headbands.

1. Each day Carl delivers four times as many newspapers as Mark. Carl delivers 108 papers. How many does Mark deliver?

   $4m = 108$

   $m =$ ■ Mark delivers ■ newspapers.

2. At the carnival Jake won three times as many things as Darryl won. Jake won 12 things. How many things did Darryl win?

3. In craft class Ed made twice as many things as Burt. Ed made 10 things. How many things did Burt make?

4. Evelyn made 5 times as many cookies as Kim. Evelyn made 120 cookies. How many did Kim make?

5. Some girls in Mr. Morgan's class made paper flowers to decorate for a party. Julia made 4 times as many flowers as Sandra. Julia made 24 flowers. How many flowers did Sandra make? Sandra made 3 times as many as Frieda. How many flowers did Frieda make?

6. During her vacation Frances hiked twice as far as Janice hiked during hers. Frances hiked 34 miles. How far did Janice hike?

Using the Distributive Law

The distributive law says that $mp + np = (m + n)\,p$

We can use this law to simplify expressions.

$$4m + 2m + 3m + m = (4 + 2 + 3 + 1)m$$
$$= 10m$$

We can also use the distributive law to simplify subtraction problems.

First, rewrite the problem as an addition problem.

$$6m - 4m = 6m + (-4)m \qquad r - 6r = r + (-6)r$$
$$= \big(6 + (-4)\big)m \qquad = \big(1 + (-6)\big)r$$
$$= 2m \qquad = -5r$$

Use the distributive law to simplify these expressions.

1. a. $6m + 5m$
   b. $-2r + 8r$
   c. $9s - 4s$
   d. $3t - 3t$
   e. $f - 7f$

2. a. $2p + 4p$
   b. $-w + 9w$
   c. $8r - r$
   d. $6e + 9e$
   e. $-7m - 4m$

3. a. $6p + 6p$
   b. $g + 4g$
   c. $3d + (-8)d$
   d. $-7k - 3k$
   e. $9t - 9t$

We can simplify expressions if the distributive law applies.

We can simplify: $3m + 4m = (3 + 4)m$
$= 7m$

We can not simplify: $3m + 4p$ or $3 + 4p$

Simplify the expressions to which we can apply the distributive law.

4. a. $8m + 9$
   b. $6r + 3r$
   c. $-5t - 8s$
   d. $-6m - 4m$
   e. $9t + 3t + 4t$

5. a. $2r - 7m$
   b. $8g - 7g$
   c. $9s + 2a$
   d. $4a + 9b$
   e. $8r + 5s + 7t$

6. a. $-4c - 9c$
   b. $r + 4$
   c. $-8n + (-7)$
   d. $4g - 2f$
   e. $6a + 5a - 2a$

Write an equation for each problem. Answer each question.

Mrs. Lang had a party for the children in the neighborhood. Three Martin children, five Thomas children, one Larsen child and four Olsen children came. Mrs. Lang bought 26 favors so that she could give the same number of favors to each child. How many favors did each child get?

Let f equal the number of favors each child got.

$3f + 5f + f + 4f = 26$

$(3 + 5 + 1 + 4)f = 26$

$13f = 26$

$f = 2$ Each child received 2 favors.

1. Mr. Norris made feather hats for the people in the play. He had 28 feathers. He used the same number of feathers to make each hat. He made 3 hats for the princesses, 2 hats for the knights, 5 hats for the knaves, and 4 hats for the princes. How many feathers did he put in each hat?

2. The business manager bought sets of science reference books for the schools in the district. He ordered 4 sets for Lincoln School, 3 sets for Lowell School, 6 sets for Greenway School, and 2 sets for Naco School. One hundred twenty books arrived. How many books were in each set?

3. Franklin was selling tickets to the Boy Scout Carnival. He sold 2 tickets to the Perez family, 8 tickets to the McArthurs, 5 tickets to the Clevelands, and 1 ticket to Mrs. Frey. He collected $12.00. How much did each ticket cost?

4. Edith collected silver spoons from each state she visited during her summer vacation. She bought 2 spoons in Wisconsin, 2 spoons in Illinois, 1 spoon in Ohio and 3 spoons in Minnesota. She spent $14.00 on the spoons. What was the average price of each spoon?

We can add the same number to both sides of an equation without changing the value of the equation.

$$\begin{array}{rl} x - 6 = & 9 \\ + \quad 6 = & 6 \\ \hline x \quad = & 15 \end{array} \qquad \begin{array}{rl} 2m - 4 = & 12 \\ + \quad 4 = & 4 \\ \hline 2m \quad = & 16 \\ m \quad = & 8 \end{array}$$

Solve each equation using addition.

1. $m - 9 = 10$
2. $n - 14 = 32$
3. $g - 4 = 0$
4. $3t - 2 = 7$
5. $5k - 4 = 26$
6. $4b - 3 = 31$
7. $2g - 3 = 17$
8. $6m - 2 = 10$
9. $9b - 9 = 18$

We can subtract the same number from both sides of an equation without changing the value of the equation.

$$\begin{array}{rl} x + 6 = & 14 \\ - \quad 6 = & 6 \\ \hline x \quad = & 8 \end{array} \qquad \begin{array}{rl} 2m + 3 = & 11 \\ - \quad 3 = & 3 \\ \hline 2m \quad = & 8 \\ m \quad = & 4 \end{array}$$

Solve each equation using subtraction.

10. $m + 9 = 18$
11. $r + 7 = 12$
12. $s + 1 = 3$
13. $2g + 3 = 13$
14. $5f + 4 = 14$
15. $3p + 2 = 20$
16. $4a + 5 = 33$
17. $6k + 2 = 38$
18. $8t + 6 = 14$

Solve each equation. First decide whether to use addition or subtraction.

19. $t - 8 = 14$
20. $s + 7 = 13$
21. $g - 14 = 38$
22. $3r + 5 = 20$
23. $2f - 8 = 16$
24. $5b + 1 = 31$
25. $4k - 9 = 27$
26. $2m - 7 = 7$
27. $7g + 4 = 39$

28. Which equations in Exercises 19-27 did you solve by using addition?
29. Which equations in Exercises 19-27 did you solve by using subtraction?

Since we have studied negative numbers, we can add a negative number to both sides of an equation instead of subtracting a positive number.

$$\begin{array}{rl} x + 6 = & 14 \\ + \quad -6 = & -6 \\ \hline x \quad = & 8 \end{array} \qquad \begin{array}{rl} 5d - 8 = & 7 \\ + \quad 8 = & 8 \\ \hline 5d \quad = & 15 \\ d \quad = & 3 \end{array}$$

Decide what number to add to both sides of the equation in order to solve the problem, and then solve each equation.

1. $m + 9 = 18$
2. $r + 7 = 12$
3. $s + 3 = 1$
4. $2g + 3 = 13$
5. $5f + 4 = 14$
6. $3p + 2 = 20$
7. $6m - 2 = 10$
8. $9b - 9 = 18$
9. $g - 4 = 0$

First simplify each expression. Then solve and check each equation.

$m + 2m + 5m + 3 = 19$, so

$$\begin{array}{rl} 8m + 3 = & 19 \\ -3 = & -3 \\ \hline 8m \quad = & 16 \\ m \quad = & 2 \end{array}$$

We can check that $m = 2$ by substituting in the equation.

$$\begin{aligned} m + 2m + 5m + 3 &= 2 + (2 \times 2) + (5 \times 2) + 3 \\ &= 2 + 4 + 10 + 3 \\ &= 19 \end{aligned}$$

10. $6f + f + f + 2 = 26$
11. $4 + m + m + 5 = 15$
12. $8a + 6a + 0 = 42$
13. $3r + r + 2 + 6 = 32$
14. $6p + (-4) + (-8) = 24$
15. $4q - 3q + 2q = 48$
16. $3s + 2s - 8 = 27$
17. $5n + 3n + 7n = 48$
18. $q + q - 6 = 20$
19. $8n + n + 7n = 64$
20. $2 + 4 + s + s + s = 30$
21. $-8 - 5 + g + 2g = 2$
22. $3s + s + (-s) - 8 = 10$
23. $6t - 5t = -3$
24. $7r + 4r - 9r = 16$
25. $4p + p - 2 = 13$

Use an equation to solve each problem. Tell what each variable stands for.

Linda and Marianne were carving figures out of soap. Linda carved 1 more than twice as many figures as Marianne. Together the girls carved 16 figures. How many did each carve?

Let f be the number of figures Marianne carved.

$2f + 1$ is the number of figures Linda carved.

$$f + 2f + 1 = 16$$

So
$$\begin{aligned} 3f + 1 &= 16 \\ -1 &= -1 \\ \hline 3f &= 15 \\ f &= 5 \end{aligned}$$

Marianne carved 5 figures.

Linda carved $(2 \times 5) + 1$ or 11 figures.

1. Together, Mr. Montez and his cousin took to market 30 bushels of string beans from their vegetable farms. Mr. Montez took 2 more than 3 times as many bushels as his cousin. How many bushels did each man take?

2. Jacob's coin collection is 5 less than 3 times as big as Esther's. Together they have 71 coins in their collections. How many coins are in each collection?

3. Don, Pierre, and Will earned money by working in their neighbor's yards. Don earned 2 times as much money as Will. Pierre earned 4 times as much money as Will. Together they earned $28. How much did each boy earn?

4. Jeannie, Vivian, and Clara had a contest standing on their heads. Vivian stood on her head 1 second more than twice as long as Clara. Jeannie stood on her head 3 times as long as Clara. Together they stood on their heads 7 seconds. How long did each girl stand on her head?

Checkout

Find each sum.

1.	2.	3.	4.
82	92	346	739
36	18	285	817
45	55	497	564
85	73	364	786

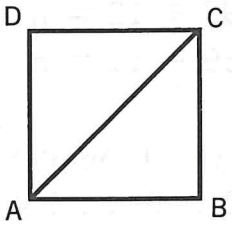
Find each product.

5.	6.	7.	8.
486 × 7	672 × 9	72 × 36	927 × 48

Complete each problem.

9. $\square - \frac{2}{5} = \frac{3}{10}$
10. $\frac{10}{12} - \square = \frac{1}{12}$
11. $\square - \frac{3}{4} = \frac{1}{12}$
12. $\frac{5}{9} - \square = \frac{2}{9}$
13. $\square - \frac{1}{3} = \frac{2}{9}$
14. $\frac{9}{10} - \square = \frac{1}{10}$
15. $\frac{7}{8} - \square = \frac{1}{8}$
16. $\square - \frac{3}{4} = \frac{1}{8}$
17. $\frac{4}{12} - \square = \frac{1}{12}$

18. a. Draw a segment AB.
    b. Construct a square ABCD.
    c. Draw diagonal AC.
    d. Construct the circle with diagonal AC as diameter.
    e. Do vertices B and D lie on the circle?



Change all or part of each addition problem to a multiplication problem. Then, if possible, leave out the multiplication sign.

19. 2 + 2 + 2 + 2
20. 5 + m + m + 5 + 5
21. 16 + 16 + 16
22. 9 + 9 + s + s + 9 + s

Part 12 Using Division

1. Find each quotient. Check each problem by multiplication.

a. $40 \div 8 =$ ▢	e. $96 \div 12 =$ ▢	i. $81 \div 9 =$ ▢
b. $28 \div 7 =$ ▢	f. $63 \div 9 =$ ▢	j. $84 \div 7 =$ ▢
c. $36 \div 4 =$ ▢	g. $42 \div 7 =$ ▢	k. $90 \div 10 =$ ▢
d. $72 \div 9 =$ ▢	h. $56 \div 8 =$ ▢	l. $144 \div 12 =$ ▢

2. Solve each equation. Use the distributive law for multiplication to check each answer.

$46 \div 7 = n$

$n = 6\frac{4}{7}$

Check: $7 \times 6\frac{4}{7} = 7 \times \left(6 + \frac{4}{7}\right)$

$= (7 \times 6) + \left(7 \times \frac{4}{7}\right)$

$= 42 + 4$

$= 46$

a. $38 \div 7 = n$	e. $70 \div 6 = n$	i. $63 \div 8 = n$
b. $46 \div 9 = n$	f. $49 \div 8 = n$	j. $52 \div 7 = n$
c. $97 \div 12 = n$	g. $20 \div 3 = n$	k. $13 \div 5 = n$
d. $59 \div 11 = n$	h. $18 \div 7 = n$	l. $29 \div 6 = n$

3. Copy and complete each problem.

a. $3 \times 8 < 26 < 3 \times 9$	i. $8 \times \_\_ < 67 < 8 \times \_\_$
b. $6 \times \_\_ < 51 < 6 \times \_\_$	j. $7 \times \_\_ < 32 < 7 \times \_\_$
c. $8 \times \_\_ < 36 < 8 \times \_\_$	k. $9 \times \_\_ < 46 < 9 \times \_\_$
d. $4 \times \_\_ < 13 < 4 \times \_\_$	l. $6 \times \_\_ < 28 < 6 \times \_\_$
e. $5 \times \_\_ < 38 < 5 \times \_\_$	m. $6 \times \_\_ < 38 < 6 \times \_\_$
f. $7 \times \_\_ < 54 < 7 \times \_\_$	n. $8 \times \_\_ < 50 < 8 \times \_\_$
g. $9 \times \_\_ < 65 < 9 \times \_\_$	o. $4 \times \_\_ < 33 < 4 \times \_\_$
h. $5 \times \_\_ < 43 < 5 \times \_\_$	p. $5 \times \_\_ < 27 < 5 \times \_\_$

Multiples of 10, 100, and 1000

Complete each problem.

1. a. $8 \times 10 =$ ■
   b. $8 \times 100 =$ ■
   c. $8 \times 1000 =$ ■

2. a. $7 \times 10 =$ ■
   b. $7 \times 100 =$ ■
   c. $7 \times 1000 =$ ■

3. a. $16 \times 10 =$ ■
   b. $16 \times 100 =$ ■
   c. $16 \times 1000 =$ ■

4. a. $10 \times 9 =$ ■
   b. $100 \times 9 =$ ■
   c. $1000 \times 9 =$ ■

5. a. $10 \times 14 =$ ■
   b. $100 \times 14 =$ ■
   c. $1000 \times 14 =$ ■

6. a. $10 \times 98 =$ ■
   b. $100 \times 98 =$ ■
   c. $1000 \times 98 =$ ■

7. a. $10 \times 80 =$ ■
   b. $20 \times 80 =$ ■
   c. $50 \times 80 =$ ■

8. a. $100 \times 30 =$ ■
   b. $200 \times 30 =$ ■
   c. $600 \times 30 =$ ■

9. a. $1000 \times 60 =$ ■
   b. $3000 \times 60 =$ ■
   c. $7000 \times 60 =$ ■

Copy and complete each problem.

10. a. $6 \times 70 < 439 < 6 \times 80$
    b. $4 \times \_\_ < 83 < 4 \times \_\_$
    c. $8 \times \_\_ < 957 < 8 \times \_\_$
    d. $5 \times \_\_ < 67 < 5 \times \_\_$
    e. $6 \times \_\_ < 430 < 6 \times \_\_$
    f. $3 \times \_\_ < 97 < 3 \times \_\_$
    g. $9 \times \_\_ < 561 < 9 \times \_\_$
    h. $7 \times \_\_ < 576 < 7 \times \_\_$

11. a. $40 \times \_\_ < 598 < 40 \times \_\_$
    b. $70 \times \_\_ < 139 < 70 \times \_\_$
    c. $30 \times \_\_ < 877 < 30 \times \_\_$
    d. $60 \times \_\_ < 468 < 60 \times \_\_$
    e. $90 \times \_\_ < 527 < 90 \times \_\_$
    f. $10 \times \_\_ < 74 < 10 \times \_\_$
    g. $50 \times \_\_ < 362 < 50 \times \_\_$
    h. $80 \times \_\_ < 694 < 80 \times \_\_$

Divide and check each problem. You need not give the reason for each step.

$$\begin{array}{r} 57 \\ 8\overline{)462} \\ \underline{400} \\ 62 \\ \underline{56} \\ 6 \end{array}$$

because:

$8 \times 50 < 462 < 8 \times 60$

$462 - 400 = 62$

$8 \times 7 < 62 < 8 \times 8$

$62 - 56 = 6$

$462 \div 8 = 57\frac{6}{8}$

Check:

$$\begin{aligned} 8 \times 57\tfrac{6}{8} &= 8 \times \left(57 + \tfrac{6}{8}\right) \\ &= (8 \times 57) + \left(8 \times \tfrac{6}{8}\right) \\ &= 456 + 6 \\ &= 462 \end{aligned}$$

1. $4\overline{)652}$	5. $8\overline{)24986}$	9. $5\overline{)4329}$	13. $8\overline{)52439}$
2. $9\overline{)837}$	6. $5\overline{)9263}$	10. $8\overline{)649}$	14. $5\overline{)6245}$
3. $7\overline{)2431}$	7. $9\overline{)52413}$	11. $7\overline{)3428}$	15. $9\overline{)3890}$
4. $6\overline{)3743}$	8. $3\overline{)8775}$	12. $4\overline{)56874}$	16. $6\overline{)4002}$

Find the answer to each question. Show all of your work.

17. a. Marta bought 7 packages of stamps for her stamp albums. There were 165 stamps in each package. How many stamps did she buy?

    b. She put the same number of stamps into each of 3 albums. How many stamps did she put into each album?

18. One summer Rex practiced swimming 336 hours in 8 weeks in order to win at the swimming meet. How many hours did he practice each week?

19. Joyce and Jenny found 1248 shells on the beach. When they got home, they divided the shells evenly between themselves. Then Joyce divided her shells among 4 friends. How many shells did each of Joyce's friends get?

Divide and check each problem.

$$\begin{array}{r} 92 \\ 30\overline{)2783} \\ \underline{2700} \\ 83 \\ \underline{60} \\ 23 \end{array}$$

because:

$30 \times 90 < 2783 < 30 \times 100$

$2783 - 2700 = 83$

$30 \times 2 < 83 < 30 \times 3$

$83 - 60 = 23$

Check:

$$\begin{array}{r} 92 \\ \times 30 \\ \hline 2760 \\ + \ 23 \\ \hline 2783 \end{array}$$

$2783 \div 30 = 92\frac{23}{30}$

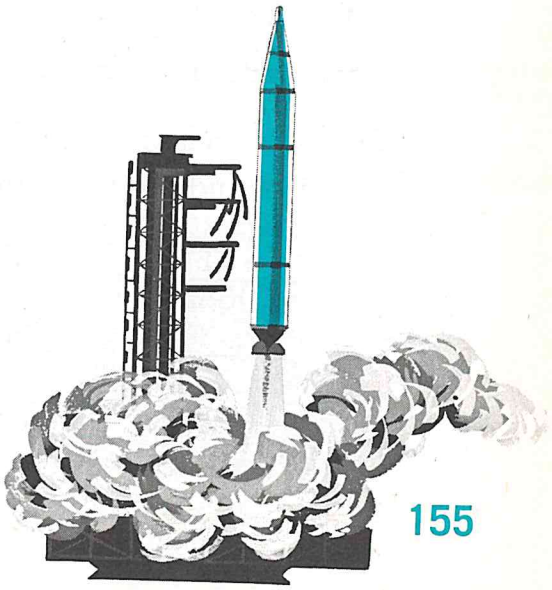
1. $80\overline{)532}$
2. $50\overline{)4683}$
3. $40\overline{)24786}$
4. $70\overline{)9481}$
5. $30\overline{)7950}$
6. $90\overline{)64702}$
7. $60\overline{)28339}$
8. $50\overline{)4582}$
9. $70\overline{)68347}$
10. $30\overline{)9549}$
11. $80\overline{)47294}$
12. $60\overline{)375}$
13. $70\overline{)5486}$
14. $40\overline{)8469}$
15. $60\overline{)28734}$
16. $20\overline{)4862}$

Answer each question. Show all of your work.

17. A real estate office in one city sold 30 houses in one month. The total selling price of the houses was $704,850. What was the average cost of each house?

18. The greatest distance the moon travels from the earth is about 252,710 miles. If a rocket travels this distance in 60 hours, what is the average number of miles it goes each hour?

19. In 1963 the *Faith 7* spacecraft orbited the Earth 20 times in 1880 minutes. What was its average time for 1 orbit?



When we divide by a two-digit number that is not a multiple of 10, we use the nearest multiple of 10 as an estimate for the divisor.

For 61, we use 60.
For 65, we may use 70.
For 67, we use 70.

Look at each problem. Tell what number we use as an estimate to help us decide the number of times the divisor will divide the dividend.

1. $23\overline{)478}$
2. $97\overline{)2463}$
3. $38\overline{)8527}$
4. $45\overline{)7694}$
5. $65\overline{)4463}$
6. $53\overline{)2853}$
7. $86\overline{)9047}$
8. $79\overline{)3579}$
9. $42\overline{)8734}$
10. $25\overline{)12158}$
11. $81\overline{)32681}$
12. $33\overline{)40082}$
13. $92\overline{)5624}$
14. $73\overline{)3090}$
15. $27\overline{)8025}$
16. $59\overline{)33626}$

In some of these problems, the estimate is wrong. Find these and give the correct estimate.

17.

	Divisor	Estimate
a.	46	50
b.	23	20
c.	35	30
d.	17	10
e.	89	90

18.

	Divisor	Estimate
a.	55	60
b.	32	30
c.	91	100
d.	64	70
e.	83	80

19.

	Divisor	Estimate
a.	76	70
b.	68	60
c.	27	20
d.	11	10
e.	98	100

20. For each of the problems in Exercices 1-16, show the first step in dividing, and the following multiplication, as in the example.

$$\begin{array}{r} 1 \\ 43\overline{)6825} \\ \underline{4300} \end{array}$$

The estimate for 43 is 40.

because $40 \times 100 < 6825 < 40 \times 200$
$43 \times 100 = 4300$

We multiply by the divisor, not by its estimate.

We often need to divide several times.

We check our answer in division by multiplication.

Step 1:

$$\begin{array}{r} 1 \\ 49\overline{)8532} \\ 4900 \\ \hline 3632 \end{array}$$

Step 2:

$$\begin{array}{r} 17 \\ 49\overline{)8532} \\ 4900 \\ \hline 3632 \\ 3430 \\ \hline 202 \end{array}$$

Step 3:

$$\begin{array}{r} 174 \\ 49\overline{)8532} \\ 4900 \\ \hline 3632 \\ 3430 \\ \hline 202 \\ 196 \\ \hline 6 \end{array}$$

Check:

$$\begin{array}{r} 174 \\ \times\ 49 \\ \hline 1566 \\ 6960 \\ \hline 8526 \\ +6 \\ \hline 8532 \end{array}$$

$8532 \div 49 = 174\frac{6}{49}$

Copy and complete each problem.

1. $62\overline{)3483}$ — 3100, 383

2. $74\overline{)2632}$ — 412, 42

3. $58\overline{)782}$ — 580, 202, 28

4. $22\overline{)8954}$ — 8800, 154, 0

5. $49\overline{)5763}$ — 4900, 863, 373

6. $82\overline{)43582}$ — 41000, 2582

7. $34\overline{)6308}$

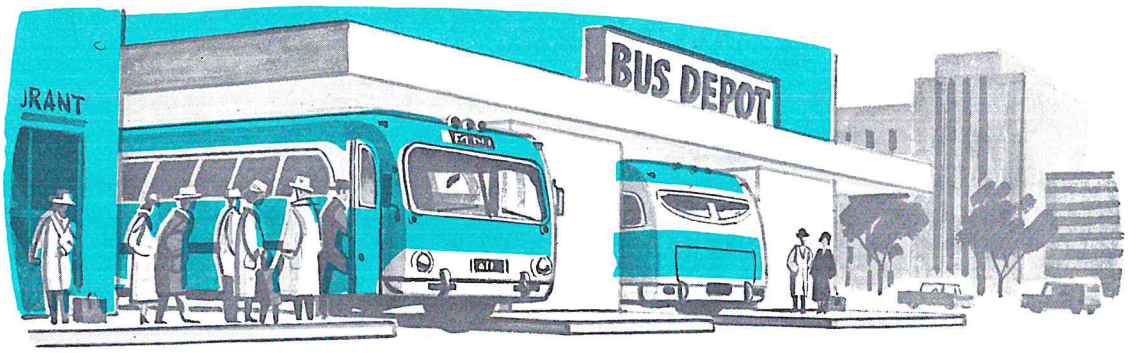
8. $49\overline{)8532}$

9. $38\overline{)4682}$

10. $41\overline{)7843}$

11. $83\overline{)39950}$

12. $69\overline{)15956}$



Solve each problem. Show all of your work.

1. One day 43 people spent a total of 587 hours traveling on a bus. What was the average number of hours traveled per person?

2. The 17 children on the bus had a total of 81 toys with them. What was the average number of toys brought on the trip by each child?

3. In 24 miles, one boy counted 2976 telephone poles. What was the average number of poles per mile?

4. Another boy watched for freight trains so he could count their cars. He saw 11 freight trains and counted a total of 1771 cars. What was the average number of cars per train?

5. During the trip, the bus passed through 28 small towns and 4 cities. The total population of the towns was 134,026 people. The total population of the cities was 109,276. What was the average population per town? What was the average population per city?

6. On the back of each seat there was a pocket for booklets. There were 62 seats on the bus. There was a total of 372 booklets in the pockets. What was the average number of booklets per pocket?

7. Twenty-two people got off the bus at one city. There were 110 people waiting to greet them. What was the average number of greeters per person who got off the bus there?

Sometimes we must show 0 tens or 0 ones in our quotient.

$$\begin{array}{r} 306 \\ 43\overline{)13158} \\ \underline{12900} \\ 258 \\ \underline{258} \\ 0 \end{array} \qquad \begin{array}{r} 420 \\ 56\overline{)23520} \\ \underline{22400} \\ 1120 \\ \underline{1120} \\ 0 \end{array}$$

Copy and complete each problem.

1. $43\overline{)34658}$
2. $94\overline{)84988}$
3. $73\overline{)29711}$
4. $34\overline{)13908}$
5. $64\overline{)21120}$
6. $71\overline{)35926}$
7. $53\overline{)48230}$
8. $62\overline{)6799}$
9. $92\overline{)28267}$
10. $52\overline{)22360}$
11. $84\overline{)59398}$
12. $43\overline{)21758}$

Sometimes the method of using our estimate for the divisor does not work.

$$\begin{array}{r} 1 \\ 12\overline{)23} \\ \underline{12} \end{array}$$

The estimate for 12 is 10.
We know: $10 \times 2 < 23 < 10 \times 3$,
but $12 \times 2 = 24$ and $24 > 23$.
So we use 1: $12 \times 1 = 12$

2 does not work because $12 \times 2 = 24$ and $24 > 23$. We must use a lesser whole number, which in this case is 1.

Copy each problem. Give the estimate for the divisor. Find out which of the estimates gives a quotient that is too great.

13. a. $14\overline{)38}$ b. $23\overline{)62}$ c. $52\overline{)360}$
14. a. $34\overline{)75}$ b. $42\overline{)122}$ c. $11\overline{)53}$
15. a. $63\overline{)248}$ b. $32\overline{)98}$ c. $12\overline{)31}$
16. a. $51\overline{)158}$ b. $44\overline{)83}$ c. $33\overline{)168}$

Sometimes we must use a greater number for our estimated divisor.

```
   3
17)54
   51
```

The estimate for 17 is 20.

We know: $20 \times 2 < 54 < 20 \times 3$, but $17 \times 2 = 34$.
So we use 3: $3 \times 17 = 51$

2 does not work because $17 \times 2 = 34$. $54 - 34 = 20$, the remainder, which is greater than 17, the divisor. We use a greater whole number, which in this case is 3.

Copy each problem. Give the estimate for the divisor. Find out which of the estimates gives a quotient that is too small.

1. $28\overline{)115}$
2. $47\overline{)386}$
3. $16\overline{)54}$
4. $39\overline{)159}$
5. $59\overline{)298}$
6. $25\overline{)186}$
7. $28\overline{)95}$
8. $37\overline{)153}$

Copy and complete each problem.

9. $86\overline{)529}$
   516

10. $42\overline{)2673}$
    2520

    126

11. $27\overline{)638}$
    540

    98

12. $63\overline{)1342}$

    82

    19

13. $47\overline{)5734}$
    4700

    1034

    94

14. $74\overline{)4689}$
    4440

    249

15. $22\overline{)9876}$
    8800

    1076

    196

16. $39\overline{)2769}$
    2730

    39

Find each quotient.

17. $69\overline{)458}$
18. $21\overline{)783}$
19. $54\overline{)507}$
20. $32\overline{)695}$
21. $58\overline{)929}$
22. $26\overline{)488}$
23. $43\overline{)927}$
24. $77\overline{)582}$

Copy and complete each problem.

1. a. $5 \times 500 < 2863 < 5 \times 600$
   b. $9 \times \_\_ < 8476 < 9 \times \_\_$
   c. $8 \times \_\_ < 5164 < 8 \times \_\_$
   d. $6 \times \_\_ < 2580 < 6 \times \_\_$
   e. $5 \times \_\_ < 4156 < 5 \times \_\_$
   f. $7 \times \_\_ < 5431 < 7 \times \_\_$
   g. $3 \times \_\_ < 1970 < 3 \times \_\_$

2. a. $4 \times 7000 < 29{,}142 < 4 \times 8000$
   b. $8 \times \_\_ < 41{,}838 < 8 \times \_\_$
   c. $6 \times \_\_ < 52{,}110 < 6 \times \_\_$
   d. $5 \times \_\_ < 96{,}300 < 5 \times \_\_$
   e. $7 \times \_\_ < 58{,}394 < 7 \times \_\_$
   f. $9 \times \_\_ < 75{,}640 < 9 \times \_\_$
   g. $6 \times \_\_ < 44{,}852 < 6 \times \_\_$

Copy and complete each problem. Check by multiplication.

3.
```
      159
400)63845
    40000
    23845
    20000
     3845
     3600
      245
```

4.
```
200)143827
   140000
     3827
     ■■
     1827
     ■■
       27
```

5.
```
300)23849
   21000
     ■■
     ■■
```

6.
```
600)436215
   420000
    16215
    12000
     ■■
     ■■
     ■■
```

7.
```
900)526384
   450000
    76384
    72000
     4384
     3600
      ■■
```

8.
```
500)63894
     ■■
    13894
     ■■
     3894
     ■■
      ■■
```

9.
```
700)876532
   700000
   176532
     ■■
    36532
     ■■
     1532
     ■■
      ■■
```

10.
```
800)98732
   80000
     ■■
     ■■
      ■■
     ■■
      332
```

11.
```
500)83145
     ■■
    33145
     ■■
     3145
     ■■
      145
```

12.
```
800)632946
   560000
     ■■
      946
     ■■
      146
```

13.
```
600)568246
     ■■
      ■■
    24000
     4246
     ■■
      ■■
```

Find each answer. Then check each problem by multiplication.

1. $700\overline{)2865}$
2. $100\overline{)14382}$
3. $500\overline{)846235}$
4. $400\overline{)9852}$
5. $500\overline{)47815}$
6. $900\overline{)60497}$
7. $900\overline{)20064}$
8. $600\overline{)67825}$
9. $800\overline{)54789}$
10. $800\overline{)99462}$
11. $200\overline{)78888}$
12. $700\overline{)30014}$

Find each answer. Show all of your work.

13. A basketweaver made 300 baskets in 1345 hours. What was the average time he spent on each basket?

14. A glassblower made 400 glasses in 5200 minutes and 200 pitchers in 4600 minutes. What was the average time he spent on each glass? What was the average time he spent on each pitcher?

15. A man wove 500 straw hats in 23,500 minutes. What was the average time he spent on each hat? How many hours did he work on the hats?

16. An adobe-brick maker made 900 bricks in 6287 minutes. What was the average number of minutes it took to make each brick?

17. A potter spent 44,400 minutes making 600 pottery bowls. What is the average number of minutes he spent making each bowl?

18. The potter's wife painted the bowls. It took her 2483 hours to paint all the bowls. What was the average number of hours she spent painting each bowl?

19. A sandalmaker spent 9 hours every day making sandals. He made 200 pairs of sandals in 54,000 minutes. What was the average number of minutes he spent on each pair of sandals?

Copy and complete each problem.

1. 478)26584
   23900
   2684
   ■
   294

2. 259)68429
   51800
   ■
   15540
   1089
   ■
   53

3. 375)94729
   ■
   19729
   ■
   979
   ■
   229

4. 739)265497
   ■
   43797
   ■
   ■
   ■
   196

5. 834)572144

6. 452)86529

7. 942)452638

8. 430)638429

9. 579)205371

10. 914)47632

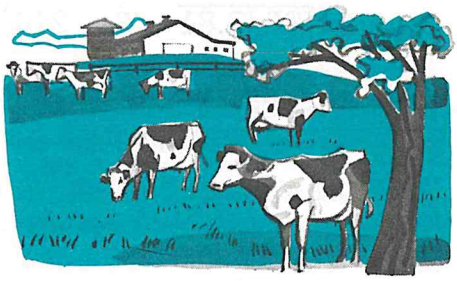
11. 659)24589

12. 337)63728

13. Mr. Sutton sent his 5000 cattle to market in trucks. Each truck could hold an average of 145 cattle. How many trucks did he need to transport the cattle?

14. Mr. Murray raised 698 turkeys weighing a total of 11,748 pounds. What was the average weight of each turkey?

15. Mr. Jordan has 564 dairy cows. In 90 days the cows gave a total of 304,560 gallons of milk.



   a. What was the average number of gallons of milk given by each cow in that time?

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16. Mr. Flores sent 259 hogs to market. The hogs weighed a total of 217,560 pounds. What was the average weight of each of the hogs?

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When we divide amounts of money, we must show dollars and cents in the quotient. We divide in the same way.

If the remainder is more than half of the divisor, we add 1¢ to the quotient.

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$$
\begin{array}{r}
\$\ \ 2.19 \\
438\overline{)\$962.85} \\
\underline{876\ 00} \\
86\ 85 \\
\underline{43\ 80} \\
43\ 05 \\
\underline{39\ 42} \\
3\ 63
\end{array}
$$

or about \$2.20

$$
\begin{array}{r}
\$\ \ 4.22 \\
87\overline{)\$367.24} \\
\underline{348\ 00} \\
19\ 24 \\
\underline{17\ 40} \\
1\ 84 \\
\underline{1\ 74} \\
10
\end{array}
$$

or about \$4.22

Find each quotient. Round off the answer to the nearest cent.

1.	$45\overline{)\$468.29}$	4.	$870\overline{)\$600.13}$	7.	$18\overline{)\$485.14}$	10.	$742\overline{)\$121.45}$
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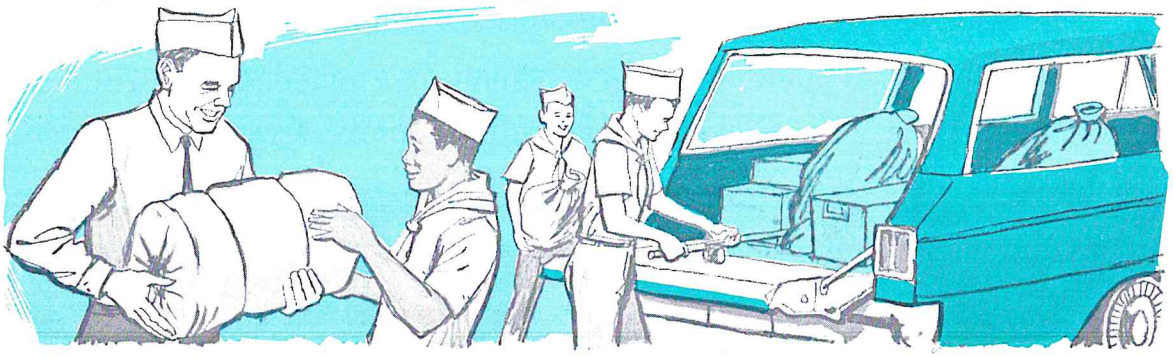

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Find each sum or difference.

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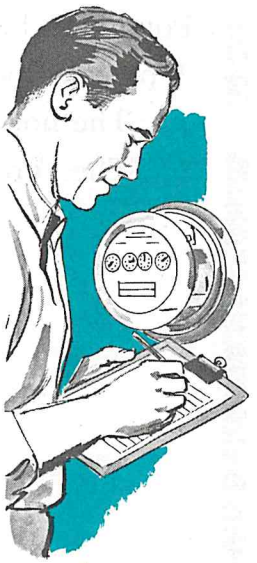

Complete each sentence.

5. The prime numbers between 80 and 90 are ■.
6. The factors of 24 are ■.
7. The greatest common factor of 35 and 45 is ■.
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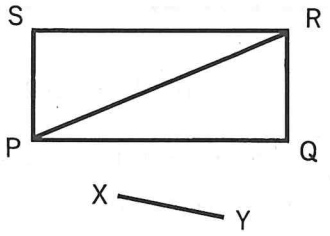
9. The only even number that is prime is ■.

Choose ∪ or ∩ or −.

10. $\{2, 7, 10\}$ ● $\{4, 9, 10, 14\} = \{10\}$
11. $\{3, 8, 13\}$ ● $\{4, 8, 12\} = \{3, 4, 8, 12, 13\}$
12. $\{1, 3, 5\}$ ● $\{2, 4, 8\} = \{\ \}$
13. $\{9, 10, 11\}$ ● $\{10, 11, 12\} = \{9\}$

14. a. Draw segments PQ and XY.
    b. Construct a rectangle PQRS with base PQ and two sides equal in length to $\overline{XY}$.
    c. Draw diagonal PR.
    d. Construct the circle with diagonal PR as diameter.
    e. Do vertices Q and S of the rectangle lie on the circle?
    f. Draw diagonal QS.
    g. Is $\overline{QS}$ a diameter of the circle?



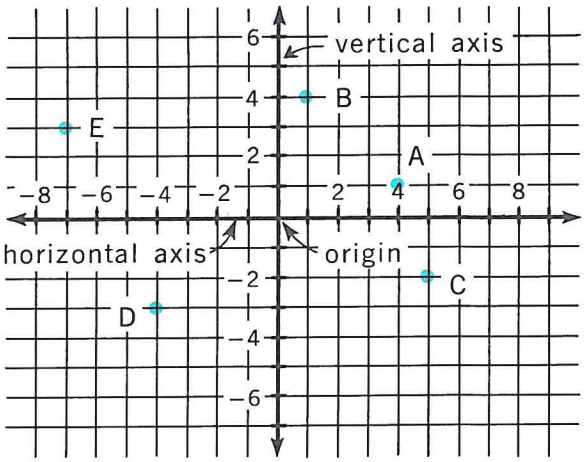
Part 13 A Coordinate System

Points can be located on a **coordinate system.**

A point is located by giving the two numbers which are its **coordinates.**

The horizontal coordinate is measured on the horizontal axis.

The vertical coordinate is measured on the vertical axis.



The coordinates of A are 4 and 1. We write: A (4, 1)

The coordinates of B are 1 and 4. We write: B (1, 4)

We always write the horizontal coordinate first.

Complete each problem.

1. a. The coordinates of point C are ▪ and −2.
   b. We write this: C(▪, −2)
2. a. The coordinates of point D are −4 and ▪.
   b. We write this: D(−4, ▪)
3. a. The coordinates of point E are ▪ and 3.
   b. We write this: E(▪, 3)

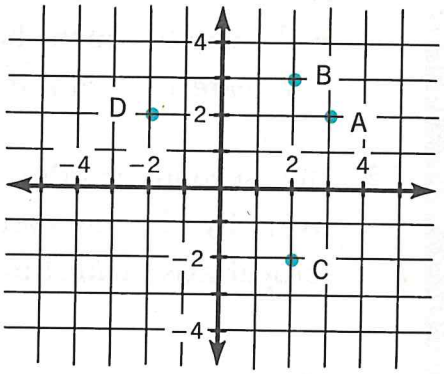
Ordered Pairs

These points are shown on the coordinate system:

A (3, 2) C (2, −2)
B (2, 3) D (−2, 2)

Point A is not the same as point B.
Point C is not the same as point D.

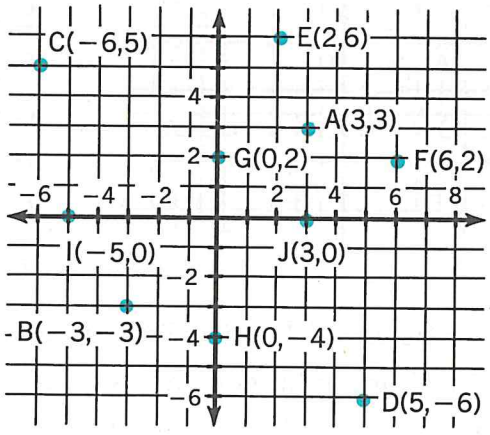
We call (3, 2) an **ordered pair** because it matters in which order we give the coordinates. We always give the horizontal coordinate first in an ordered pair.



1. In each example, do the coordinates locate the same point?
   a. (3, 4) (4, 3)
   b. (−1, 2) (2, −1)
   c. (3, 6) (3, 6)
   d. (−4, 1) (1, −4)
   e. (−6, −7) (−6, −7)
   f. (−8, −5) (−5, −8)

2. Copy the table. Use the ordered pairs shown on the coordinate system to complete the table.

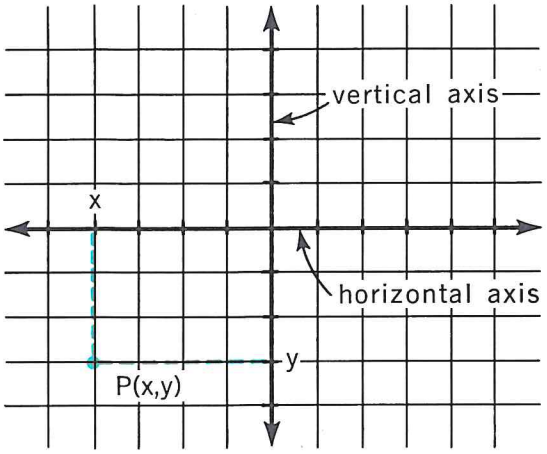
point	horizontal coordinate	vertical coordinate
A	3	3
B		−3
C	−6	
D		
E	2	
F		
G	0	
H		
I		0
J		



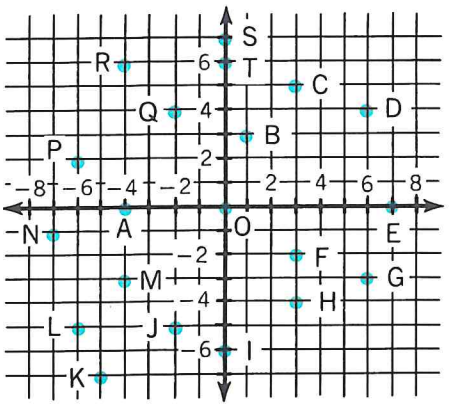
Given a point P we can find the coordinates (x, y) which locate it.

1. The first coordinate, x, is given by the number on the horizontal axis nearest to point P.

2. The second coordinate, y, is given by the number on the vertical axis nearest to point P.



1. Sue found the coordinates of the points on the coordinate system shown below. Sometimes she made mistakes and gave the coordinates in the wrong order. For which points did she make mistakes? Write the correct coordinates for these points.



A (0, −4)
B (1, 3)
C (5, 3)
D (4, 6)
E (7, 0)
F (−2, 3)
G (−3, 6)
H (−4, 3)
I (0, −6)
J (−2, −5)
K (−7, −5)
L (−6, −5)
M (−3, −4)
N (−7, −1)
O (0, 0)
P (−6, 2)
Q (−2, 4)
R (6, −4)
S (0, 7)
T (0, 6)

2. Choose **horizontal** or **vertical** to complete each sentence.
   a. If the horizontal coordinate of a point is zero, the point lies on the ▒ axis.
   b. If the vertical coordinate of a point is zero, the point lies on the ▒ axis.

Copy and complete each problem.

1. a. $5 \times 500 < 2863 < 5 \times 600$
   b. $9 \times \_\_ < 8476 < 9 \times \_\_$
   c. $8 \times \_\_ < 5164 < 8 \times \_\_$
   d. $6 \times \_\_ < 2580 < 6 \times \_\_$
   e. $5 \times \_\_ < 4156 < 5 \times \_\_$
   f. $7 \times \_\_ < 5431 < 7 \times \_\_$
   g. $3 \times \_\_ < 1970 < 3 \times \_\_$

2. a. $4 \times 7000 < 29{,}142 < 4 \times 8000$
   b. $8 \times \_\_ < 41{,}838 < 8 \times \_\_$
   c. $6 \times \_\_ < 52{,}110 < 6 \times \_\_$
   d. $5 \times \_\_ < 96{,}300 < 5 \times \_\_$
   e. $7 \times \_\_ < 58{,}394 < 7 \times \_\_$
   f. $9 \times \_\_ < 75{,}640 < 9 \times \_\_$
   g. $6 \times \_\_ < 44{,}852 < 6 \times \_\_$

Copy and complete each problem. Check by multiplication.

3.
```
      159
400)63845
    40000
    23845
    20000
     3845
     3600
      245
```

4.
```
200)143827
   140000
     3827
     ■■■
     1827
     ■■■
       27
```

5.
```
300)23849
   21000
     ■■■
     ■■■
```

6.
```
600)436215
   420000
    16215
    12000
     ■■■
     ■■■
```

7.
```
900)526384
   450000
    76384
    72000
     4384
     3600
      ■■■
```

8.
```
500)63894
     ■■■
    13894
     ■■■
     3894
     ■■■
     ■■■
```

9.
```
700)876532
   700000
   176532
     ■■■
    36532
     ■■■
     1532
     ■■■
     ■■■
```

10.
```
800)98732
   80000
     ■■■
     ■■■
     ■■■
     ■■■
      332
```

11.
```
500)83145
     ■■■
    33145
     ■■■
     3145
     ■■■
      145
```

12.
```
800)632946
   560000
     ■■■
     946
     ■■■
     146
```

13.
```
600)568246
     ■■■
     ■■■
    24000
     4246
     ■■■
     ■■■
```

Find each answer. Then check each problem by multiplication.

1.	$700\overline{)2865}$	4.	$400\overline{)9852}$	7.	$900\overline{)20064}$	10.	$800\overline{)99462}$
2.	$100\overline{)14382}$	5.	$500\overline{)47815}$	8.	$600\overline{)67825}$	11.	$200\overline{)78888}$
3.	$500\overline{)846235}$	6.	$900\overline{)60497}$	9.	$800\overline{)54789}$	12.	$700\overline{)30014}$

Find each answer. Show all of your work.

13. A basketweaver made 300 baskets in 1345 hours. What was the average time he spent on each basket?

14. A glassblower made 400 glasses in 5200 minutes and 200 pitchers in 4600 minutes. What was the average time he spent on each glass? What was the average time he spent on each pitcher?

15. A man wove 500 straw hats in 23,500 minutes. What was the average time he spent on each hat? How many hours did he work on the hats?

16. An adobe-brick maker made 900 bricks in 6287 minutes. What was the average number of minutes it took to make each brick?

17. A potter spent 44,400 minutes making 600 pottery bowls. What is the average number of minutes he spent making each bowl?

18. The potter's wife painted the bowls. It took her 2483 hours to paint all the bowls. What was the average number of hours she spent painting each bowl?

19. A sandalmaker spent 9 hours every day making sandals. He made 200 pairs of sandals in 54,000 minutes. What was the average number of minutes he spent on each pair of sandals?

When we divide by a three-digit number that is not a multiple of 100 we use the nearest multiple of 100 as an estimate for the divisor.

For 347, we use 300 to estimate.

For 360, we use 400 to estimate.

For 350, we usually use 400 to estimate.

1. If each of these numbers is a divisor, what number do we use as an estimate?

a. 629	**c.** 285	**e.** 318	**g.** 472	**i.** 214
b. 930	**d.** 747	**f.** 365	**h.** 894	**j.** 605

2. Give the estimate for the divisor. Then show the first division and multiplication.

a. $826\overline{)3549}$	**c.** $333\overline{)66723}$	**e.** $920\overline{)44682}$	**g.** $281\overline{)497294}$
b. $301\overline{)6991}$	**d.** $640\overline{)98642}$	**f.** $564\overline{)73650}$	**h.** $422\overline{)528319}$

3. Copy and complete each problem.

a. $429\overline{)46836}$

$$\begin{array}{r} 42900 \\ \hline 3936 \\ \square \\ \hline 75 \end{array}$$

b. $240\overline{)76939}$

$$\begin{array}{r} 72000 \\ \hline 4939 \\ \square \\ \hline \square \end{array}$$

c. $569\overline{)446380}$

$$\begin{array}{r} 398300 \\ \hline 48080 \\ \square \\ \hline 2560 \\ \square \\ \hline 284 \end{array}$$

d. $395\overline{)986548}$

$$\begin{array}{r} 790000 \\ \hline 196548 \\ 158000 \\ \hline \square \\ \square \\ \hline \square \\ \square \\ \hline \square \end{array}$$

4. The Girl Scout summer camp had craft classes. Two hundred sixty girls came to the classes. These materials were provided:

a. 12,480 yards of raffia	**d.** 325,000 beads
b. 520 pounds of clay	**e.** 3120 feet of wire
c. 6240 inches of leather strips	**f.** 6500 yards of yarn

What was the average number of each thing provided for each girl?

Sometimes we must use a lesser number for our estimated quotient.

$$\begin{array}{r} 6 \\ 442\overline{)29680} \\ \underline{26520} \end{array}$$

The estimate for 442 is 400.

We know: $400 \times 70 < 29680 < 400 \times 80$, but

$442 \times 70 = 30940$ and $30940 > 29680$.

So we use 60: $442 \times 60 = 26520$

70 does not work because $442 \times 70 = 30940$ and $30940 > 29680$. We must use a lesser number of tens which in this case is 6 tens or 60.

Copy each problem and make the first estimation for each problem. Tell when the estimate gives a quotient that is too great.

1.	2.	3.	4.
a. $436\overline{)33245}$	a. $349\overline{)9875}$	a. $836\overline{)74128}$	a. $942\overline{)595200}$
b. $745\overline{)45390}$	b. $618\overline{)30246}$	b. $214\overline{)68724}$	b. $526\overline{)258904}$
c. $533\overline{)25987}$	c. $127\overline{)6425}$	c. $316\overline{)218457}$	c. $849\overline{)22485}$

Sometimes we must use a greater number for our estimated quotient. The estimate for 389 is 400.

$$\begin{array}{r} \\ 389\overline{)2759} \\ \underline{2723} \end{array}$$

We know: $400 \times 6 < 2759 < 400 \times 7$, but $389 \times 6 = 2334$

6 does not work because $389 \times 6 = 2334$ and $2759 - 2334 = 425$ for a remainder which is greater than the divisor 389. We must use a greater whole number which in this case is 7: $389 \times 7 = 2723$.

Copy the problems and make the first estimation for each problem. Tell when the estimate gives a quotient that is too small.

5.	6.	7.	8.
a. $268\overline{)17849}$	a. $464\overline{)1900}$	a. $758\overline{)23100}$	a. $387\overline{)15722}$
b. $572\overline{)23786}$	b. $682\overline{)3466}$	b. $884\overline{)49876}$	b. $190\overline{)1548}$

Copy and complete each problem.

1. $478\overline{)26584}$

$$\begin{array}{r} 23900 \\ \hline 2684 \\ \blacksquare \\ \hline 294 \end{array}$$

2. $259\overline{)68429}$

$$\begin{array}{r} 51800 \\ \hline \blacksquare \\ 15540 \\ \hline 1089 \\ \blacksquare \\ \hline 53 \end{array}$$

3. $375\overline{)94729}$

$$\begin{array}{r} \blacksquare \\ \hline 19729 \\ \blacksquare \\ \hline 979 \\ \blacksquare \\ \hline 229 \end{array}$$

4. $739\overline{)265497}$

$$\begin{array}{r} \blacksquare \\ \hline 43797 \\ \blacksquare \\ \hline \blacksquare \\ \blacksquare \\ \hline 196 \end{array}$$

5. $834\overline{)572144}$

6. $452\overline{)86529}$

7. $942\overline{)452638}$

8. $430\overline{)638429}$

9. $579\overline{)205371}$

10. $914\overline{)47632}$

11. $659\overline{)24589}$

12. $337\overline{)63728}$

13. Mr. Sutton sent his 5000 cattle to market in trucks. Each truck could hold an average of 145 cattle. How many trucks did he need to transport the cattle?

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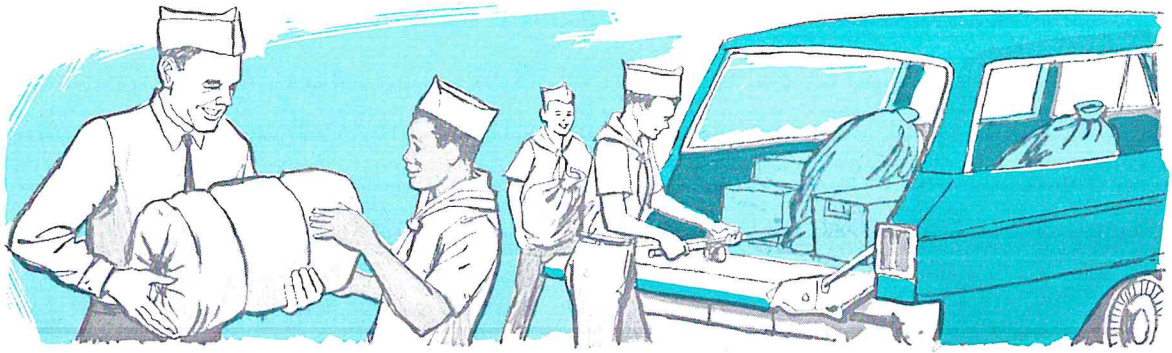

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3. $38\frac{1}{2} - 16\frac{5}{8}$
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Complete each sentence.

5. The prime numbers between 80 and 90 are ■.
6. The factors of 24 are ■.
7. The greatest common factor of 35 and 45 is ■.
8. The least common multiple of 8 and 12 is ■.
9. The only even number that is prime is ■.

Choose ∪ or ∩ or −.

10. $\{2, 7, 10\}$ ● $\{4, 9, 10, 14\} = \{10\}$
11. $\{3, 8, 13\}$ ● $\{4, 8, 12\} = \{3, 4, 8, 12, 13\}$
12. $\{1, 3, 5\}$ ● $\{2, 4, 8\} = \{\ \}$
13. $\{9, 10, 11\}$ ● $\{10, 11, 12\} = \{9\}$

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    b. Construct a rectangle PQRS with base PQ and two sides equal in length to $\overline{XY}$.
    c. Draw diagonal PR.
    d. Construct the circle with diagonal PR as diameter.
    e. Do vertices Q and S of the rectangle lie on the circle?
    f. Draw diagonal QS.
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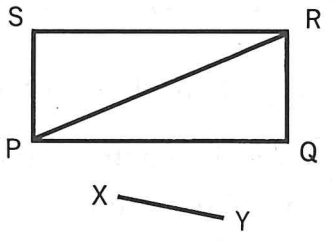

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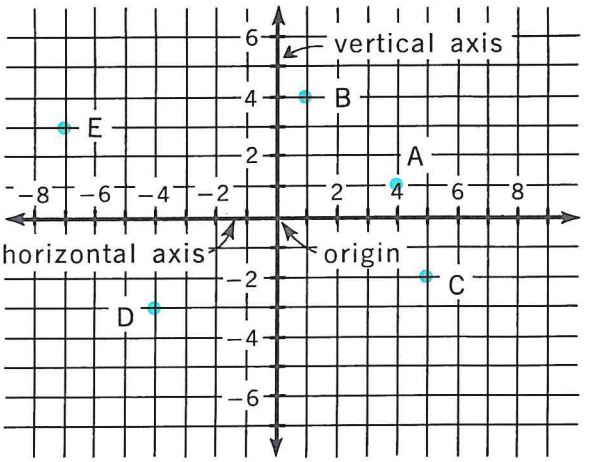


The coordinates of A are 4 and 1. We write: A (4, 1)

The coordinates of B are 1 and 4. We write: B (1, 4)

We always write the horizontal coordinate first.

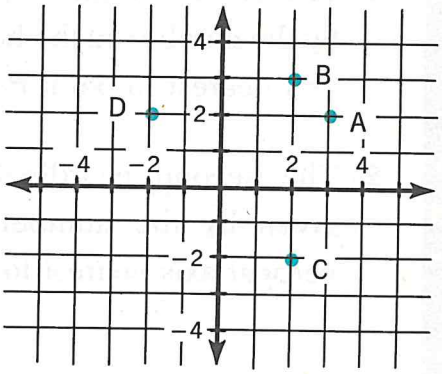
Complete each problem.

1. a. The coordinates of point C are ▪ and −2.
   b. We write this: C(▪, −2)
2. a. The coordinates of point D are −4 and ▪.
   b. We write this: D(−4, ▪)
3. a. The coordinates of point E are ▪ and 3.
   b. We write this: E(▪, 3)

Ordered Pairs

These points are shown on the coordinate system:

A (3, 2) C (2, −2)
B (2, 3) D (−2, 2)



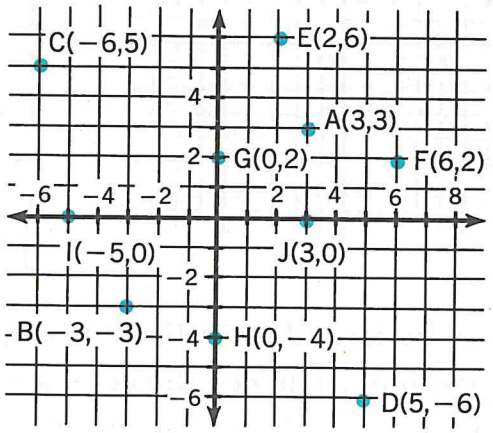
Point A is not the same as point B.
Point C is not the same as point D.

We call (3, 2) an **ordered pair** because it matters in which order we give the coordinates. We always give the horizontal coordinate first in an ordered pair.

1. In each example, do the coordinates locate the same point?
   a. (3, 4) (4, 3)
   b. (−1, 2) (2, −1)
   c. (3, 6) (3, 6)
   d. (−4, 1) (1, −4)
   e. (−6, −7) (−6, −7)
   f. (−8, −5) (−5, −8)

2. Copy the table. Use the ordered pairs shown on the coordinate system to complete the table.

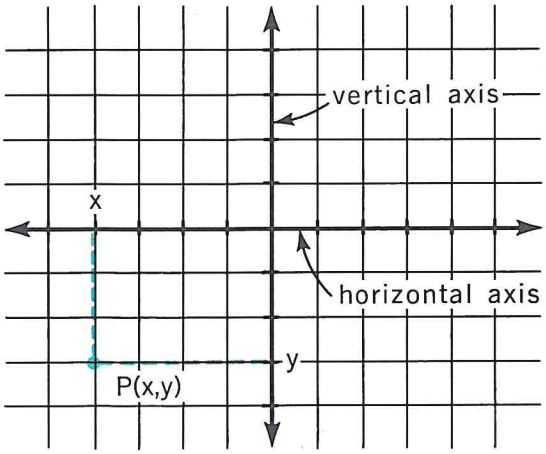
point	horizontal coordinate	vertical coordinate
A	3	3
B		−3
C	−6	
D		
E	2	
F		
G	0	
H		
I		0
J		



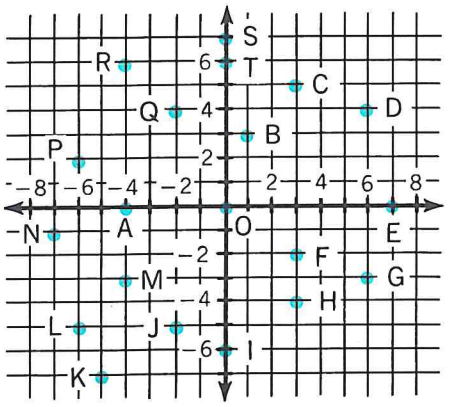
Given a point P we can find the coordinates (x, y) which locate it.

1. The first coordinate, x, is given by the number on the horizontal axis nearest to point P.

2. The second coordinate, y, is given by the number on the vertical axis nearest to point P.



1. Sue found the coordinates of the points on the coordinate system shown below. Sometimes she made mistakes and gave the coordinates in the wrong order. For which points did she make mistakes? Write the correct coordinates for these points.



A (0, −4)	K (−7, −5)
B (1, 3)	L (−6, −5)
C (5, 3)	M (−3, −4)
D (4, 6)	N (−7, −1)
E (7, 0)	O (0, 0)
F (−2, 3)	P (−6, 2)
G (−3, 6)	Q (−2, 4)
H (−4, 3)	R (6, −4)
I (0, −6)	S (0, 7)
J (−2, −5)	T (0, 6)

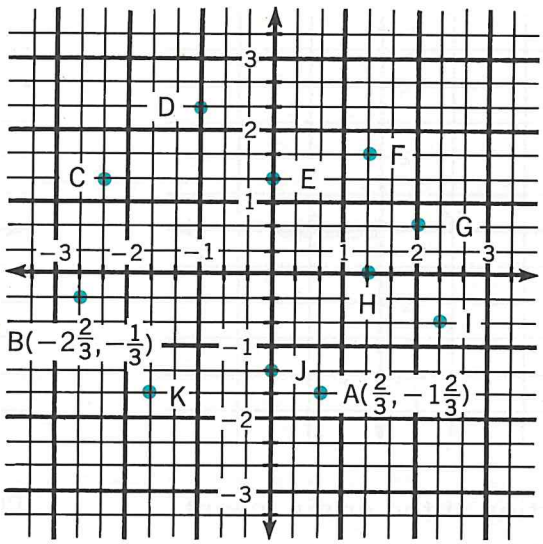
2. Choose **horizontal** or **vertical** to complete each sentence.

a. If the horizontal coordinate of a point is zero, the point lies on the ▒ axis.

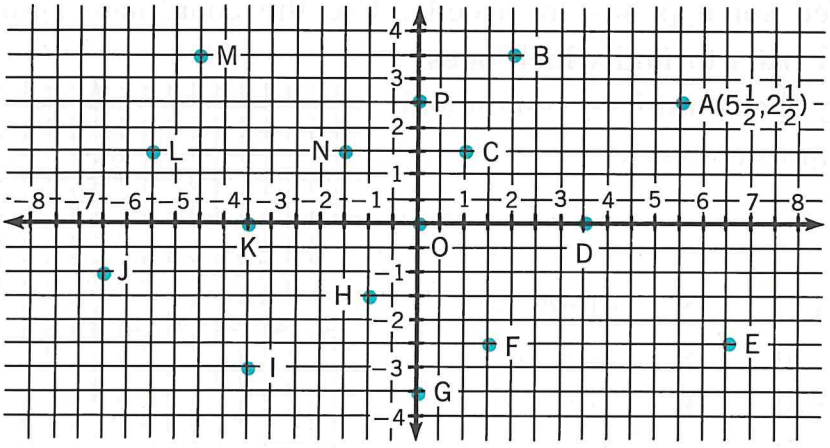
b. If the vertical coordinate of a point is zero, the point lies on the ▒ axis.

When using fractions as coordinates, we use a larger scale. Here, 3 squares equal 1 unit. Which point corresponds to each of the ordered pairs? The coordinates for points A and B are shown.

1. $(-1, 2\frac{1}{3})$
2. $(1\frac{1}{3}, 1\frac{2}{3})$
3. $(0, -1\frac{1}{3})$
4. $(2, \frac{2}{3})$
5. $(-2\frac{1}{3}, 1\frac{1}{3})$
6. $(0, 1\frac{1}{3})$
7. $(-1\frac{2}{3}, -1\frac{2}{3})$
8. $(2\frac{1}{3}, -\frac{2}{3})$
9. $(1\frac{1}{3}, 0)$



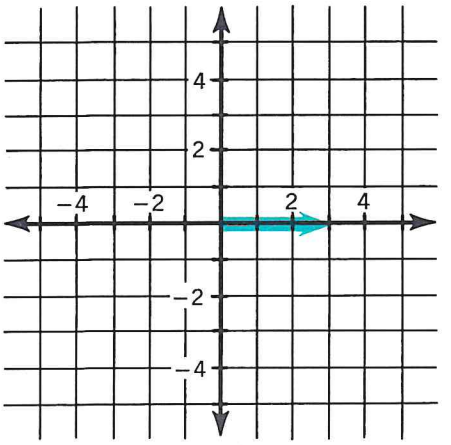
10. On this coordinate system 2 squares equal 1 unit. The coordinates for point A are shown. Give the coordinates for the other labeled points.



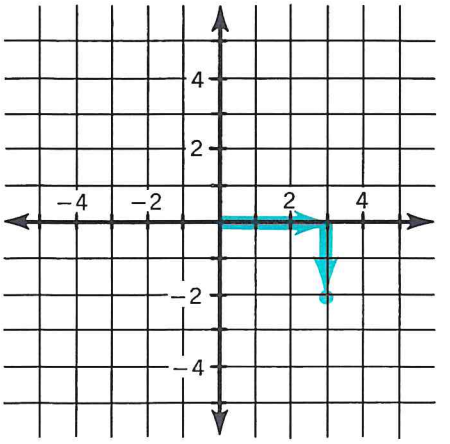
11. On a coordinate system where:
    a. 3 squares equal 1 unit, each square equals ■ unit.
    b. 2 squares equal 1 unit, each square equals ■ unit.

Plotting Points

Given an ordered pair $(3, -2)$ we can plot point P that corresponds to it.

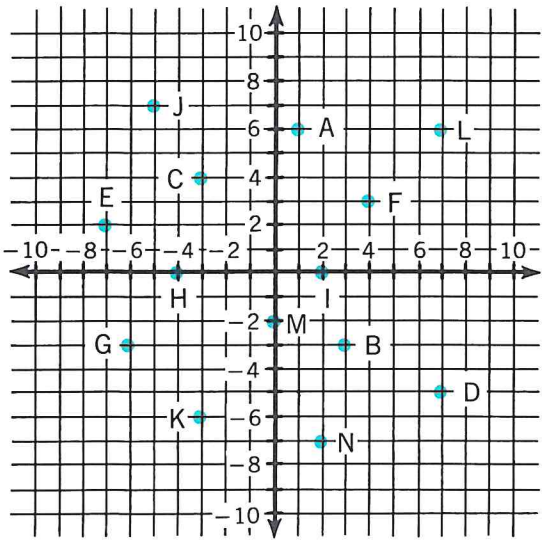


Starting at the origin, count off 3 horizontal units in the positive direction.



Then count off 2 vertical units in the negative direction.

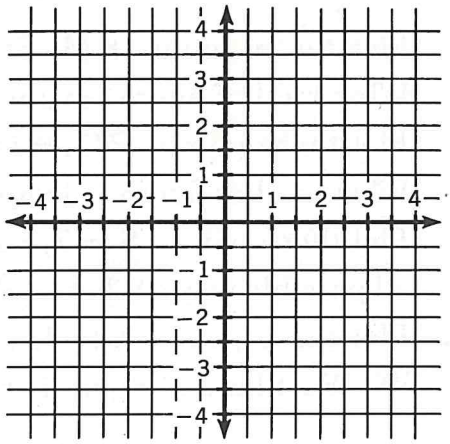
Ken plotted some points incorrectly. Use the coordinate system and list of ordered pairs to find which ones. Then copy the coordinate system and plot the points correctly.



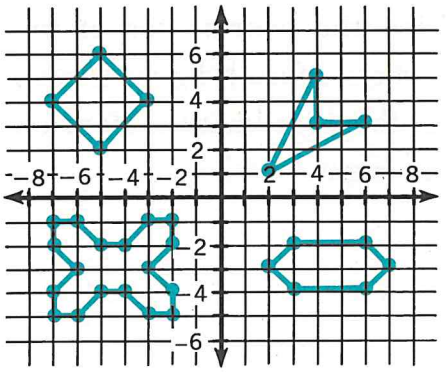
1. A (1, 6)
2. B (−3, 3)
3. C (4, −3)
4. D (7, −5)
5. E (−7, 2)
6. F (3, 4)
7. G (6, 3)
8. H (−4, 0)
9. I (0, 2)
10. J (−5, 7)
11. K (−6, −3)
12. L (6, 7)
13. M (−2, 0)
14. N (2, −7)

Copy the coordinate system and plot these points on it.

1. A $\left(-1\frac{1}{2}, 2\frac{1}{2}\right)$
2. B $\left(-3\frac{1}{2}, -2\frac{1}{2}\right)$
3. C $\left(-1, -1\frac{1}{2}\right)$
4. D $\left(-2\frac{1}{2}, -1\right)$
5. E $\left(1\frac{1}{2}, 2\frac{1}{2}\right)$
6. F $\left(-3\frac{1}{2}, 0\right)$
7. G $\left(-2\frac{1}{2}, 2\right)$
8. H $\left(3, -2\frac{1}{2}\right)$
9. I $\left(3\frac{1}{2}, 1\right)$
10. J $(1, -2)$
11. K $\left(0, 1\frac{1}{2}\right)$
12. L $(-4, 2)$
13. M $\left(2\frac{1}{2}, 0\right)$
14. N $\left(0, -\frac{1}{2}\right)$
15. O $(0, 0)$
16. P $\left(\frac{1}{2}, 3\right)$



17. Give the coordinates of the vertices of each figure shown on the coordinate system on the right.



18. Draw a coordinate system showing both axes from -7 to 7. Let 2 squares equal 1 unit. On it, plot and label these points.

A $\left(7, \frac{1}{2}\right)$ C $\left(-4\frac{1}{2}, \frac{1}{2}\right)$ E $(-6, -4)$ G $\left(6, -2\frac{1}{2}\right)$ I $\left(-6\frac{1}{2}, 4\frac{1}{2}\right)$

B $(2, -3)$ D $\left(0, 4\frac{1}{2}\right)$ F $\left(-1, -6\frac{1}{2}\right)$ H $\left(-3\frac{1}{2}, 6\frac{1}{2}\right)$ J $\left(-1\frac{1}{2}, -5\frac{1}{2}\right)$

a. Draw segment AH. Does point D lie on $\overline{AH}$? If not, check to make sure points A, D, and H were plotted correctly.

b. Draw segment EG. Point B should lie on $\overline{EG}$.

c. Draw segment IF. Which other plotted points lie on this segment?

1. Draw a coordinate system showing both axes from −10 to 10. Plot these 4 sets of points on this system.

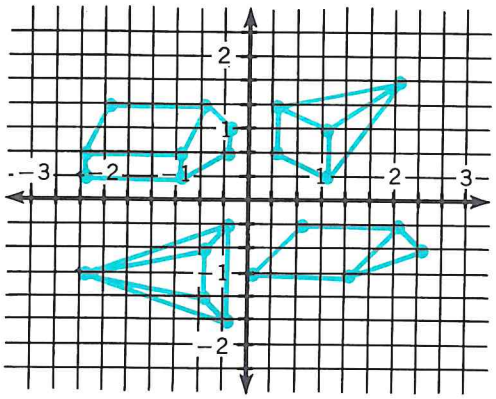
   a. Plot these points: K (4, 8) L (8, 4) M (2, 0). Draw $\overline{KL}$, $\overline{LM}$, and $\overline{KM}$. What kind of figure have you drawn?

   b. Plot these points: P (1, −1) Q (10, −7) R (4, −3). Connect the points. What kind of figure is figure PQR?

   c. Plot these points: S (−7, −2) T (−4, −5) U (−7, −8). Connect them in all possible ways. What kind of figure have you drawn?

   d. Plot these points: V (−4, 7) W (−9, 2) X (−1, 9). Draw $\overline{VW}$, $\overline{WX}$, and $\overline{VX}$. What kind of figure is figure WVX?

2. Give the coordinates of the vertices of each figure shown on the coordinate system on the right.



3. Draw a coordinate system showing both axes from −8 to 8. Plot these 4 sets of points on this system.

   a. Plot these points: A (2, 8) B (7, 8) C (7, 2) D (2, 2). Draw $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, and $\overline{AD}$. What kind of figure have you drawn?

   b. Plot these points: E (3, −1) F (6, −3) G (4, −6) H (1, −4). Draw $\overline{EF}$, $\overline{FG}$, $\overline{GH}$, and $\overline{HE}$. What kind of figure is figure EFGH?

   c. Plot these points: I (−2, −4) J (−5, −7) K (−6, −8) L (−7, −3). Draw $\overline{IJ}$, $\overline{JK}$, $\overline{KL}$, and $\overline{LI}$. What kind of figure is figure IJKL?

   d. Plot these points: M (−8, 2) N (−6, 4) O (−5, 5) P (−2, 8). Connect the points. What kind of figure have you drawn?

Checkout

Complete each problem.

1. 6 lb. 4 oz.
   −2 lb. 8 oz.

2. 7 gal. 3 qt.
   +2 gal. 2 qt.

3. 4 yd. 1 ft.
   −1 yd. 2 ft.

4. $\{n: 2 < n < 6\} \cup \{n: 3 < n < 5\} \cup \{n: 4 < n < 9\} =$ ■

5. $\{n: 11 < n < 14\} \cap \{n: 10 < n < 16\} \cap \{13, 15, 17, 19\} =$ ■

6. $\{n: 3 < n < 7\} - \{n: 4 < n < 6\} =$ ■

7. Which of these numbers are prime?
   a. 37 b. 55 c. 74 d. 18

8. Find the L.C.M. of each set of numbers.
   a. $\{22, 44, 11\}$ b. $\{25, 40, 32\}$

9. Find the G.C.F. of each set of numbers.
   a. $\{33, 66, 22\}$ b. $\{18, 24, 54\}$

10. 846
    × 27

11. 968
    × 35

12. 8972
    −4685

13. 6084
    −2496

14. The floor of the gymnasium measures 90 ft. by 80 ft. What is the area of the floor?

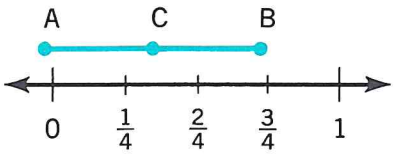
15. A fish tank at the aquarium is shaped like a rectangular prism. If its height is 11 ft. and its volume is 4530 cubic feet, what is the area of its base?

16. Another tank measures 15 ft. by 10 ft. by 8 ft. How many cubic feet of water are needed to fill it?

Part 14 Using Fractions

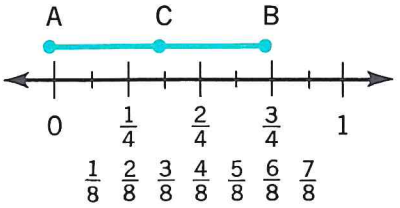
Segment AB is $\frac{3}{4}$ unit long.
C separates $\overline{AB}$ into 2 segments of equal length.

The measure of $\overline{AC}$ is $\frac{1}{2}$ of $\frac{3}{4}$ unit.



To measure $\overline{AC}$, we need a number line showing eighths.

Point C corresponds to $\frac{3}{8}$. The measure of $\overline{AC}$ is $\frac{3}{8}$ unit.

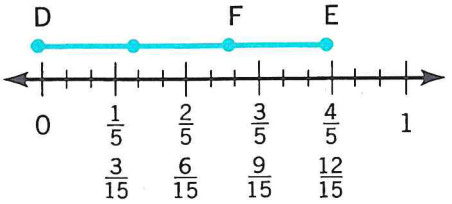


1. Find a segment whose length is

   a. $\frac{4}{5}$ unit

   b. $\frac{2}{3} \times \frac{4}{5}$ unit

   c. $\frac{8}{15}$ unit

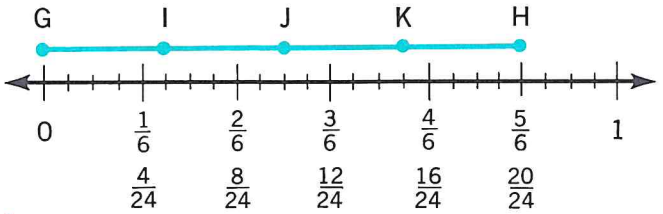


2. Find a segment whose length is

   a. $\frac{5}{6}$ unit

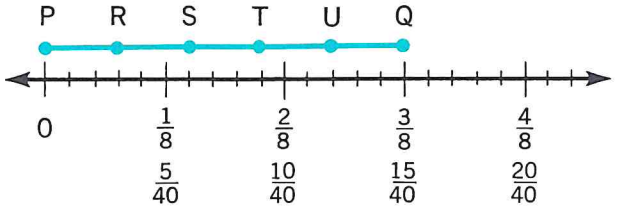
   b. $\frac{1}{4} \times \frac{5}{6}$ unit

   c. $\frac{5}{24}$ unit



3. Find a segment with P as an endpoint, whose length is

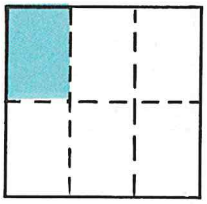
   a. $\frac{3}{8}$ unit b. $\frac{2}{5} \times \frac{3}{8}$ unit



Areas of Fractional Parts

1. The area of each square is 1 sq. unit. What is the area of the shaded part of each square?

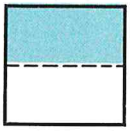
Example:

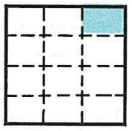


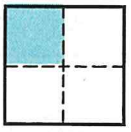
$\frac{1}{6}$ of 1 square unit is shaded.

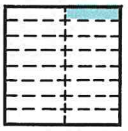
$\frac{1}{6} \times 1 = \frac{1}{6}$

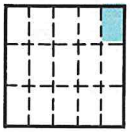
So, $\frac{1}{6}$ square unit is shaded.

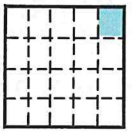
a. 

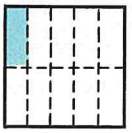
b. 

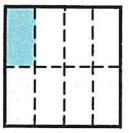
c. 

d. 

e. 

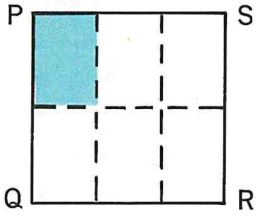
f. 

g. 

h. 

2. Use the squares to help you complete the chart.

Example:



The measure of $\overline{PQ}$ is 1 unit, so the length of the shaded rectangle is $\frac{1}{2}$ unit.

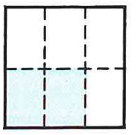
The measure of $\overline{PS}$ is 1 unit, so the width of the shaded rectangle is $\frac{1}{3}$ unit.

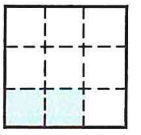
From the example, Exercise 1, its area is $\frac{1}{6}$ sq. unit.

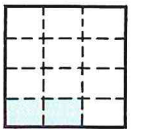
Square	a	b	c	d	e	f	g	h
Length of shaded part (in units)	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{2}$				$\frac{1}{2}$	
Width of shaded part (in units)		$\frac{1}{3}$			$\frac{1}{5}$			
Area of shaded part (in sq. units)		$\frac{1}{12}$		$\frac{1}{16}$				$\frac{1}{8}$

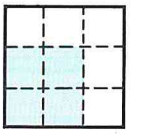
1. The area of each square is 1 sq. unit. What is the area of the shaded part of each square?

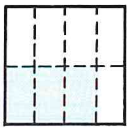
For square a: $\frac{2}{6} \times 1 = \frac{2}{6}$ so $\frac{2}{6}$ sq. unit is shaded.

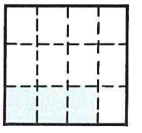
a. 

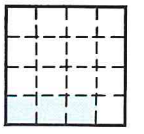
b. 

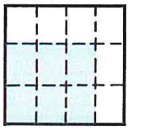
c. 

d. 

e. 

f. 

g. 

h. 

2. Give the length and width of the shaded part of each square.

For square a, the length is $\frac{2}{3}$ unit, width $\frac{1}{2}$ unit.

3. Use the results of Exercises 1 and 2 to write an equation for the area for each shaded part.

For square a, the equation is $\frac{1}{2} \times \frac{2}{3} = \frac{2}{6}$.

4. Find each product.

a. $\frac{1}{3} \times \frac{1}{4} =$ ▢

b. $\frac{1}{2} \times \frac{1}{5} =$ ▢

c. $\frac{2}{3} \times \frac{2}{3} =$ ▢

d. $\frac{1}{3} \times \frac{1}{5} =$ ▢

e. $\frac{3}{4} \times \frac{1}{4} =$ ▢

f. $\frac{1}{2} \times \frac{1}{8} =$ ▢

g. $\frac{2}{3} \times \frac{1}{4} =$ ▢

h. $\frac{3}{4} \times \frac{1}{2} =$ ▢

i. $\frac{3}{4} \times \frac{2}{3} =$ ▢

To multiply fractions,

1. Multiply the numerators.
2. Multiply the denominators.
3. If possible, express the product as a whole number or a mixed number.
4. Express the fraction in lowest terms.

$$\frac{3}{4} \times \frac{16}{9} = \frac{3 \times 16}{4 \times 9} = \frac{48}{36} = 1\frac{12}{36} = 1\frac{1}{3}$$

1. Which of the following products are equal to 1? to 3? to $\frac{1}{2}$? to $\frac{2}{3}$?

a. $5 \times \frac{1}{10}$	e. $\frac{4}{4} \times \frac{5}{5}$	i. $\frac{2}{3} \times \frac{9}{2}$	m. $\frac{1 \times 2}{3 \times 1}$
b. $\frac{1}{2} \times \frac{4}{3}$	f. $\frac{3 \times 5}{5 \times 6}$	j. $\frac{8}{7} \times \frac{7}{8}$	n. $\frac{4}{6} \times \frac{3}{2}$
c. $\frac{2}{3} \times \frac{3}{2}$	g. $\frac{1}{3} \times 9$	k. $\frac{1}{4} \times 2$	o. $\frac{1}{4} \times \frac{24}{2}$
d. $15 \times \frac{1}{5}$	h. $\frac{2 \times 2}{2 \times 3}$	l. $\frac{2}{5} \times \frac{5}{3}$	p. $\frac{2}{4} \times \frac{4}{3}$

2. Find each product. Express all answers in lowest terms.

a. $\frac{1}{2} \times \frac{4}{5} =$ ■	g. $\frac{4}{9} \times \frac{3}{8} =$ ■	m. $\frac{4}{5} \times \frac{7}{10} =$ ■
b. $\frac{2}{3} \times \frac{3}{7} =$ ■	h. $\frac{7}{8} \times \frac{4}{9} =$ ■	n. $\frac{3}{2} \times \frac{6}{7} =$ ■
c. $\frac{1}{3} \times \frac{7}{8} =$ ■	i. $\frac{3}{10} \times \frac{2}{5} =$ ■	o. $\frac{9}{8} \times \frac{2}{3} =$ ■
d. $\frac{3}{4} \times \frac{4}{5} =$ ■	j. $\frac{11}{12} \times \frac{3}{11} =$ ■	p. $\frac{4}{11} \times \frac{11}{3} =$ ■
e. $\frac{2}{5} \times \frac{5}{4} =$ ■	k. $\frac{9}{2} \times \frac{1}{3} =$ ■	q. $\frac{3}{8} \times \frac{3}{8} =$ ■
f. $\frac{6}{7} \times \frac{11}{12} =$ ■	l. $\frac{11}{4} \times \frac{2}{5} =$ ■	r. $\frac{9}{10} \times \frac{9}{10} =$ ■

3. a. At 9 A.M., a 36 foot pole casts a shadow $\frac{5}{4}$ the height of the pole. How long is the shadow?

$\frac{5}{4} \times 36 =$ ■ The shadow is ■ feet long.

b. At noon the shadow is $\frac{1}{8}$ the height of the pole. How long is the shadow?

c. At 5 P.M. the shadow is $\frac{3}{2}$ the height of the pole. What is its length?

4. Jellybeans cost 32¢ per lb. Jeff bought $\frac{3}{8}$ lb., Mike bought $\frac{1}{2}$ lb. and Steve bought $\frac{9}{16}$ lb. How much did each have to pay?

5. Al drank $\frac{3}{4}$ qt. of milk. Pete drank $\frac{3}{4}$ as much as Al. How much did Pete drink?

$\frac{3}{4} \times \frac{3}{4} =$ ■ Pete drank ■ qt. of milk.

1. Which of the following are equal to 1?

a. $\frac{4}{4}$ c. $\frac{7}{7}$ e. $\frac{6 \times 6}{6}$ g. $\frac{3}{3}$ i. $\frac{10}{10}$

b. $\frac{6}{3}$ d. $\frac{3 \times 2}{6}$ f. $\frac{1 \times 4}{4 \times 1}$ h. $\frac{9}{8}$ j. $\frac{0 \times 4}{4}$

For any number m, $m \neq 0$,

$$m \div m = 1$$

2. Find each product.

a. $1 \times \frac{2}{3} =$ ▢ d. $\frac{3}{3} \times 8 =$ ▢ g. $\frac{6}{6} \times \frac{7}{9} =$ ▢

b. $\frac{4}{5} \times 1 =$ ▢ e. $\frac{4}{4} \times \frac{1}{4} =$ ▢ h. $\frac{1}{10} \times \frac{2}{2} =$ ▢

c. $6 \times 1 =$ ▢ f. $\frac{3}{8} \times \frac{5}{5} =$ ▢ i. $\frac{7}{20} \times 1 =$ ▢

For any number m,

$$m \times 1 = 1 \times m = m.$$

3. Find each product.

a. $\frac{1}{5} \times 5 =$ ▢ d. $\frac{1}{3} \times 3 =$ ▢ g. $\frac{9}{10} \times \frac{10}{9} =$ ▢

b. $6 \times \frac{1}{2} =$ ▢ e. $\frac{7}{8} \times \frac{8}{7} =$ ▢ h. $\frac{21}{4} \times \frac{4}{21} =$ ▢

c. $8 \times \frac{1}{8} =$ ▢ f. $\frac{5}{2} \times \frac{2}{3} =$ ▢ i. $\frac{3}{5} \times \frac{4}{5} =$ ▢

For any number a, if $a \neq 0$, we can find a number b so that:

$$a \times b = 1$$

The number a is the **multiplicative inverse** of b, and b is the multiplicative inverse of a.

4. Find the multiplicative inverse of each number.

$\frac{3}{2}$ is the multiplicative inverse of $\frac{2}{3}$ because $\frac{2}{3} \times \frac{3}{2} = 1$.

a. $\frac{4}{5}$ c. $\frac{1}{2}$ e. 19 g. $\frac{3}{4}$ i. $\frac{4}{6}$

b. 6 d. $\frac{3}{8}$ f. $\frac{6}{5}$ h. $\frac{7}{10}$ j. 5

1. Choose $=$ or $\neq$.

a. $\left(\frac{7}{8} \times \frac{8}{7}\right) \times \frac{4}{5} \bullet \frac{4}{5}$

b. $\left(\frac{2}{5} \times \frac{5}{2}\right) \times \frac{3}{5} \bullet \frac{7}{10}$

c. $\left(4 \times \frac{1}{4}\right) \times \frac{1}{2} \bullet \frac{1}{2}$

d. $\frac{2}{3} \bullet \frac{4}{6} \times \left(\frac{1}{9} \times 9\right)$

e. $\frac{5}{6} \bullet \frac{5}{6} \times \left(\frac{7}{2} \times \frac{2}{7}\right)$

f. $\frac{3}{8} \bullet \frac{3}{4} \times \left(2 \times \frac{1}{2}\right)$

2. True or False?

a. If $m = 2$, then $3 \times m = 3 \times 2$

b. If $n = 4$, then $5 \times n = 5 \times 4$

c. If $p = 9$, then $2 \times p = 2 \times 9$

d. If $s = 6$, then $7 \times s = 7 \times 6$

Multiplying both sides of a true equation by the same number yields a true equation.

If $t = 8$, then $7 \times t = 7 \times 8$.

3. For each pair of equations, tell whether or not n equals m.

a.
$$n = \frac{2}{3} \times 5$$
$$\frac{2}{3} \times \frac{3}{2} \times m = \frac{2}{3} \times 5$$

b.
$$n = \frac{1}{2} \times 3$$
$$\frac{1}{2} \times 2 \times m = \frac{1}{2} \times 3$$

c.
$$n = \frac{1}{3} \times \frac{1}{5}$$
$$3 \times \frac{2}{3} \times m = \frac{1}{3} \times \frac{1}{5}$$

d.
$$n = 4 \times \frac{5}{6}$$
$$4 \times \frac{1}{4} \times m = 4 \times \frac{3}{8}$$

4. True or false?

a. If $2 \times d = 9$, then $\frac{1}{2} \times 2 \times d = \frac{1}{2} \times 9$.

b. If $\frac{1}{2} \times e = 6$, then $2 \times \frac{1}{2} \times e = 2 \times 6$.

c. If $\frac{3}{4} \times f = 7$, then $\frac{4}{3} \times \frac{3}{4} \times f = \frac{4}{3} \times 7$.

d. If $4 \times g = 5$, then $\frac{1}{4} \times 4 \times g = 5$.

e. If $\frac{3}{2} \times h = 1$, then $\frac{2}{3} \times \frac{3}{2} \times h = \frac{2}{3} \times 1$.

5. In Exercise 4, solve the last equation in each example to find d, e, f, g, and h.

To solve the equation $\frac{2}{3} \times b = \frac{1}{4}$ we must find a value for "b" which makes the equation true.

1. Multiply both sides of the equation by $\frac{3}{2}$.
2. Simplify.
3. Check: $\frac{2}{3} \times \frac{3}{8} = \frac{1}{4}$

We choose $\frac{3}{2}$ because $\frac{2}{3} \times \frac{3}{2} = 1$.

$$\frac{3}{2} \times \frac{2}{3} \times b = \frac{3}{2} \times \frac{1}{4}$$
$$b = \frac{3}{8}$$

Solve and check each equation.

1. $\frac{1}{3} \times w = 4$
2. $5 \times v = \frac{2}{3}$
3. $\frac{3}{5} \times t = \frac{5}{6}$
4. $\frac{1}{2} \times s = 4$
5. $6 \times r = \frac{3}{7}$
6. $\frac{2}{5} \times j = \frac{3}{5}$
7. $\frac{7}{8} \times k = \frac{9}{10}$
8. $\frac{1}{6} \times c = \frac{3}{4}$
9. $\frac{3}{7} \times m = \frac{5}{8}$
10. $\frac{1}{5} \times n = \frac{7}{10}$
11. $\frac{3}{7} \times p = \frac{1}{14}$
12. $\frac{9}{10} \times q = 7$
13. $8 \times h = \frac{7}{8}$
14. $\frac{1}{7} \times e = \frac{3}{5}$
15. $\frac{4}{5} \times f = 3$

For which equations is the solution incorrect? Give the correct solution.

16. $4 \times p = 3,\ p = \frac{3}{4}$
17. $5 \times t = \frac{4}{7},\ t = \frac{20}{7}$
18. $\frac{3}{10} \times r = 6,\ r = 20$
19. $\frac{2}{3} \times j = \frac{8}{9},\ j = \frac{4}{3}$
20. $\frac{1}{5} \times k = \frac{5}{6},\ k = \frac{1}{6}$
21. $\frac{2}{7} \times m = \frac{3}{4},\ m = \frac{5}{4}$

Solve and check each equation.

$$b \times \frac{4}{5} = \frac{1}{3}$$
$$b \times \frac{4}{5} \times \frac{5}{4} = \frac{1}{3} \times \frac{5}{4}$$
$$b = \frac{5}{12}$$

Check: $\frac{5}{12} \times \frac{4}{5} = \frac{20}{60} = \frac{1}{3}$

22. $m \times \frac{1}{3} = 6$
23. $y \times \frac{3}{8} = \frac{4}{5}$
24. $k \times 5 = 3$
25. $p \times \frac{3}{4} = \frac{2}{3}$
26. $r \times 7 = \frac{9}{14}$
27. $\frac{4}{7} \times q = 4$
28. $5 \times j = \frac{9}{10}$
29. $s \times \frac{5}{6} = \frac{6}{5}$
30. $d \times \frac{7}{9} = 3$
31. $6 \times e = \frac{18}{19}$
32. $\frac{3}{10} \times t = \frac{10}{3}$
33. $b \times \frac{1}{6} = 7$

Division of Fractions

1. What is the multiplicative inverse of each number?

a. $\frac{1}{2}$	c. $\frac{4}{5}$	e. 3	g. $\frac{7}{8}$	i. $\frac{9}{10}$	k. $\frac{11}{3}$
b. 8	d. $\frac{1}{6}$	f. 23	h. 1	j. $\frac{3}{5}$	l. 10

We can multiply the numerator and the denominator of a fraction by the same number to find an equal fraction.

We know that $\frac{5}{6} = \frac{5 \times 2}{6 \times 2}$.

We also know that $\frac{5}{6} = 5 \div 6$.

So $5 \div 6 = (5 \times 2) \div (6 \times 2)$.

We can multiply the dividend and the divisor by the same number without changing our quotient.

2. Copy each problem. Give a reason for each step.

$m = 3 \div \frac{5}{8}$

$= \left(3 \times \frac{8}{5}\right) \div \left(\frac{5}{8} \times \frac{8}{5}\right)$ because multiplying dividend and divisor by $\frac{8}{5}$ doesn't change the result.

$= \frac{24}{5} \div 1$ because $3 \times \frac{8}{5} = \frac{24}{5}$ and $\frac{5}{8} \times \frac{8}{5} = 1$

$= \frac{24}{5}$ because $\frac{24}{5} \div 1 = \frac{24}{5}$

$= 4\frac{4}{5}$ because $\frac{24}{5} = 4\frac{4}{5}$

a. $y = 5 \div \frac{1}{2}$

$= (5 \times 2) \div \left(\frac{1}{2} \times 2\right)$

$= 10 \div 1$

$= 10$

b. $j = \frac{1}{4} \div \frac{1}{3}$

$= \left(\frac{1}{4} \times 3\right) \div \left(\frac{1}{3} \times 3\right)$

$= \frac{3}{4} \div 1$

$= \frac{3}{4}$

Complete each problem.

$$\frac{2}{3} \div \frac{5}{7} = \left(\frac{2}{3} \times \frac{7}{5}\right) \div \left(\frac{5}{7} \times \frac{7}{5}\right)$$
$$= \frac{14}{15} \div 1$$
$$= \frac{14}{15}$$

1. $\frac{3}{5} \div \frac{1}{6} =$ ▢
2. $\frac{4}{7} \div \frac{1}{10} =$ ▢
3. $\frac{7}{8} \div \frac{3}{4} =$ ▢
4. $\frac{4}{5} \div 6 =$ ▢
5. $\frac{3}{4} \div \frac{2}{3} =$ ▢
6. $\frac{3}{10} \div \frac{5}{6} =$ ▢
7. $3 \div \frac{4}{5} =$ ▢
8. $\frac{1}{5} \div \frac{1}{6} =$ ▢
9. $\frac{1}{9} \div \frac{3}{8} =$ ▢
10. $\frac{2}{7} \div 8 =$ ▢
11. $5 \div \frac{9}{10} =$ ▢
12. $\frac{1}{6} \div 4 =$ ▢

We can omit some steps:

Instead of: $\frac{1}{5} \div \frac{2}{3} = \left(\frac{1}{5} \times \frac{3}{2}\right) \div \left(\frac{2}{3} \times \frac{3}{2}\right)$
$= \frac{3}{10} \div 1$
$= \frac{3}{10}$

We write: $\frac{1}{5} \div \frac{2}{3} = \frac{1}{5} \times \frac{3}{2}$
$= \frac{3}{10}$

To divide fractions, multiply by the multiplicative inverse of the divisor. Check by multiplication.

$\frac{2}{5} \div \frac{6}{7} = \frac{2}{5} \times \frac{7}{6}$
$= \frac{14}{30}$
$= \frac{7}{15}$

Check: $\frac{7}{15} \times \frac{6}{7} = \frac{42}{105}$
$= \frac{2}{5}$

Complete each problem.

13. $\frac{7}{8} \div 5 =$ ▢
14. $\frac{4}{5} \div \frac{4}{5} =$ ▢
15. $\frac{3}{4} \div \frac{5}{9} =$ ▢
16. $\frac{9}{10} \div 2 =$ ▢
17. $3 \div \frac{7}{8} =$ ▢
18. $\frac{15}{16} \div 5 =$ ▢
19. $\frac{2}{7} \div \frac{2}{5} =$ ▢
20. $\frac{1}{10} \div \frac{2}{3} =$ ▢
21. $9 \div \frac{1}{6} =$ ▢
22. $\frac{1}{2} \div \frac{3}{10} =$ ▢
23. $\frac{1}{7} \div 4 =$ ▢
24. $\frac{4}{9} \div \frac{5}{6} =$ ▢
25. $\frac{2}{5} \div \frac{1}{8} =$ ▢
26. $\frac{11}{12} \div \frac{1}{3} =$ ▢
27. $14 \div \frac{1}{2} =$ ▢

If $a \div b = c$, then $b = a \div c$ and $a = b \times c$.

We use these facts to help solve equations.

$y \div \frac{4}{5} = \frac{1}{3}$

because if $a \div b = c$ then $a = b \times c$

$y = \frac{4}{5} \times \frac{1}{3}$

$= \frac{4}{15}$

$\frac{1}{2} \div s = \frac{2}{3}$

because if $a \div b = c$ then $b = a \div c$

$s = \frac{1}{2} \div \frac{2}{3}$

$= \frac{1}{2} \times \frac{3}{2}$

$= \frac{3}{4}$

Solve each equation.

1. $x \div \frac{2}{3} = \frac{6}{7}$
2. $m \div \frac{2}{5} = 4$
3. $p \div \frac{1}{3} = \frac{3}{4}$
4. $s \div \frac{1}{4} = \frac{4}{5}$
5. $w \div \frac{5}{6} = \frac{1}{3}$
6. $y \div 3 = \frac{2}{3}$
7. $e \div \frac{3}{5} = 3$
8. $b \div \frac{2}{7} = \frac{1}{7}$
9. $r \div \frac{5}{6} = \frac{1}{6}$
10. $\frac{3}{5} \div w = \frac{3}{5}$
11. $6 \div t = 1$
12. $\frac{11}{12} \div v = \frac{1}{3}$
13. $4 \div s = 5$
14. $\frac{1}{6} \div r = \frac{5}{6}$
15. $\frac{3}{4} \div q = \frac{1}{8}$
16. $\frac{4}{7} \div k = \frac{8}{9}$
17. $\frac{3}{5} \div n = \frac{2}{3}$
18. $3 \div p = 7$
19. $\frac{2}{3} \div f = \frac{1}{8}$
20. $\frac{11}{12} \div g = \frac{1}{3}$
21. $v \div \frac{1}{5} = \frac{5}{6}$
22. $\frac{1}{2} \div n = \frac{9}{10}$
23. $p \div \frac{3}{8} = \frac{1}{4}$
24. $\frac{7}{8} \div m = \frac{2}{3}$
25. $\frac{5}{6} \div e = \frac{4}{5}$
26. $n \div 4 = \frac{7}{8}$
27. $\frac{1}{4} \div j = \frac{3}{8}$

Solve each equation.

$m = \frac{2}{7} \div \frac{2}{3}$

$= \frac{2}{7} \times \frac{3}{2}$

$= \frac{6}{14}$

$= \frac{3}{7}$

28. $m = \frac{7}{8} \div \frac{1}{3}$
29. $y = \frac{2}{3} \div 5$
30. $x = \frac{6}{7} \div \frac{5}{6}$
31. $n = \frac{8}{9} \div \frac{2}{3}$
32. $p = \frac{1}{3} \div \frac{5}{8}$
33. $r = 6 \div \frac{3}{4}$
34. $e = \frac{3}{7} \div 8$
35. $a = \frac{7}{10} \div \frac{1}{3}$
36. $c = \frac{2}{5} \div 7$
37. $f = \frac{8}{9} \div \frac{9}{8}$
38. $t = 7 \div \frac{3}{8}$
39. $s = \frac{3}{5} \div \frac{2}{7}$
40. $b = \frac{2}{3} \div \frac{3}{2}$
41. $w = \frac{1}{2} \div \frac{7}{8}$
42. $v = \frac{3}{4} \div \frac{5}{6}$

We can multiply and divide fractions.

To multiply or divide mixed numbers:

1. Express any mixed numbers as improper fractions.
2. Follow the rules for fractions.

$$2\tfrac{1}{5} \times 3\tfrac{6}{7} = \tfrac{11}{5} \times \tfrac{27}{7} = \tfrac{11 \times 27}{5 \times 7} = \tfrac{297}{35} = 8\tfrac{17}{35}$$

because $2\tfrac{1}{5} = \tfrac{11}{5}$ and $3\tfrac{6}{7} = \tfrac{27}{7}$

$$2\tfrac{1}{5} \div 3\tfrac{6}{7} = \tfrac{11}{5} \div \tfrac{27}{7} = \tfrac{11}{5} \times \tfrac{7}{27} = \tfrac{77}{135}$$

Complete each problem.

1. $4\frac{1}{2} \times 3$
2. $7\frac{2}{3} \times \frac{1}{3}$
3. $4 \times 8\frac{3}{8}$
4. $\frac{1}{5} \times 9\frac{2}{5}$
5. $3\frac{1}{5} \times 1\frac{4}{5}$
6. $2\frac{1}{6} \times 3\frac{5}{9}$
7. $4\frac{2}{3} \times 1\frac{4}{5}$
8. $6\frac{1}{8} \times 2\frac{3}{4}$
9. $2\frac{7}{10} \times 1\frac{2}{5}$
10. $3\frac{5}{8} \div 4$
11. $6\frac{1}{7} \div \frac{1}{2}$
12. $5\frac{9}{10} \div \frac{2}{5}$
13. $4 \div 2\frac{3}{7}$
14. $6\frac{5}{6} \div 2\frac{2}{3}$
15. $5\frac{3}{8} \div 3\frac{3}{4}$
16. $1\frac{4}{7} \div 4\frac{1}{7}$
17. $6\frac{1}{4} \div 2\frac{3}{8}$
18. $4\frac{5}{6} \div 1\frac{7}{10}$

19. John lived $2\frac{1}{3}$ miles from school. Jim lived only $\frac{3}{4}$ as far away from school as John did. How many miles from school did Jim live?

20. Pam goes to school on a bus. The trip takes $\frac{3}{4}$ hour. Dawn makes the trip by bike, taking $\frac{1}{3}$ the time Pam's trip takes. How many minutes does it take Dawn to get to school?

21. On Jan. 31, 1958 just seven minutes after lift-off, America's first satellite was placed in orbit. The first stage of the rocket burned out in about $\frac{3}{10}$ of the time taken to get into orbit. Approximately how many seconds did it take for the first stage to burn out?

We can multiply mixed numbers by using the distributive law for multiplication.

$7\frac{2}{5} \times 4 = \left(7 + \frac{2}{5}\right) \times 4$ because $7\frac{2}{5} = 7 + \frac{2}{5}$

$= (7 \times 4) + \left(\frac{2}{5} \times 4\right)$ because multiplication is distributive

$= 28 + 1\frac{3}{5}$ because $7 \times 4 = 28$ and $\frac{2}{5} \times 4 = 1\frac{3}{5}$

$= 29\frac{3}{5}$ because $28 + 1\frac{3}{5} = 29\frac{3}{5}$

1. Find each product. Follow the examples given above. Give a reason for each step.

a. $3 \times 5\frac{1}{8} =$ ■

b. $2\frac{1}{3} \times 4 =$ ■

c. $6\frac{1}{2} \times 8 =$ ■

d. $2 \times 1\frac{1}{4} =$ ■

e. $3 \times 2\frac{3}{8} =$ ■

f. $6\frac{1}{4} \times 5 =$ ■

g. $6 \times 7\frac{1}{2} =$ ■

h. $4\frac{1}{2} \times 8 =$ ■

i. $18 \times 4\frac{1}{3} =$ ■

2. Complete the reason for each step.

a. $2\frac{1}{3} \times 4\frac{5}{6} = \left(2 + \frac{1}{3}\right) \times \left(4 + \frac{5}{6}\right)$

because $2\frac{1}{3} = 2 + \frac{1}{3}$ and $4\frac{5}{6} = 4 +$ ■

b. $= \left((2 + \frac{1}{3}) \times 4\right) + \left((2 + \frac{1}{3}) \times \frac{5}{6}\right)$

because multiplication is ■

c. $= \left((2 \times 4) + (\frac{1}{3} \times 4)\right) + \left((2 \times \frac{5}{6}) + (\frac{1}{3} \times \frac{5}{6})\right)$

because multiplication is ■

d. $= \left(8 + \frac{4}{3}\right) + \left(\frac{5}{3} + \frac{5}{18}\right)$

because $2 \times 4 = 8$, $\frac{1}{3} \times 4 = \frac{4}{3}$, $2 \times \frac{5}{6} = \frac{5}{3}$, and $\frac{1}{3} \times \frac{5}{6} =$ ■

e. $= 8 + \left(\frac{4}{3} + \frac{5}{3}\right) + \frac{5}{18}$ because addition is ■

f. $= 8 + 3 + \frac{5}{18}$ because $\frac{4}{3} + \frac{5}{3} = \frac{9}{3} =$ ■

g. $= 11\frac{5}{18}$ because $8 + 3 + \frac{5}{18} =$ ■

Estimating Answers

Before solving a problem, we can **estimate** our answer.

Our problem: $2\frac{2}{3} \times \frac{1}{2}$

We know that $2 \times \frac{1}{2} = 1$ and that $3 \times \frac{1}{2} = 1\frac{1}{2}$

So: $1 < 2\frac{2}{3} \times \frac{1}{2} < 1\frac{1}{2}$

In each problem is a, b, or c closest to the exact answer. Do not find the exact product or quotient.

1. a. $0 < \left(2 \times 4\frac{1}{5}\right) < 10$
   b. $8 < \left(2 \times 4\frac{1}{5}\right) < 10$
   c. $5 < \left(2 \times 4\frac{1}{5}\right) < 10$

2. a. $6 < \left(3 \times 2\frac{1}{3}\right) < 7\frac{1}{2}$
   b. $3 < \left(3 \times 2\frac{1}{3}\right) < 9$
   c. $6 < \left(3 \times 2\frac{1}{3}\right) < 9$

3. a. $2 < \left(6 \div 2\frac{1}{2}\right) < 6$
   b. $0 < \left(6 \div 2\frac{1}{2}\right) < 20$
   c. $2 < \left(6 \div 2\frac{1}{2}\right) < 3$

4. a. $1\frac{3}{4} < \left(7\frac{9}{10} \div 4\right) < 2$
   b. $1 < \left(7\frac{9}{10} \div 4\right) < 2$
   c. $0 < \left(7\frac{9}{10} \div 4\right) < 3$

In each problem is a, b, c, or d the closest estimate? Do not work out the exact products or quotients.

5. $4 \times 3\frac{7}{8} =$ ■	a. 12	b. $7\frac{7}{8}$	c. 16	d. $\frac{7}{8}$
6. $9 \div 1\frac{4}{5} =$ ■	a. 9	b. $4\frac{1}{2}$	c. 10	d. 8
7. $2\frac{1}{4} \times 5\frac{9}{10} =$ ■	a. 9	b. 12	c. 8	d. 18
8. $7\frac{2}{3} \div 2 =$ ■	a. 4	b. $3\frac{1}{2}$	c. 9	d. $4\frac{1}{2}$
9. $6\frac{5}{6} \times 4\frac{2}{7} =$ ■	a. 20	b. 24	c. 28	d. 35
10. $8 \div 1\frac{4}{5} =$ ■	a. 2	b. 6	c. 4	d. $2\frac{1}{2}$
11. $9 \times 2\frac{1}{4} =$ ■	a. 18	b. $4\frac{1}{2}$	c. 27	d. 11

1. Clara has some material in the shape of a rectangle which measures $\frac{5}{8}$ by $\frac{4}{8}$ yd. Ida has some material in the shape of a rectangle which measures $\frac{3}{5}$ yd. by $\frac{2}{3}$ yd.
   a. Which girl has more material?
   b. How much more?

2. Mr. Carey has $2\frac{1}{2}$ boxes of candy bars. If each box contains 20 bars when filled, how many candy bars does Mr. Carey have?

3. Vicki and her mother drove to the city. After driving for 30 minutes, her mother said that they had driven $\frac{3}{5}$ of the distance. If the city was 40 miles away from their home, how far had they driven?

4. Tom picked $3\frac{1}{2}$ bushels of apples. If each bushel weighed 60 pounds, how many pounds of apples did Tom pick?

5. At a special "$\frac{1}{4}$ off" sale, Mr. Kent bought a chair that formerly sold for $68.
   a. How much did he get off?
   b. How much did he pay for the chair?

6. Mildred's mother cut $5\frac{1}{4}$ yd. of toweling into 3 pieces that were each the same length. Find the length of each piece.

7. Jack's father drove his automobile for $3\frac{2}{3}$ hours at an average rate of 48 miles per hour. How many miles did he drive?

8. Barbie walked $\frac{7}{8}$ mi. Jean walked $\frac{1}{3}$ the distance Barbie walked. How far did Jean walk?

Checkout

Find each sum or difference.

1. $18\frac{1}{2} - 9\frac{2}{3}$

2. $7\frac{1}{3} + 4\frac{1}{4}$

3. $9\frac{3}{5} - 6\frac{1}{2}$

4. $28\frac{5}{6} + 2\frac{3}{4}$

5. $24\frac{3}{10} - 8\frac{3}{5}$

6. $9\frac{7}{8} + 6\frac{3}{4}$

7. $8\frac{2}{3} - 4\frac{7}{9}$

8. $34\frac{3}{5} - 16\frac{7}{10}$

Complete each problem.

9. $\frac{2}{3} \times 8 =$ ▢

10. $3\frac{1}{2} \times 4 =$ ▢

11. $\frac{9}{10} \div 1 =$ ▢

12. $14 \div \frac{7}{9} =$ ▢

13. $\frac{2}{5} \times \frac{3}{4} =$ ▢

14. $1 \times \frac{2}{5} =$ ▢

15. $\frac{5}{6} \div 2 =$ ▢

16. $\frac{8}{9} \div \frac{5}{9} =$ ▢

17. $\frac{7}{8} \times 6 =$ ▢

18. $\frac{3}{4} \div \frac{2}{9} =$ ▢

19. $1\frac{3}{4} \times 1\frac{1}{3} =$ ▢

20. $16 \div \frac{1}{2} =$ ▢

21. Each side of a hexagon measures $1\frac{2}{3}$ yd. What is its perimeter in yards?

22. Doug walked $1\frac{1}{4}$ miles on Monday, $3\frac{1}{2}$ miles on Tuesday, and $2\frac{3}{4}$ miles on Wednesday. How far did he walk all together on these three days?

23. Mrs. Koski purchased $5\frac{1}{2}$ yd. of material. If she cut it into 4 pieces of equal length, how long was each piece?

24. Mrs. Martinez bought $2\frac{3}{16}$ times as much candy as Mrs. Wong, who bought $2\frac{7}{8}$ lb. About how much candy did Mrs. Martinez buy?

Checkout

Find each difference.

1. $\begin{array}{r} 846 \\ -279 \\ \hline \end{array}$

2. $\begin{array}{r} 604 \\ -439 \\ \hline \end{array}$

3. $\begin{array}{r} 700 \\ -238 \\ \hline \end{array}$

4. $\begin{array}{r} 9042 \\ -2687 \\ \hline \end{array}$

Find each quotient.

5. $4\overline{)928}$

6. $7\overline{)2947}$

7. $9\overline{)3258}$

8. $6\overline{)4352}$

Perform each operation and find each answer set.

9. $\{n: 3 < n < 10\} \cup \{n: 5 < n < 11\} =$ ■

10. $\{n: 73 < n < 77\} \cap \{n: 72 < n < 77\} =$ ■

11. $\{n: 100 < n < 105\} \cup \{n: 99 < n < 103\} =$ ■

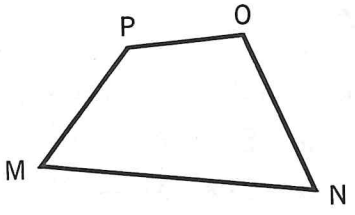
12. $\{n: 17 < n < 23\} - \{n: 16 < n < 22\} =$ ■

13. $\{n: 6 < n < 11\} \cap \{n: n < 6\} =$ ■

14. $\{n: 8 < n < 15\} - \{n: 9 < n < 14\} =$ ■

15. Walter bicycled 1.5 mi. on Monday, 2.4 mi. on Tuesday, 2.8 mi. on Wednesday and 1.9 mi. on Thursday. How far did he bicycle all together?

16. a. Construct a quadrilateral MNOP with the shape of the figure shown.



b. Draw diagonal MO.

c. Construct the circle with diagonal MO as diameter.

d. Do vertices N and P of the quadrilateral lie on the circle?

e. Draw diagonal NP.

f. Is NP a diameter of the circle?

Part 15 Using Decimals

1. Read these aloud.

We read "3.016" as "three and sixteen thousandths".

a. 7.05 b. 9.32 c. 6.008 d. .344 e. 2.101 f. .01

2. Express each decimal in Exercise 1 as a fraction.

$$3.016 = 3 + \frac{16}{1000}$$
$$= \frac{3000}{1000} + \frac{16}{1000}$$
$$= \frac{3016}{1000}$$

3. List these decimals in order from greatest to least:
8.000, .088, .808, .008, .800, 8.808, .080, 8.088

We can multiply decimals and whole numbers.

$$6 \times .18 = 6 \times \frac{18}{100}$$
$$= \frac{6 \times 18}{100}$$
$$= \frac{108}{100}$$
$$= 1\frac{8}{100}$$
$$= 1.08$$

$$3.2 \times 7 = \frac{32}{10} \times 7$$
$$= \frac{32 \times 7}{10}$$
$$= \frac{224}{10}$$
$$= 22\frac{4}{10}$$
$$= 22.4$$

4. Find each product.

a. $5 \times .7 =$ ■ c. $.45 \times 3 =$ ■ e. $4 \times 9.3 =$ ■

b. $6 \times .31 =$ ■ d. $.21 \times 2 =$ ■ f. $1.08 \times 10 =$ ■

5. Choose $<$ or $=$ or $>$.

a. $.004 \times 2$ ● $.04 \times 2$ c. $.400 \times 2$ ● $.40 \times 2$

b. $.040 \times 2$ ● $.04 \times 2$ d. $.04 \times 2$ ● $.40 \times 2$

Instead of multiplying fractions and whole numbers we can count the number of decimal places in the factors to find how many decimal places the product will have. $27 \times \underbrace{4.832}_{3 \text{ places}} = \underbrace{130.464}_{3 \text{ places}}$

1. How many decimal places has each product?

a. $6 \times .38$	**d.** $.81 \times 13$	**g.** $.42 \times 9$	**j.** 5.63×18
b. $8 \times .467$	**e.** $.374 \times 9$	**h.** 1.7×601	**k.** $45 \times .444$
c. 4×5.92	**f.** 15.9×22	**i.** $.831 \times 14$	**l.** 26×2.6

Instead of: $1.4 \times 2 = \frac{14}{10} \times 2$

$= \frac{14 \times 2}{10}$

$= \frac{28}{10}$

$= 2\frac{8}{10}$

$= 2.8$

We write:

$$\begin{array}{r} 1.4 \\ \times\ \ 2 \\ \hline 2.8 \end{array}$$

one place

2. Find each product. Make sure that you include the decimal point when you copy the problem. Check that your answer has the correct number of decimal places.

a. $\begin{array}{r} .28 \\ \times\ \ 3 \\ \hline \end{array}$	**b.** $\begin{array}{r} 4.5 \\ \times 17 \\ \hline \end{array}$	**c.** $\begin{array}{r} 31 \\ \times .18 \\ \hline \end{array}$	**d.** $\begin{array}{r} 777 \\ \times .43 \\ \hline \end{array}$	**e.** $\begin{array}{r} 568 \\ \times .47 \\ \hline \end{array}$
f. $\begin{array}{r} 69 \\ \times .4 \\ \hline \end{array}$	**g.** $\begin{array}{r} 9.32 \\ \times\ 16 \\ \hline \end{array}$	**h.** $\begin{array}{r} 2.8 \\ \times 32 \\ \hline \end{array}$	**i.** $\begin{array}{r} .326 \\ \times\ 15 \\ \hline \end{array}$	**j.** $\begin{array}{r} 9.26 \\ \times\ 61 \\ \hline \end{array}$

Multiplication is commutative. For any numbers a and b,

$$a \times b = b \times a.$$

$23 \times 3.79 = 3.79 \times 23$ $\qquad$ $.31 \times 4256 = 4256 \times .31$

Multiplying Two Decimals

We can multiply decimals by decimals.

$$.3 \times .14 = \frac{3}{10} \times \frac{14}{100} = \frac{3 \times 14}{10 \times 100} = \frac{42}{1000} = .042$$

$$1.6 \times .7 = \frac{16}{10} \times \frac{7}{10} = \frac{16 \times 7}{10 \times 10} = \frac{112}{100} = 1.12$$

1. Find each product. Follow the examples above.

a. $.1 \times .1 =$ ▢
b. $.01 \times .1 =$ ▢
c. $.001 \times .1 =$ ▢
d. $.01 \times .01 =$ ▢
e. $.001 \times .01 =$ ▢
f. $.001 \times .001 =$ ▢
g. $.04 \times .03 =$ ▢
h. $.111 \times .08 =$ ▢
i. $.222 \times .001 =$ ▢

When multiplying fractions whose denominators are multiples of ten, the total number of zeros in the factors tells us how many zeros the product will have.

$$\frac{204}{100} \times \frac{1}{10} = \frac{204}{1000}$$

2 zeros 1 zero 3 zeros

The total number of decimal places in the factors tells us how many decimal places the product will have.

$$\underbrace{2.04}_{2 \text{ places}} \times \underbrace{.1}_{1 \text{ place}} = \underbrace{.204}_{3 \text{ places}}$$

2. How many decimal places will each product have?

a. $.7 \times .3$
b. $.5 \times .08$
c. $.8 \times .8$
d. $.351 \times .2$
e. $.2 \times 3.4$
f. $9.1 \times .11$
g. $.003 \times .002$
h. $.21 \times .004$

1. Choose the correct solution for each equation. You may choose the same solution more than once.

Solutions:

m = .004	m = .4	m = 40
m = .04	m = 4	m = 400

a. .3 × m = .12
b. .3 × m = 12
c. .3 × m = 1.2
d. .03 × m = 1.2
e. .03 × m = 12
f. .03 × m = .12

2. Choose the correct solution for each equation. You may choose the same solution more than once.

Solutions:

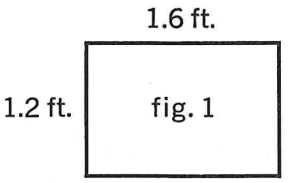
s = .0031	s = .31	s = 31
s = .031	s = 3.1	s = 310

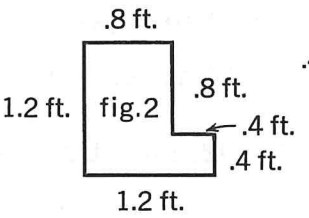
a. s × .23 = .0713
b. s × .23 = 7.13
c. s × 2.3 = .713
d. s × 2.3 = .00713
e. s × .023 = .0713
f. s × .023 = .713

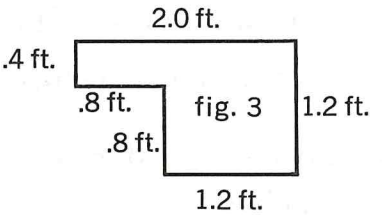
3. Eric walks .76 mile. Jeff walks .6 the distance Eric walks. How far does Jeff walk?

4. A rectangle measures .92 inch by .4 inch. What is its area?

5. Find the perimeter of each figure.







6. Which figure in Exercise 5 has the greatest perimeter?

Instead of:

$$.12 \times 1.8 = \frac{12}{100} \times \frac{18}{10} = \frac{12 \times 18}{100 \times 10} = \frac{216}{1000} = .216$$

We write:

$$\begin{array}{r} .12 \\ \times\ 1.8 \\ \hline .096 \\ .120 \\ \hline .216 \end{array}$$

← 2 decimal places
← 1 decimal place
← 3 decimal places

Make sure your answer has the correct number of decimal places.

Find each product. Follow the example on the right in the box.

1.	.34 ×2.1	2.	9.32 × .6	3.	.55 ×.08	4.	3.15 ×2.14	5.	1.76 ×1.05
6.	6.8 ×1.3	7.	.31 ×.04	8.	2.79 × .11	9.	.876 × .05	10.	7.38 ×.534
11.	.73 × .9	12.	.69 ×.44	13.	3.54 × .6	14.	.721 × .39	15.	.462 ×.312

16. Mary's garden is in the shape of a rectangle measuring 7.85 ft. by 4.2 ft. What is its area?

17. Mr. Clark buys .5 lb. of nails at $.16 per lb., .6 lb. of nails at $.15 per lb. and .4 lb. of nails at $.20 per lb.
   a. How many pounds of nails does he buy all together?
   b. How much does he pay for each kind?
   c. How much does his total bill come to?
   d. How much change should he receive from $10?

18. Bobby says .3 × .2, 3 × .02, and .03 × 2 are all names for the same number.
   a. Is he right?
   b. If so, what is the number?

Find each product.

$$\begin{array}{r} 3.14 \leftarrow \text{2 decimal places} \\ \times\ .125 \leftarrow \text{3 decimal places} \\ \hline .01570 \\ .06280 \\ .31400 \\ \hline .39250 \leftarrow \text{5 decimal places} \end{array}$$

$$\begin{array}{r} 6.85 \leftarrow \text{2 decimal places} \\ \times\ \ .3 \leftarrow \text{1 decimal place} \\ \hline 2.055 \leftarrow \text{3 decimal places} \end{array}$$

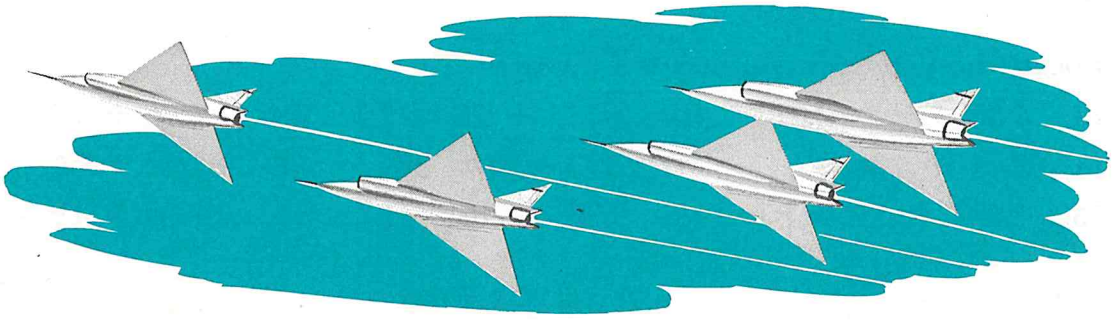
1. $.685 \times .3 =$ ■
2. $14.8 \times .76 =$ ■
3. $.32 \times .5 =$ ■
4. $7.8 \times 6 =$ ■
5. $.35 \times .5 =$ ■
6. $.93 \times .46 =$ ■
7. $9.03 \times .25 =$ ■
8. $.34 \times .9 =$ ■
9. $.834 \times .64 =$ ■
10. $1.02 \times .68 =$ ■
11. $72.3 \times .8 =$ ■
12. $31.6 \times .82 =$ ■
13. $.61 \times 3.20 =$ ■
14. $.610 \times 3.2 =$ ■
15. $.703 \times .84 =$ ■
16. $5.35 \times .19 =$ ■
17. $.886 \times 3.1 =$ ■
18. $.0304 \times .7 =$ ■

19. If 1.6 cu. ft. of plastic will make 597.85 sq. ft. of plastic film, how many sq. ft. of film will 4.8 cu. ft. of plastic make?

20. If a 1 cu. ft. cylinder can contain 36.8 cubic feet of compressed gas, how many cu. ft. of gas can a 2.85 cu. ft. cylinder contain?

21. Mr. Carlson used 16.8 gallons of gasoline. If he paid $.31 per gallon, how much did the gasoline cost him?

22. A jet pilot flying at 600 miles per hour travels about 545.5 ft. during the time he blinks his eyes once. About how far will he travel while blinking his eyes 6 times?



Division of Decimals

We can divide a decimal by a whole number.

$$28.8 \div 9 = 28\tfrac{8}{10} \div 9 = \tfrac{288}{10} \div 9 = \tfrac{32}{10} = 3.2$$

$$.3232 \div 16 = \tfrac{3232}{10000} \div 16 = \tfrac{202}{10000} = .0202$$

Complete each problem.

1. $19\overline{)5.7}$
2. $22\overline{).132}$
3. $38\overline{)1.52}$
4. $11\overline{).044}$
5. $37\overline{)2.96}$
6. $55\overline{).0165}$
7. $44\overline{)52.8}$
8. $14\overline{).266}$
9. $73\overline{).0803}$
10. $68\overline{)47.6}$
11. $20\overline{)19.80}$
12. $15\overline{).285}$

Instead of:

$$28.8 \div 9 = 28\tfrac{8}{10} \div 9 = \tfrac{288}{10} \div 9 = \tfrac{32}{10} = 3.2$$

We write:

$$\begin{array}{r} 3.2 \\ 9\overline{)28.8} \\ \underline{27.0} \\ 1.8 \\ \underline{1.8} \\ .0 \end{array}$$

one place

When dividing a decimal by a whole number, we can count the number of decimal places in the dividend to find how many decimal places the quotient will have.

Locate the decimal point in each quotient.

13. $4.38 \div 2 = 219$
14. $37.2 \div 4 = 93$
15. $1.044 \div 12 = 87$
16. $.0437 \div 23 = 19$
17. $.333 \div 9 = 37$
18. $9.36 \div 78 = 12$
19. $1.428 \div 17 = 84$
20. $403.2 \div 64 = 63$
21. $140.6 \div 19 = 74$

Find each quotient. Check by multiplication.

$$\begin{array}{r} .0202 \\ 16\overline{).3232} \\ .3200 \\ \hline .0032 \\ .0032 \\ \hline .0000 \end{array}$$

Check:

$$\begin{array}{r} .0202 \\ \times\ 16 \\ \hline .1212 \\ .2020 \\ \hline .3232 \end{array}$$

$$\begin{array}{r} \$2.40 \\ 3\overline{)\$7.20} \\ 6.00 \\ \hline 1.20 \\ 1.20 \\ \hline 0.00 \end{array}$$

Check:

$$\begin{array}{r} \$2.40 \\ \times\ 3 \\ \hline \$7.20 \end{array}$$

1. $4\overline{)82.52}$
2. $5\overline{)4.625}$
3. $3\overline{)458.4}$
4. $9\overline{)8.433}$
5. $14\overline{)88.48}$
6. $18\overline{)273.6}$
7. $43\overline{)6.149}$
8. $28\overline{)59.92}$
9. $6\overline{)\$3.72}$
10. $5\overline{)\$8.60}$
11. $4\overline{)\$9.32}$
12. $8\overline{)\$39.84}$
13. $24\overline{)\$44.16}$
14. $16\overline{)\$32.64}$
15. $29\overline{)\$41.47}$
16. $41\overline{)\$88.15}$

Choose $<$ or $=$ or $>$.

17. 1 ● 1.0
18. 2.0 ● 2.1
19. 7.00 ● 7.0
20. 3.0 ● 3.3
21. 6.9 ● 6.90
22. 4.2 ● .42
23. 5.8 ● 5.80
24. .90 ● 9.0
25. .61 ● .16
26. .08 ● .080
27. .20 ● 0.2
28. .006 ● .060

Writing zeros in a numeral to the right of a decimal point does not change the value of a number. Sometimes we use more decimal places in order to get a remainder of zero.

$1 \div 8 = \blacksquare$ In each step we write a "0".

Step 1:

$$\begin{array}{r} .1 \\ 8\overline{)1.0} \\ .8 \\ \hline .2 \end{array}$$

Step 2:

$$\begin{array}{r} .12 \\ 8\overline{)1.00} \\ .8 \\ \hline .20 \\ .16 \\ \hline .04 \end{array}$$

Step 3:

$$\begin{array}{r} .125 \\ 8\overline{)1.000} \\ .8 \\ \hline .20 \\ .16 \\ \hline .040 \\ .040 \\ \hline .000 \end{array}$$

$1 \div 8 = .125$

Check:

$$\begin{array}{r} .125 \\ \times\ 8 \\ \hline 1.000 \end{array}$$

Find each quotient. Add more decimal places to the dividend if you need to. Check by multiplication.

1. $7 \div 8 =$
2. $1 \div 2 =$
3. $.3 \div 15 =$
4. $1 \div 4 =$
5. $.4 \div 8 =$
6. $.1 \div 25 =$
7. $5 \div 8 =$
8. $.02 \div 4 =$
9. $.03 \div 3 =$
10. $.12 \div 5 =$
11. $2.7 \div 6 =$
12. $.7 \div 14 =$
13. $.52 \div 5 =$
14. $.078 \div 3 =$
15. $.009 \div 45 =$

Locate the decimal point in each quotient.

16. $.075 \div 3 = 25$
17. $19.6 \div 2 = 98$
18. $.136 \div 17 = 8$
19. $3 \div 4 = 75$
20. $1 \div 2 = 5$
21. $3 \div 8 = 375$
22. $.6 \div 12 = 5$
23. $.04 \div 5 = 8$
24. $.001 \div 2 = 5$

Copy and complete the table. On the left, multiply the number from column A as indicated. On the right, divide the number from column A, as indicated.

multiply by			A	divide by		
1000	100	10		10	100	1000
400	40	4	.4	.04	.004	.0004
		5	.5			
			.23		.0023	
80			.08			
			.672	.0672		
		5.04	.504			
			.083		.00083	
	31.69		.3169			

Complete each problem.

25. $.37 \times \frac{1}{10} =$
26. $.37 \div 10 =$
27. $.498 \div 100 =$
28. $.498 \times \frac{1}{100} =$
29. $3.578 \times \frac{1}{1000} =$
30. $3.578 \div 1000 =$
31. $3.578 \times .001 =$
32. $26.43 \times .01 =$
33. $6 \div 10 =$
34. $8.3 \times \frac{1}{100} =$
35. $3.21 \div 100 =$
36. $4.76 \div 1000 =$

Multiply each decimal by 10, 100, or 1000 to get the least whole number.

1. a. .4 b. .785 c. .32

2. a. .08 b. .301 c. .67

3. a. .55 b. .003 c. .086

4. a. .709 b. .084 c. .09

5. a. .012 b. .327 c. .273

We find equal fractions by multiplying the numerator and the denominator by the same number.

$$\frac{3}{4} = \frac{3 \times 2}{4 \times 2} = \frac{6}{8}$$

When we multiply the numerator and the denominator by the same number, we are multiplying by one. We can write this another way:

$$\frac{3}{4} = 3 \div 4 \quad \text{so} \quad 3 \div 4 = (3 \times 2) \div (4 \times 2)$$

When we divide, we can multiply the dividend and the divisor by the same number without changing the value of the expression.

$$\begin{aligned} .408 \div .24 &= (.408 \times 100) \div (.24 \times 100) \\ &= 40.8 \div 24 \end{aligned}$$

When we multiply the dividend and the divisor by the same number, we are multiplying by one.

Choose $=$ or $\neq$.

6. a. $.32 \div .4 \bigcirc (.32 \times 10) \div (.4 \times 10)$

b. $.6 \div .3 \bigcirc (.6 \times 10) \div (.3 \times 10)$

c. $.5 \div .78 \bigcirc (.5 \times 10) \div (.78 \times 100)$

d. $.09 \div .6 \bigcirc (.09 \times 100) \div (.6 \times 10)$

Rewrite each expression so the divisor is a whole number.

$$\begin{aligned} 3.172 \div .2 &= (3.172 \times 10) \div (.2 \times 10) \\ &= 31.72 \div 2 \end{aligned}$$

7. a. $6.3 \div .3$ b. $.48 \div .6$

8. a. $.04 \div .5$ b. $1 \div .25$

9. a. $.06 \div 1.2$ b. $.221 \div .17$

10. a. $8.88 \div .296$ b. $.326 \div 1.63$

We can divide decimals:

1. Rewrite each expression so the divisor is a whole number.

$$.04 \div 2.5 = (.04 \times 10) \div (2.5 \times 10)$$
$$= .4 \div 25$$

2. Divide. Keep the decimal points in a straight line.

$$\begin{array}{r} .016 \\ 25\overline{)\,.400} \\ \underline{.25} \\ .150 \\ \underline{.150} \\ .000 \end{array}$$

$.04 \div 2.5 = .016$

3. Check:

$$\begin{array}{r} .016 \\ \times\ 2.5 \\ \hline .0080 \\ .0320 \\ \hline .0400 \end{array}$$

Find each quotient. Check by multiplication.

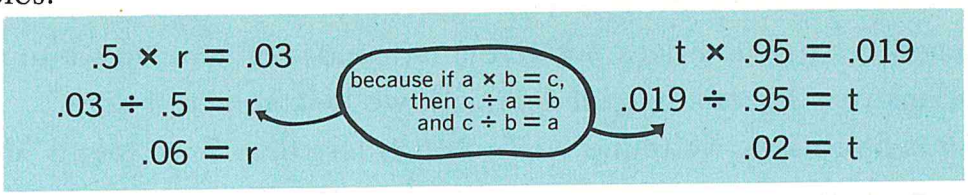
1. $1.54 \div .07 =$
2. $2.40 \div 1.5 =$
3. $.038 \div .19 =$
4. $1.035 \div .09 =$
5. $.625 \div 2.5 =$
6. $12.8 \div 3.2 =$
7. $9.6 \div .08 =$
8. $.9 \div .45 =$
9. $.003 \div 7.5 =$
10. $.165 \div .55 =$
11. $.025 \div .4 =$
12. $.522 \div .18 =$
13. $.1064 \div 1.9 =$
14. $6.32 \div .008 =$
15. $.0087 \div .058 =$
16. $10256 \div .128 =$
17. $.006 \div .04 =$
18. $.0161 \div 2.3 =$

19. The area of a rectangle is 40.32 sq. ft. Its width is 4.2 ft.
    a. What is its length?
    b. What is its perimeter?

20. Michael traveled 829.25 miles in 2.5 days while on vacation last summer. What was the average distance he traveled per day?

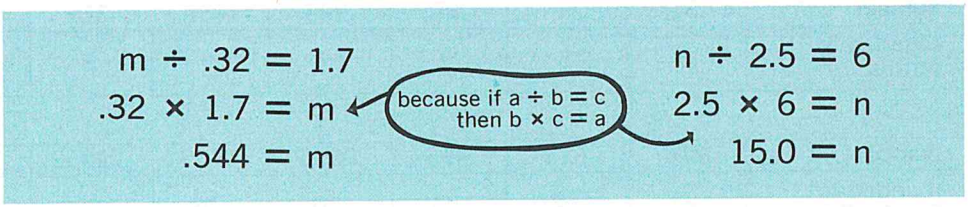
21. George traveled 1177.42 miles in 3.4 days while on vacation. What was the average distance he traveled per day?

Solve each equation by rewriting it as an equation using division. Follow the examples.



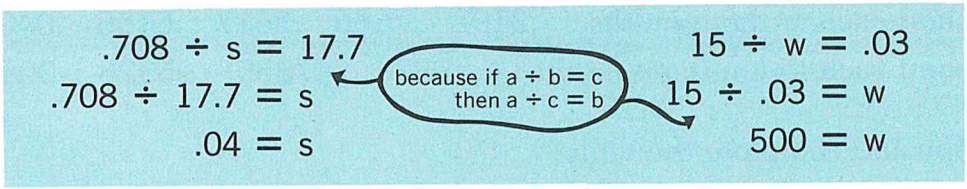
1. 1.6 × v = .8
2. .7 × s = .56
3. .96 × z = 1.2
4. 1.4 × n = .42
5. .5 × p = 4.5
6. 1.7 × m = .51
7. .26 × k = 7.8
8. 3.5 × x = 1.05
9. 7 × w = 49
10. .03 × q = 3.6
11. 60 × y = 4.2
12. .5 × e = .03

Solve each equation by rewriting it as an equation using multiplication. Follow the examples.



13. x ÷ .25 = .3
14. t ÷ .6 = .08
15. w ÷ 3.8 = 5.1
16. k ÷ .06 = 2.1
17. p ÷ .8 = .72
18. r ÷ 12 = .09
19. e ÷ .17 = 3
20. f ÷ .05 = 8.8
21. d ÷ .32 = 1.6

If a ÷ b = c, then a ÷ c = b. Use this fact to rewrite and solve these equations. Follow the examples.



22. 7.5 ÷ k = 3
23. .001 ÷ m = .2
24. .39 ÷ j = 1.3
25. 1.44 ÷ r = 1.2
26. 2.8 ÷ t = .7
27. .018 ÷ p = .09
28. 6.408 ÷ x = .08
29. .729 ÷ c = .9
30. .085 ÷ f = .17

Rounding Off Decimals

Sometimes division does not give a remainder of zero, so we round off our answer. In rounding off decimals, we look at a "test digit". If it is 5 or more, we round "up". If it is less than 5 we round "down".

Rounding to:	Test digit is found here:
tenths (or 1 decimal)	hundredths' place
hundredths (or 2 decimals)	thousandths' place
thousandths (or 3 decimals)	ten-thousandths' place
ten-thousandths (or 4 decimals)	hundred-thousandths' place

The test digits are marked like this: $3\overset{\downarrow}{.}$

number	.31	.47	.85	.985	.451	.319	.0639
rounded to tenths	.3	.5	.9	1.0	.5	.3	.1

number	.216	.823	.495	.0975	.3649	.72859	.14577
rounded to hundredths	.22	.82	.50	.10	.36	.73	.15

number	.0614	.3945	.8888	.39393	.68259	.476299	.008899
rounded to thousandths	.061	.395	.889	.394	.683	.476	.009

1. Round each to tenths : .25 .31 .781 .939 .073 .058
2. Round each to hundredths : .714 .895 .3765 .4139 .0084
3. Round each to thousandths : .3192 .7666 .3617 .5155 .040506
4. Round each to ten-thousandths : .79823 .47855 .95326 .0069923

5. Copy and complete the table.

Round each to	.3275	.4153	.6506	.5328	.4905	.2134	.7692	.8995
1 decimal	.3				.5			
2 decimals	.33		.65				.77	
3 decimals	.328					.213		

When our answer is not exact, we use three dots to show that we could continue our division.

The more places we use, the more nearly exact is the quotient.

$.25 \div 1.2 = (.25 \times 10) \div (1.2 \times 10)$

$= 2.5 \div 12$

$.25 \div 1.2 = .2$ (to 1 decimal)

$.25 \div 1.2 = .21$ (to 2 decimals)

$.25 \div 1.2 = .208$ (to 3 decimals)

$.25 \div 1.2 = .2083$ (to 4 decimals)

$$
\begin{array}{r}
.20833\ldots \\
12\overline{)2.50000} \\
\underline{2.4} \\
.100 \\
\underline{.096} \\
.0040 \\
\underline{.0036} \\
.00040 \\
\underline{.00036} \\
.00004
\end{array}
$$

To find $.25 \div 1.2$ to 5 decimals, we must divide to find what test digit is in the sixth place.

1. Find each quotient to the nearest tenth. (Remember, you must divide to find the test digit in the hundredths' place.)

a. $.631 \div 8$	**d.** $.8 \div .34$	**g.** $.031 \div .2$	**j.** $.8342 \div .6$
b. $.007 \div .1$	**e.** $.082 \div .9$	**h.** $9.32 \div 6.5$	**k.** $.0908 \div .37$
c. $.2 \div .25$	**f.** $1.2 \div .76$	**i.** $8.08 \div .3$	**l.** $.7601 \div .09$

2. Find each quotient to the nearest hundredth.

a. $.85 \div .7$	**d.** $5.632 \div .7$	**g.** $37.8 \div .24$	**j.** $.538 \div .08$
b. $.025 \div .03$	**e.** $.8213 \div .99$	**h.** $4.06 \div .52$	**k.** $.024 \div 1.7$
c. $9.32 \div 4.4$	**f.** $.102 \div .035$	**i.** $9.37 \div .02$	**l.** $3.55 \div 6.2$

3. Find each quotient to the nearest thousandth.

a. $.85 \div .3$	**d.** $9.3 \div .63$	**g.** $.0802 \div 4.1$	**j.** $.612 \div .006$
b. $.76 \div 4.1$	**e.** $.706 \div .54$	**h.** $.7203 \div .55$	**k.** $.076 \div 1.2$
c. $.08 \div .02$	**f.** $.1133 \div .6$	**i.** $.892 \div .074$	**l.** $3.98 \div .07$

Instead of:

$.288 \div .16 = (.288 \times 100) \div (.16 \times 100) = 28.8 \div 16$ $\quad 16\overline{)28.8}$

We write: $.16\overline{).28.8}$

We use loops, ⌣, to show where the new decimal points will be. The decimal point in the answer is located directly above the new decimal point in the dividend. Notice that we have omitted the decimal point in the division steps.

We know: $.288 \div .16 = 1.8$

$$\begin{array}{r} 1.8 \\ .16\overline{).28.8} \\ 16\,0 \\ \hline 12\,8 \\ 12\,8 \\ \hline 0 \end{array}$$

Check:

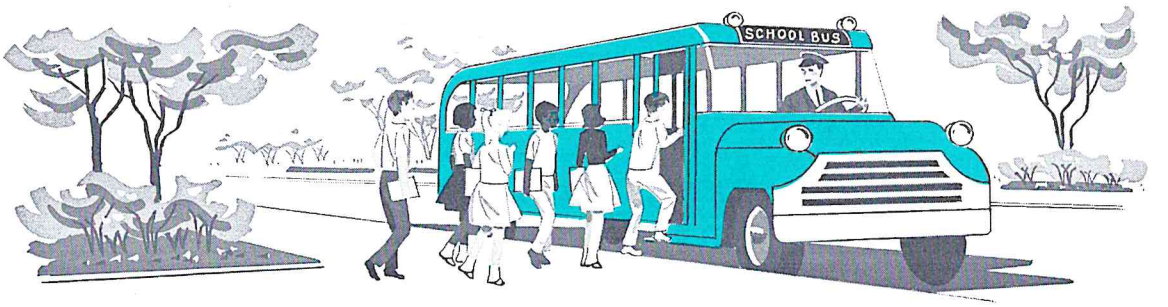
$$\begin{array}{r} 1.8 \\ \times .16 \\ \hline .108 \\ .180 \\ \hline .288 \end{array}$$

Find each answer. Check by multiplication. Follow the example.

1. $.07\overline{)3.024}$
2. $.5\overline{).38}$
3. $3.2\overline{)6.08}$
4. $.14\overline{).49}$
5. $.62\overline{).1302}$
6. $5.4\overline{)2.322}$
7. $1.1\overline{)6.38}$
8. $2.5\overline{)1.915}$
9. $.77\overline{).6391}$
10. $.34\overline{).0323}$
11. $.98\overline{).147}$
12. $.08\overline{).1096}$
13. $.91\overline{)4.277}$
14. $.46\overline{).2116}$
15. $.69\overline{).0897}$
16. $.28\overline{)2.1}$

17. Mr. Ryan's car averages 17.6 miles per gallon of gas. How many gallons will the car use in going 316.8 miles?

18. The school bus averages 7.2 miles per gallon. How many gallons will the bus use in going 316.8 miles?



When we round off our answer, we get an approximate answer. To get an exact answer, or to check our answer, we must remember what the remainder is.

$$
\begin{array}{r}
.0522\ldots \\
.67\overline{).03.5000} \\
\underline{3\,35} \\
150 \\
\underline{134} \\
160 \\
\underline{134} \\
26
\end{array}
$$

remainder: 26 hundred-thousandths

$.035 \div .67 = .0522\frac{26}{67}$ (exact answer)
$.035 \div .67 = .0522$ (to 4 decimals)
$.035 \div .67 = .052$ (to 3 decimals)
$.035 \div .67 = .05$ (to 2 decimals)

To check:

$$
\begin{array}{r}
.0522 \\
\times\ \ .67 \\
\hline
.003654 \\
.03132 \\
\hline
.034974
\end{array}
\qquad
\begin{array}{r}
.034974 \\
+.000026 \\
\hline
.035000
\end{array}
$$

remainder

Give each answer to 2 decimals. Check each problem by multiplication.

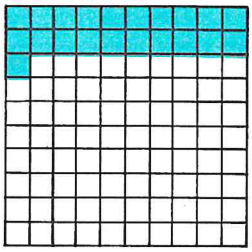
1. $3.5\overline{).7034}$
2. $.07\overline{).92}$
3. $.41\overline{)7.46}$
4. $.68\overline{).8103}$
5. $.44\overline{)5.5}$
6. $.52\overline{)2.3}$
7. $6.1\overline{)7.68}$
8. $.89\overline{).934}$
9. $.37\overline{).28}$
10. $2.8\overline{)6.73}$
11. $.06\overline{)9.2}$
12. $.16\overline{)1.483}$
13. $1.7\overline{)7.8}$
14. $3.3\overline{).089}$
15. $9.4\overline{).097}$
16. $.82\overline{).008}$
17. $.76\overline{).349}$
18. $.59\overline{)7.64}$
19. $8.3\overline{).0721}$
20. $6.6\overline{).0886}$

21. In 1930 Lindbergh flew about 2500 miles in 14.4 hours. Find the average rate per hour.

22. A commercial jet airliner flies about 2500 miles in 4.02 hours. Find the average rate per hour.

Per cent

The region is divided into 100 squares the same size.



21 of the 100 parts are colored.

$\frac{21}{100}$ of the region is colored.

.21 of the region is colored.

21% of the region is colored.

We read "21%" as "twenty-one per cent".

1. Write each as a fraction, as a decimal, and as a per cent.

   a. 92 of 100 parts b. 23 of 25 parts c. 3 of 4 parts

2. Tell what each expression means.

   39% means 39 of 100 parts.

   a. 17% b. 49% c. 100% d. 6% e. 3% f. 5% g. 25%

3. Express each fraction or decimal as a per cent.

   a. .42 b. $\frac{14}{100}$ c. .92 d. .03 e. $\frac{48}{100}$

We can write

"134 of 1000 parts" as "$\frac{134}{1000}$" or ".134" or "13.4%".

"41 of 10,000 parts" as "$\frac{41}{10{,}000}$" or ".0041" or ".41%".

"3 of 8 parts" as "$\frac{3}{8}$" or "$\frac{375}{1000}$" or ".375" or "37.5%".

4. Write each as a fraction whose denominator is 100, 1000, or 10,000, then as a decimal and a percent.

   7 of 8 parts $= \frac{875}{1000} = .875 = 87.5\%$

   a. 478 of 1000 parts c. 1 of 16 parts e. 87 of 125 parts

   b. 9 of 10,000 parts d. 5 of 8 parts f. 576 of 625 parts

1. Which of the following are equal to .4?

a. $\frac{1}{4}$	d. .40	g. $\frac{1}{25}$	j. $\frac{3}{12}$	m. $\frac{6}{15}$
b. $\frac{4}{10}$	e. $\frac{40}{1000}$	h. $\frac{25}{100}$	k. 4%	n. $\frac{2}{50}$
c. .040	f. $\frac{2}{5}$	i. 25%	l. $\frac{4}{100}$	o. 40%

2. Which of the expressions in Exercise 1 are equal to .04?

3. Which of the expressions in Exercise 1 are equal to .25?

4. Bob has read 37 pages of a 50-page book. Jeff has read 75% of the same book. Who has read more?

5. A newspaper account of a basketball game gave this information:

Name	Brown	Casey	Daly	Hall	Levine	Haines	Pitts
% of Shots made	30%	25%	40%	25%	56%	32%	80%
Shots	20	24	10	20	25	25	20

a. How many baskets did each make?

Brown: 30% of 20 = .30 × 20 = 6.00
Brown made 6 baskets.

b. Each basket is worth 2 points. If the team also scores 9 points on foul shots, what is the total score?

c. The opposing team scored 126 points. Who won? By how many points?

6. Solve each problem.

a. 13% of 20	c. 23% of 85	e. 45% of 60	g. 83% of 25
b. 19% of 34	d. 72% of 91	f. 92% of 57	h. 68% of 17

Solve each problem.

4.5% of .3 = .045 × .3 = .0135

1. 2.8% of 34 2. 21% of 18.6 3. 71.4% of 9 4. 13.4% of .048

5. The swimming pool holds 2255.5 cubic feet of water. The pool is 75% full. How much water is in the pool? 75% = .75
2255.5 × .75 = ■ The pool contains ■ cubic feet of water.

6. Mr. Murphy borrowed $1700 at 7.5% interest. How much will the interest be? 7.5% of $1700 = .075 × $1700 = ■

7. Every three months, one bank gives 4.85% interest on savings, another gives 4.9% interest. In three months, which will earn more, $300 at 4.85% or $290 at 4.9%? How much more?

8. A store advertised a sale with all merchandise "10% off". Find the sale price of each item to the nearest cent.

A $7.98 sweater would be reduced by $.80 because 10% of $7.98 = $.80 (to the nearest cent). $7.98 − $.80 = $7.18; so the sale price of the sweater would be $7.18.

item	coat	jacket	shoes	sweater	dress	slacks	skirt	shirt	hat
price	$39.00	$15.50	$9.98	$7.50	$15.79	$12.53	$8.23	$3.25	$10

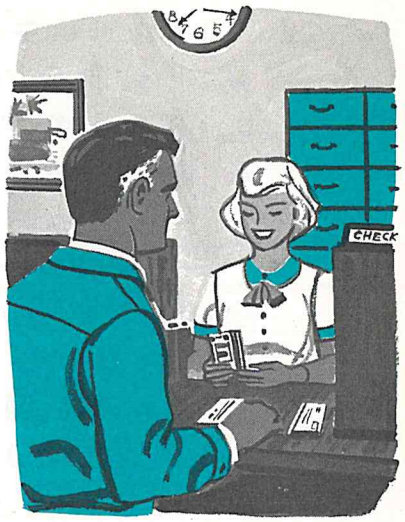
9. In one store, Mrs. Fisher found a coat priced at $50 marked "25% off". In another store, she found exactly the same kind of coat priced at $40 and marked "10% off". To save money, which coat should she buy?

10. Jerry's weekday schedule is shown below. Use it to find how many hours he spends at each activity.

activity	sleeping	eating	school	play	homework	chores
% of day	35%	12.5%	23%	20.3%	5.2%	4%

1. Mr. Chang borrowed \$900 at 7% interest. How much money will the interest be?

$$7\% \text{ of } \$900 = .07 \times \$900 = \blacksquare$$



2. A savings bank gives 5% interest every three months. In three months, how much will \$100 earn?

3. Mr. Bell earns \$7500 a year. He spends 23% on rent, 21% on food, and 13% on clothing.
   a. How much does he spend on rent?
   b. How much does he spend on food?
   c. How much does he spend on clothing?

4. Doug has read 77 pages of a 100-page book; Keith has read 150 pages of a 200-page book. Who has read the greater per cent of his book?

5. On a history quiz Jean had 20 of 25 questions correct. She missed 3 of 10 words on a spelling quiz. On which quiz did she have a greater per cent of correct answers?

6. On a science quiz Jeff had 44 of 50 problems correct. Steve had 86% of the 50 problems correct. Who had more problems correct?

7. Terry saves 50% of all the money she earns. Last month she earned \$11. How much did she save?

8. Abe has read 80% of his 325-page book. Jim has read 76% of his 275-page book.
   a. Who has read more pages?
   b. How many more?

Checkout

1. Choose $<$ or $=$ or $>$.

   a. $.2 + .1 \bigcirc .2 + .01$

   b. $.2 \times 3 \bigcirc 6 \times .1$

   c. $4 + .1 \bigcirc .4 + 1$

   d. $5 - 4.1 \bigcirc 5 - .41$

   e. $4.8 + .4 \bigcirc 4 \times 1.3$

   f. $3 - .19 \bigcirc 2 \times 1.4$

2. Which products have the decimal points located in the wrong place? Correct these problems.

   a. $.2 \times .3 = .6$

   b. $.1 \times .4 = .04$

   c. $.31 \times .5 = .155$

   d. $1.6 \times .73 = 11.68$

   e. $.6 \times .81 = .486$

   f. $2.5 \times .3 = .75$

   g. $4.42 \times .2 = 8.84$

   h. $.67 \times .5 = 33.5$

   i. $.42 \times .11 = .00462$

   j. $3.08 \times .22 = 67.76$

   k. $.8 \times 4.7 = 37.6$

   l. $.111 \times .01 = .111$

3. During a basketball game, the coach kept a record of how many shots each boy took, and how many baskets he made.

Name	Pete	Mike	Jon	Ted	Paul	Chuck	Ken	Dave
Baskets	1	3	7	3	16	1	13	2
Shots	10	5	20	10	25	4	20	5
% baskets								

   a. For each boy, find the per cent of baskets he made.

   b. Who made the greatest per cent of shots?

   c. Each basket is worth 2 points. How many points did each boy make?

   d. Who was the high scorer?

4. There were 100 problems on a test. Joe got 99% correct, Elaine got 85% correct, and Judy got 79% correct. How many problems did each get correct? How many problems did each get incorrect?

5. Ben has read 68 pages of a 100-page book. Leon has read 75% of the same book. Who has read more? By how many pages?

Checkout

Complete each problem.

1. $\begin{array}{r} 286 \\ \times\ 9 \\ \hline \end{array}$

2. $\begin{array}{r} 796 \\ \times\ 24 \\ \hline \end{array}$

3. $\begin{array}{r} 842 \\ \times 108 \\ \hline \end{array}$

4. $\begin{array}{r} 659 \\ \times 380 \\ \hline \end{array}$

5. $7\overline{)3242}$

6. $18\overline{)3769}$

7. $73\overline{)75177}$

8. $450\overline{)67382}$

9. $\begin{array}{r} 16\frac{3}{8} \\ +\,18\frac{3}{4} \\ \hline \end{array}$

10. $\begin{array}{r} 37\frac{2}{3} \\ -\,14\frac{5}{6} \\ \hline \end{array}$

11. $\begin{array}{r} 25\frac{2}{5} \\ -\ 9\frac{7}{10} \\ \hline \end{array}$

12. $\begin{array}{r} 14\frac{7}{9} \\ +\ 8\frac{1}{6} \\ \hline \end{array}$

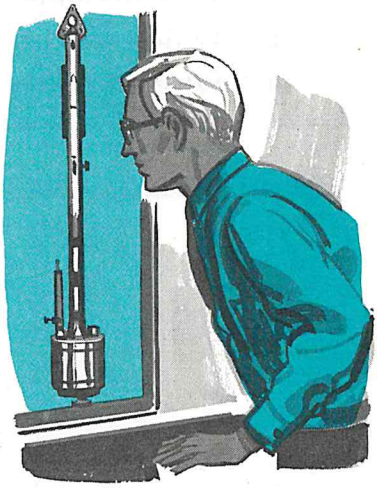
Pair each English sentence with a mathematical sentence.

13. Bert has 2 more stamps than Ted.
14. Bob has 2 less stamps than Tom.
15. Bart has more stamps than Terry.
16. Bonnie has twice as many stamps as Therese.
17. Beth has less stamps than Toni.

A. $b < t$
B. $b = 2t$
C. $b = t - 2$
D. $b > t$
E. $b = t + 2$

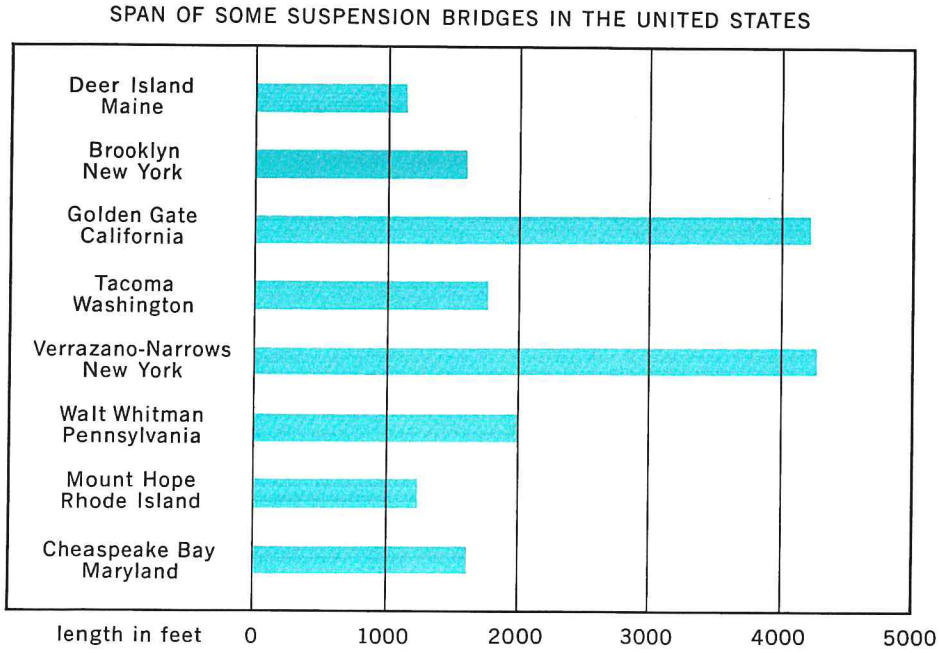
Perform each operation and find the answer set.

18. $\{n: 3 < n < 7\} \cup \{n: 2 < n < 6\} =$ ■
19. $\{n: 25 < n < 29\} \cup \{29, 28, 42\} =$ ■
20. $\{n: n < 4\} \cap \{0, 2, 4, 6, 8\} =$ ■
21. $\{n: 14 < n < 17\} \cap \{n: 10 < n < 18\} =$ ■
22. $\{n: 18 < n < 22\} - \{n: 20 < n < 24\} =$ ■
23. $\{n: 20 < n < 30\} - \{n: 10 < n < 20\} =$ ■



Part 16 Charts and Graphs

The **bar graph** below gives the span of some suspension bridges.



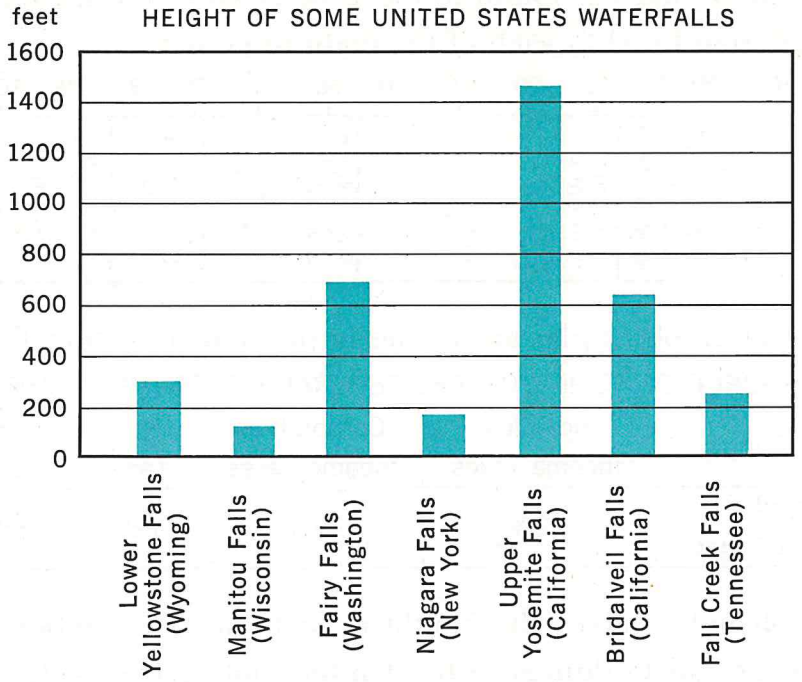
1. Study the graph above and answer these questions.
   - **a.** What is the title of the graph?
   - **b.** Give the approximate length of each bridge.
   - **c.** Which two bridges are the longest?
   - **d.** Which bridges span 2000 or more feet?

2. Make a bar graph to show the information in this table.

LENGTH OF SOME PASSENGER SHIPS

Queen Elizabeth (Britain)	1031 feet
Oceanic (Panama)	774 feet
United States (United States)	990 feet
Michelangelo (Italy)	900 feet
Nieuw Amsterdam (Netherlands)	758 feet
France (France)	1035 feet

The bar graph below shows the heights in feet of some waterfalls.



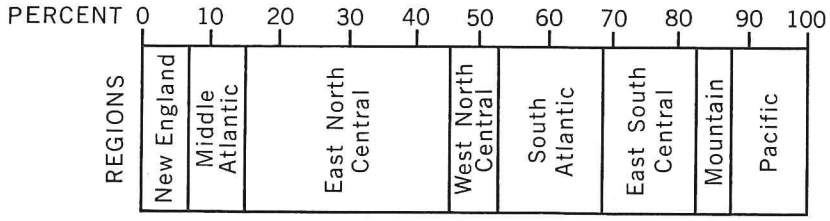
1. Study the graph above and answer these questions.
   - **a.** What is the title of the graph?
   - **b.** Give the approximate height of each waterfall.
   - **c.** Which is the highest waterfall?
   - **d.** Which waterfalls are higher than 200 feet?
   - **e.** Which waterfalls are higher than 400 feet?

2. Make a bar graph to show the information in the table below.

HEIGHT OF SOME MOUNTAINS IN THE UNITED STATES

Mt. McKinley (Alaska)	20,320 feet
Mt. Whitney (California)	14,495 feet
Pike's Peak (Colorado)	14,110 feet
Mt. Marcy (New York)	5,344 feet
Mt. Ranier (Washington)	14,410 feet
Clingman's Dome (Tennessee)	6,642 feet
Mauna Kea (Hawaii)	13,796 feet

1. This graph shows per cent distribution of population in different areas of the United States as found in the 1960 census. About what per cent of the population lived in each of the eight regions?

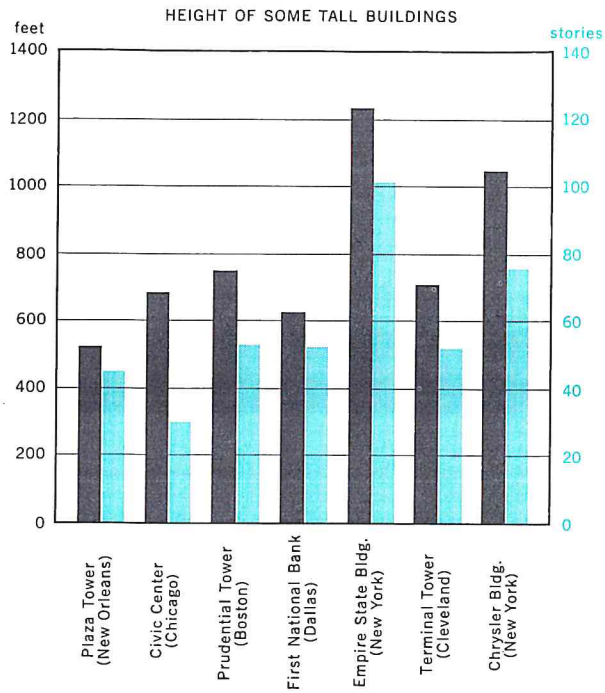


2. Make a bar graph similar to the one in problem 1 to show the following:

SOURCE OF FUNDS OF THE FEDERAL GOVERNMENT (1964)

Source	Individual Income Taxes	Corporation Income Taxes	Excise Taxes	All Other
% of total funds	53%	27%	12%	8%

3. The graph below gives the height in feet and the number of stories of some of the tall buildings in the United States. The scale at the left is used when finding the height of the buildings, and the scale at the right is used when finding the number of stories in each building.

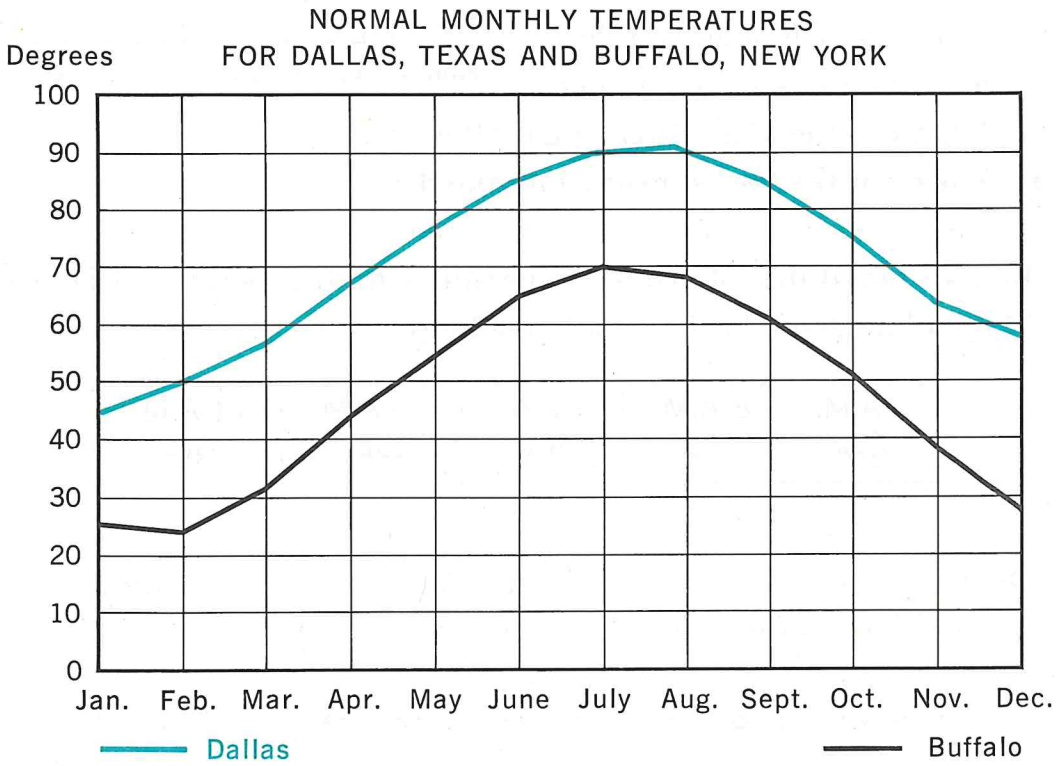


a. Which building is the tallest?
b. Which building has the most stories?
c. Which buildings are at least 700 feet tall?
d. Which buildings have at least 40 stories?
e. Make a table showing the height and number of stories of each building.

Line Graphs

A **line graph** is similar to a bar graph. The scale of the line graph is like the scale used for the bar graph. Line graphs are used when showing data such as temperatures for a given period of time where every point on the graph has meaning. Two or more lines can be shown on the same graph.

1. The following line graph shows the normal monthly temperatures for Dallas, Texas and Buffalo, New York.



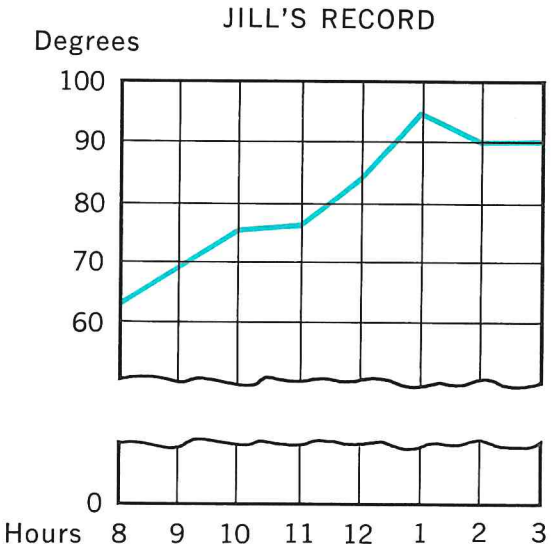
a. According to the graph which month is the coldest month in Dallas?

b. Which month is the coldest in Buffalo?

c. Which two months are considered the hottest in Dallas?

d. Does Buffalo or Dallas have the higher normal temperature throughout the year?

1. The chart at the right shows the temperatures Jill recorded for a day in July.



   a. Why is there a break in the graph between 0° and 60°?
   b. During which hours did the temperature increase?
   c. During which hours did the temperature decrease?
   d. During which hours did the temperature remain the same?
   e. What was the highest reading for the day?
   f. What was the lowest reading for the day?

2. John recorded the following temperatures for the morning of January 1. Make a line graph to show these readings.

7 A.M.	8 A.M.	9 A.M.	10 A.M.	11 A.M.
25°	28°	30°	35°	38°

3. On the graph for Exercise 2, use a different color and show these readings which John recorded for the morning of February 1.

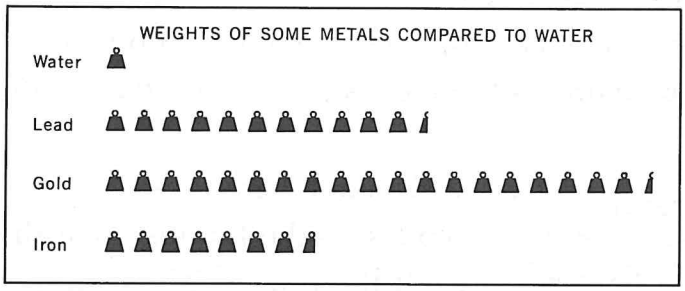
7 A.M.	8 A.M.	9 A.M.	10 A.M.	11 A.M.
36°	38°	38°	40°	44°

   a. How many degrees difference was there in the two readings at 7 o'clock?
   b. How many degrees difference was there in the two readings at 11 o'clock?

Picture Graphs

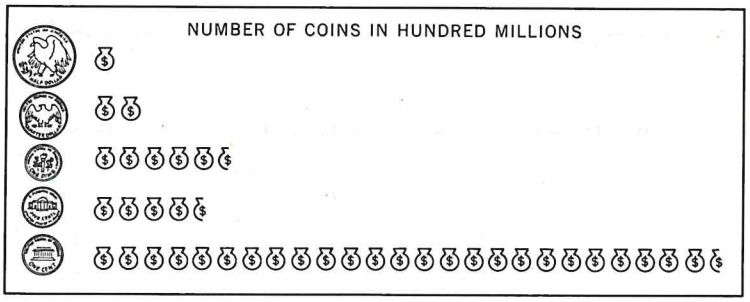
A **picture graph** or **pictograph** uses pictures or symbols instead of bars. Each picture or symbol represents the same number of things.

1. This pictograph compares the weight of lead, gold, and iron to water.



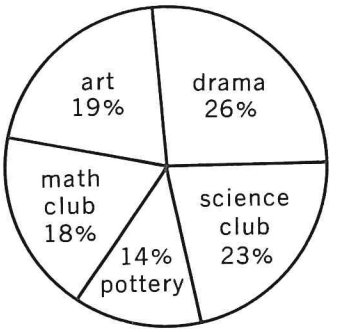
   a. Gold is about how many times as heavy as water?
   b. Lead is about how many times as heavy as water?
   c. Iron is about how many times as heavy as water?
   d. Which is heaviest, gold, iron, or lead?

2. This pictograph shows the coins that were minted in a recent year.

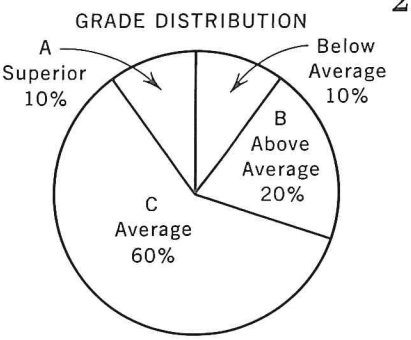


   a. About how many coins of each kind were minted during the year?
   b. What was the approximate value of the dimes minted during the year?
   c. What was the value of all the nickels that were minted during the year?
   d. How much greater was the value of the dimes minted than the value of the pennies?

Circle Graphs

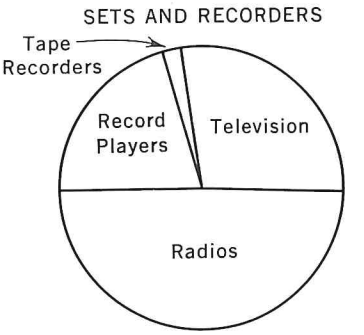


1. We can use the region inside a circle to make a graph. This graph shows what electives the sixth graders of Hall School chose.
   a. If there were 500 children in the grade, how many chose each elective?
   b. How many more children chose science club than art?
   c. How many more children chose math club than pottery?



2. This circle graph shows the distribution of grades on the last test in arithmetic.
   a. If there were 30 children in the class, how many received superior grades?
   b. How many received B?
   c. How many received C?
   d. How many received below average grades?

3. The graph at the right shows the relative number of television sets, radio sets, record players, and tape recorders in homes in the United States. In homes in the United States:
   a. Are there more television sets or radios?
   b. Are there more record players or radios?
   c. Are there more television sets or record players?



4. If the total region within the circle on the right represents 224,700,000 sets, and the region representing radio sets is 49% of the total region, about how many radios are in homes in the United States?

Checkout

Find each amount.

1. $\begin{array}{r} \$17.98 \\ 3.47 \\ 9.63 \\ 24.08 \\ \hline \end{array}$

2. $\begin{array}{r} \$34.41 \\ 2.76 \\ 13.00 \\ 24.05 \\ \hline \end{array}$

3. $\begin{array}{r} \$\ 7.36 \\ 14.23 \\ 19.98 \\ 7.36 \\ \hline \end{array}$

4. $\begin{array}{r} \$23.41 \\ 32.73 \\ 41.85 \\ 5.64 \\ \hline \end{array}$

5. $\begin{array}{r} \$21.46 \\ -18.39 \\ \hline \end{array}$

6. $\begin{array}{r} \$3.98 \\ -\ .76 \\ \hline \end{array}$

7. $\begin{array}{r} \$14.29 \\ -\ 3.41 \\ \hline \end{array}$

8. $\begin{array}{r} \$12.49 \\ -\ 7.98 \\ \hline \end{array}$

Find each quotient. Round off the answer to the nearest cent.

9. $12\overline{)\$4.76}$

10. $9\overline{)\$17.56}$

11. $20\overline{)\$39.52}$

12. $52\overline{)\$709.08}$

13. $817\overline{)\$43.16}$

14. $930\overline{)\$998.73}$

Find each sum or difference.

15. $\begin{array}{r} 8\frac{2}{3} \\ -4\frac{5}{6} \\ \hline \end{array}$

16. $\begin{array}{r} 34\frac{1}{2} \\ +17\frac{5}{8} \\ \hline \end{array}$

17. $\begin{array}{r} 14\frac{2}{5} \\ +\ 8\frac{9}{10} \\ \hline \end{array}$

18. In the first 60 days of one year, 678,456 babies were born in the United States. What is the average number of babies born each of those days?

19. There were 40 questions on a quiz. Bruce scored 80%. How many did he have correct?

20. Paula has read 30 pages of a 50 page book. Maureen has read 67 pages of a 100 page book. Who has read the greater per cent of her book?

21. Danvers spends $432,098.88 on its eight libraries. If the money is divided evenly among the libraries, how much does each library get?

Part 17 Powers

We may express a number to a power instead of multiplying the number many times.

$3^7 = 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$

We read "3^7" as "3 to the 7th power".

1. Express each power as a product.

a.	8^4	**c.**	2^6	**e.**	9^2	**g.**	6^7	**i.**	10^2	**k.**	13^4
b.	3^3	**d.**	7^5	**f.**	1^8	**h.**	5^4	**j.**	4^3	**l.**	1^2

2. Express each product as a power.

a.	$2 \times 2 \times 2 = 2^3$	**c.**	$6 \times 6 \times 6 \times 6$	**e.**	$5 \times 5 \times 5 \times 5 \times 5$
b.	16×16	**d.**	73×73	**f.**	$9 \times 9 \times 9 \times 9 \times 9 \times 9$

The number p^m has the number p as the **base** and the number m as the **exponent**.

7^3 has base 7 and exponent 3.

3. Name the base and exponent in each expression.

a.	10^2	**d.**	2^4	**g.**	1^7	**j.**	5^3	**m.**	4^6	**p.**	10^5
b.	2^2	**e.**	3^4	**h.**	1^{11}	**k.**	6^2	**n.**	10^3	**q.**	10^6
c.	2^3	**f.**	3^5	**i.**	5^2	**l.**	6^3	**o.**	10^4	**r.**	10^7

We can compute powers.

$4^3 = 4 \times 4 \times 4$
$= 4 \times 4^2$
$= 4 \times 16$
$= 64$

$4^4 = 4 \times 4 \times 4 \times 4$
$= 4 \times 4^3$
$= 4 \times 64$
$= 256$

$4^5 = 4 \times 4 \times 4 \times 4 \times 4$
$= 4 \times 4^4$
$= 4 \times 256$
$= 1024$

4. Compute each power in Exercise 3.

Any number to the first power is itself. Sometimes we do not write the exponent 1.

We write:	We read:
$n^1 = n$	n to the first power
$n^2 = n \times n$	n to the second power or n squared
$n^3 = n \times n \times n$	n to the third power or n cubed
$n^4 = n \times n \times n \times n$	n to the fourth power

1. Compute each product of powers.

$$9^1 \times 8^2 = 9 \times 64 = 576$$

a.	$7^2 \times 3^3$	**e.**	$10^4 \times 2^3$	**i.**	$18^1 \times 8^2$	**m.**	$3^2 \times 2^3 \times 1^9$
b.	$2^1 \times 4^3$	**f.**	16×9^2	**j.**	$2^7 \times 9^4$	**n.**	$26^1 \times 17^1 \times 4^2$
c.	$5^1 \times 5^2$	**g.**	$7^8 \times 2^3$	**k.**	2×6^3	**o.**	$9^3 \times 6^2 \times 1^7$
d.	$8^2 \times 8^2$	**h.**	$5^3 \times 10^2$	**l.**	$7^2 \times 7^1$	**p.**	$11^3 \times 2^4 \times 8^2$

2. An edge of a cube measures 3 in. What is its volume? A second cube has a volume 3 times as large. What is the volume of the second cube?

3. The side of a square is 2 in. long. What is its area? Another square has an area 4 times as large. What is the area of the second square? What is the length of a side of the second square?

4. I have 2 vats each containing 2 mats.
2 cats sat on each of the mats.
Each cat wore 2 funny old hats.
On each hat lay 2 thin rats.
On each rat perched 2 black bats.
On each bat hopped 2 nervous gnats.
Mats, cats, hats, rats, bats, gnats—
How many things are in my vats?

1. Express each product as a power.

$$\begin{aligned} 3^4 \times 3^3 &= (3 \times 3 \times 3 \times 3) \times (3 \times 3 \times 3) \\ &= 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \\ &= 3^7 \end{aligned}$$

a. $4^5 \times 4^2$ c. $6^4 \times 6^5$ e. $3^2 \times 3^1 \times 3^8$ g. $9^5 \times 9^3 \times 9^2$

b. $2^3 \times 2^3$ d. $8^7 \times 8^2$ f. $7^4 \times 7 \times 7^6$ h. $5^9 \times 5^4 \times 5^7$

To multiply powers that have the same base, add the exponents.

$3^5 \times 3^7 = 3^{5+7} = 3^{12}$ $10^4 \times 10^6 \times 10^5 = 10^{4+6+5} = 10^{15}$

2. Express each product as a power.

a. $4^6 \times 4^7$ c. $6^1 \times 6^4$ e. $13^2 \times 13^2 \times 13^5$ g. $23^3 \times 23^1 \times 23^9$

b. $7^3 \times 7^2$ d. $2^4 \times 2^3$ f. $10^4 \times 10^3 \times 10^7$ h. $45^8 \times 45^3 \times 45^2$

3. Complete each problem.

$3^4 \times 3^7 =$ ■	$3^4 \times 3^7 = 3^{11}$
$3^{11} \div 3^4 =$ ■	$3^{11} \div 3^4 = 3^7$
$3^{11} \div 3^7 =$ ■	$3^{11} \div 3^7 = 3^4$

a. $2^3 \times 2^2 =$ ■
$2^5 \div 2^3 =$ ■
$2^5 \div 2^2 =$ ■

b. $4^6 \times 4^4 =$ ■
$4^{10} \div 4^6 =$ ■
$4^{10} \div 4^4 =$ ■

c. $8^5 \times 8^1 =$ ■
$8^6 \div 8^5 =$ ■
$8^6 \div 8^1 =$ ■

To divide powers of the same base, subtract the exponents.

$4^{20} \div 4^5 = 4^{20-5} = 4^{15}$ $10^9 \div 10^4 = 10^{9-4} = 10^5$

4. Give each quotient as a power.

a. $6^6 \div 6^3$ d. $7^6 \div 7^5$ g. $12^{10} \div 12^2$ j. $17^{13} \div 17^5$

b. $9^7 \div 9^2$ e. $6^4 \div 6^3$ h. $24^{30} \div 24^{25}$ k. $46^6 \div 46^1$

c. $3^{13} \div 3^1$ f. $8^6 \div 8^6$ i. $31^6 \div 31^5$ l. $93^{10} \div 93^4$

Powers of Ten

We can compute powers of ten.

$10^1 = 10$ $10^2 = 100$ $10^3 = 1000$ $10^4 = 10000$

1. Compute each power.

   a. 10^6 b. 10^5 c. 10^8 d. 10^{11} e. 10^7 f. 10^9 g. 10^{10}

2. Express each number as a power of ten.

   a. 10,000
   b. 10,000,000
   c. 1,000,000
   d. 100,000,000
   e. 1,000,000,000
   f. 100

Expanded notation expresses a number in multiples of ten.

$68{,}543 = 60{,}000 + 8{,}000 + 500 + 40 + 3$

3. Express each number in expanded notation.

   a. 7382
   b. 5001
   c. 79,368
   d. 13,034
   e. 13,304
   f. 8963
   g. 321,800
   h. 202,020
   i. 202,202
   j. 31,708,775
   k. 210,396,046
   l. 8,102,734,554

Instead of using multiples of ten, we can use powers of ten in expanded notation.

$68{,}043 = (6 \times 10^4) + (8 \times 10^3) + (0 \times 10^2) + (4 \times 10) + 3$

4. Express each number of Exercise 3 in expanded notation in powers of ten.

The **digits** we use in writing numerals are 0,1,2,3,4,5,6,7,8, and 9. The digits of 20,663 are 2,0,6,6, and 3.

5. Counting from the left, give the first, third, and last digit of each number in Exercise 3.

Rounding to Powers of Ten

We can **round off** numbers to a power of ten by looking at a "test digit." If the test digit is 5 or greater, round "up". If the test digit is less than 5, round "down".

In each case the test digit is marked with an arrow.

	Number	Rounded off
To round to the nearest ten	3,625,74$\overset{\downarrow}{8}$	3,625,750
To round to the nearest hundred	3,625,7$\overset{\downarrow}{4}$8	3,625,700
To round to the nearest thousand	3,625,$\overset{\downarrow}{7}$48	3,626,000
To round to the nearest ten thousand	3,62$\overset{\downarrow}{5}$,748	3,630,000
To round to the nearest hundred thousand	3,6$\overset{\downarrow}{2}$5,748	3,600,000
To round to the nearest million	3,$\overset{\downarrow}{6}$25,748	4,000,000

1. Round each number to the nearest ten: 42, 76, 85, 105, 321, 4962.
2. Round each number to the nearest hundred: 308, 725, 988, 3751, 6091.
3. Round each number to the nearest thousand: 40,316; 6520; 41,635; 7888; 5457.
4. Round each number to the nearest million: 3,099,909; 16,542,300; 96,600,333.
5. Copy and complete this table.

Round to	Number						
nearest	2783	4169	5080	31,694	54,055	489,056	215,058
ten	2780		5080				215,060
hundred	2800						
thousand	3000				54,000		

6. The population of Barnesville was 13,680 in 1940, 15,549 in 1945, 16,721 in 1950, 16,473 in 1955, and 17,098 in 1960.
   a. What was the average population for this period?
   b. To the nearest thousand what was the population at each 5-year interval? Using these numbers, what was the average population?

Scientific Notation

Scientific notation is often used by scientists to make computations easier. Every number is expressed as a product of two factors. The first factor is a number equal to 1 or between 1 and 10. The second factor is a power of 10.

$3{,}000{,}000 = 3 \times 10^6$ $\quad 40{,}000 = 4 \times 10^4$ $\quad 200 = 2 \times 10^2$

Sometimes we use decimals:

$17{,}000 = 1.7 \times 10^4$ $\quad 309{,}127 = 3.09127 \times 10^5$

1. Express each number in scientific notation.

$$806{,}145{,}100 = 8.061451 \times 10^8$$
$$5100 = 5.1 \times 10^3$$

a.	7,000,000	**d.**	26,000	**g.**	980	**j.**	6304	**m.**	3,762,800
b.	300,000	**e.**	300	**h.**	652	**k.**	543	**n.**	450,000,000
c.	1,400,000	**f.**	7600	**i.**	4218	**l.**	9860	**o.**	93,265,000

2. Find each product. Write your answer in scientific notation.

$$(1.7 \times 10^3) \times (8 \times 10^4) = (1.7 \times 8) \times (10^3 \times 10^4)$$
$$= 13.6 \times 10^7$$
$$= 1.36 \times 10^8$$

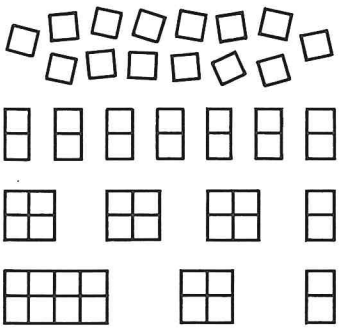
a.	$(3 \times 10^5) \times (8 \times 10^7)$	**e.**	$(2 \times 10^4) \times (1.3 \times 10^7)$
b.	$(4 \times 10^3) \times (5 \times 10^4)$	**f.**	$(8.2 \times 10^2) \times (9.3 \times 10^8)$
c.	$(2 \times 10^5) \times (1 \times 10^8)$	**g.**	$(3.1 \times 10^4) \times (5 \times 10^3)$
d.	$(1.1 \times 10^3) \times (6 \times 10^4)$	**h.**	$(7.5 \times 10) \times (5.65 \times 10^3)$

3. Choose $<$ or $=$ or $>$ without computing the product.

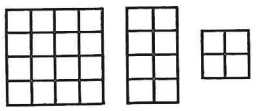
a.	$10^5 \bigcirc 2.5 \times 10^3$	**d.**	$10^1 \bigcirc 1.0 \times 10$	**g.**	$10^4 \bigcirc 1.1 \times 10^4$
b.	$10^4 \bigcirc 6.4 \times 10^4$	**e.**	$10^5 \bigcirc 1.0 \times 10^4$	**h.**	$10^7 \bigcirc 7.3 \times 10^6$
c.	$10^3 \bigcirc 7.8 \times 10^2$	**f.**	$10^8 \bigcirc 2.0 \times 10^7$	**i.**	$10^9 \bigcirc 9.2 \times 10^8$

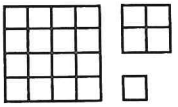
Base Two

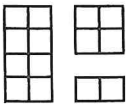
We have some blocks.
First we group them by twos.
Then we group 2 twos to make fours.
Then we group 2 fours to make eights.
We have
1 eight 1 four 1 two 0 ones.
We write:
1110_{two}
We read "1110_{two}" as
"one one one zero to base two".

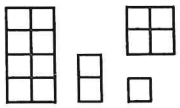


1. What numeral to base two represents each number of blocks?

a. 
11100_{two}

b. 

c. 

d. 

2. Group blocks by ones, twos, fours, eights, sixteens, and thirty-twos to find the numeral to base two for each number.

a. 6	c. 11	e. 13	g. 15	i. 27
b. 10	d. 7	f. 25	h. 32	j. 45

3. Expanded notation converts a base two numeral to base ten. Usually we do not write the "ten". ($13_{ten} = 13$) Convert these numerals to base ten.

$$\begin{aligned} 101011_{two} &= (1 \times 2^5) + (0 \times 2^4) + (1 \times 2^3) + (0 \times 2^2) + (1 \times 2) + 1 \\ &= 32 + 0 + 8 + 0 + 2 + 1 \\ &= 43 \end{aligned}$$

a. 100_{two}	c. 10000_{two}	e. 1101_{two}	g. 110101_{two}
b. 1000_{two}	d. 1010_{two}	f. 1001_{two}	h. 10110_{two}

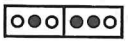
On computers, rows of lights often represent numerals. If the light is on, it represents "1". If the light is off, it represents "0". A light can be only **on** or **off**, so each light can represent only "1" or "0". We use more lights to represent other numerals to base two.

A display of lights represents a "word" in the computer. Each light represents one "bit" in the word.

Here we show words of several different lengths. The black dots mean the light is off.

3-bit word 6-bit word 12-bit word 18-bit word

Each word represents a numeral.

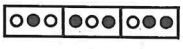
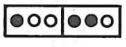
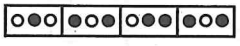


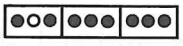
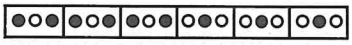
6-bit word, 101001_{two}

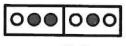
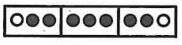
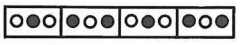
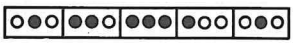
We usually do not write "0" before a numeral.

9-bit word, 11101001_{two}

1. How many bits has each word? What numeral to the base two does each word represent?

a. 
d. 
g. 
j.

b.
e.
h. 
k. 

c. 
f. 
i. 
l. 

2. Choose < or >.

a. 101_{two} ■ 110_{two} c. 1001_{two} ■ 1010_{two} e. 1000111_{two} ■ 1001011_{two}

b. 111_{two} ■ 101_{two} d. 1100_{two} ■ 1011_{two} f. 1101010_{two} ■ 1010101_{two}

How many numerals can be represented by a 6-bit word?

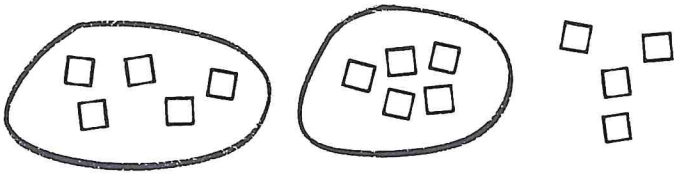
Each light can represent 2 numerals, "0" or "1". All together

$2 \times 2 \times 2 \times 2 \times 2 \times 2$ or 64 numerals can be represented.

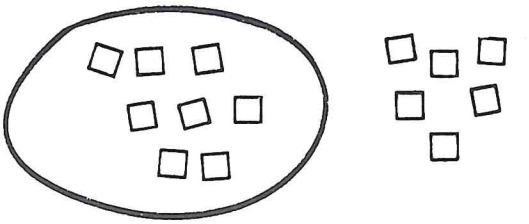
3. How many numerals can be represented by a 3-bit word? List them.
4. How many numerals can be represented by a 9-bit word?

Other Bases

We can group by fives.
We have 2 groups of five and 4 ones.
We write: 24_{five}
We read: two four base five



We can group by eights.
We have 1 group of eight and 6 ones.
We write: 16_{eight}
We read: one six base eight



1. Group blocks in ones, fives, and twenty-fives to find the numeral to base five for each number given in base ten notation.

a. 32	c. 28	e. 13	g. 42	i. 29
b. 17	d. 22	f. 9	h. 36	j. 50

2. Group blocks in ones, eights, and sixty-fours to find the numeral to base eight for each number given in base ten notation.

a. 56	c. 72	e. 15	g. 35	i. 87
b. 43	d. 24	f. 98	h. 66	j. 100

3. Group blocks in ones, sixes, and thirty-sixes to find the numeral to base six for each number given in base ten notation.

a. 28	c. 72	e. 83	g. 26	i. 64
b. 43	d. 55	f. 49	h. 34	j. 89

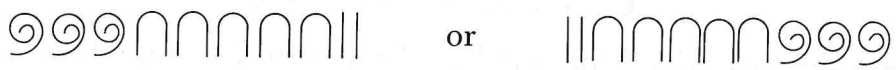
4. Convert these numerals to base ten.

a. 314_{five}	c. 742_{eight}	e. 654_{nine}	g. 221_{three}
b. 235_{six}	d. 344_{seven}	f. 342_{six}	h. 367_{eight}

Egyptian Numerals

The ancient Egyptians used base ten, but they had no symbol for zero and did not make use of place value. They used these pictures to represent numbers.

These symbols were arranged in any order in a numeral. For example, 352 could be written:


or

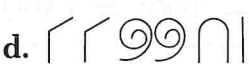
A method of expressing numbers is called a system of numeration.

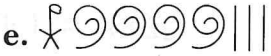
1. What number does each Egyptian numeral name?

a.

b.

c.

d. 

e. 

f.

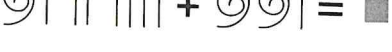
2. Write an Egyptian numeral to express each number.

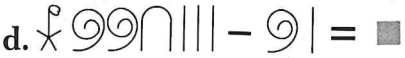
a. 428	**c.** 634	**e.** 1246	**g.** 34,682
b. 159	**d.** 752	**f.** 1995	**h.** 48,215

3. Complete each exercise using Egyptian numerals.

a.

b.

c. 

d. 

e.

f.

4. Express each sum or difference in Exercise 3 using our Hindu-Arabic numerals.

Roman Numerals

The ancient Romans used these symbols:

I	V	X	L	C	D	M
1	5	10	50	100	500	1000

They had no symbol for zero, so they did not make use of place value. Whenever a symbol for a lesser number came before a symbol for a greater number, they subtracted.

$IV = 5 - 1 = 4$ $\qquad$ $XL = 50 - 10 = 40$

$CD = 500 - 100 = 400$

Otherwise they added the numbers for which the symbols stood.

$CCXXXVII = 100 + 100 + 10 + 10 + 10 + 5 + 1 + 1 = 237$

Sometimes they had to both subtract and add. They always subtracted first.

$CDLX = (500 - 100) + 50 + 10 = 460$

$MCD = 1000 + (500 - 100) = 1400$

1. What number does each Roman numeral name?

a. CXLIV **b.** LXXVI **c.** CDLX **d.** DCXL **e.** CMXX **f.** MMCX **g.** MCDXCII **h.** MCMLXX

2. Write a Roman numeral to express each number.

a. 234 **b.** 168 **c.** 794 **d.** 287 **e.** 1437 **f.** 1607 **g.** 1980 **h.** 1513

A bar over a symbol indicated that the given number was to be multiplied by 1000.

$\overline{M} = 1000 \times 1000 = 1{,}000{,}000$

$\overline{V} = 1000 \times 5 = 5000$

3. What number does each Roman numeral name?

a. $\overline{X}$ **b.** $\overline{L}$ **c.** $\overline{M}$ **d.** $\overline{C}$

Checkout

Find each sum.

1. $4.68 + 3.25 + 5.84$

2. $\$16.38 + 2.54 + 34.19$

3. $\$24.62 + 18.59 + 3.95$

4. $\$15.75 + 3.43 + 18.26$

Find each quotient to the nearest tenth.

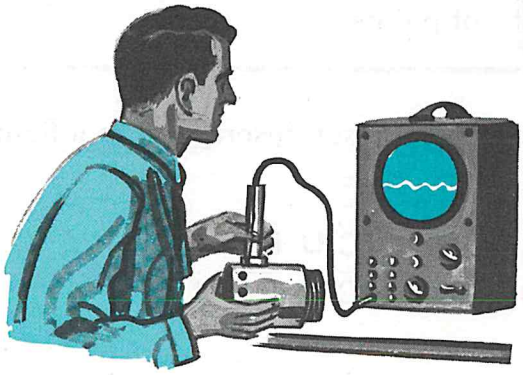
5. $20\overline{)5907}$

6. $95\overline{)34008}$

7. $620\overline{)94100}$

Choose $<$ or $=$ or $>$.

8. $3.2 \times 10 \bigcirc .32 \times 10^2$
9. $75 \div 1000 \bigcirc .075$
10. $2.4 \div 100 \bigcirc .24 \div 10^3$
11. $.81 \times 1000 \bigcirc 8.1 \times 10$
12. $.48 \times 1000 \bigcirc .048 \times 100$
13. $.03 \div 100 \bigcirc .3 \div 1000$
14. $9.09 \div 100 \bigcirc .0909 \div 10$
15. $.714 \div 10 \bigcirc 714 \div 1000$



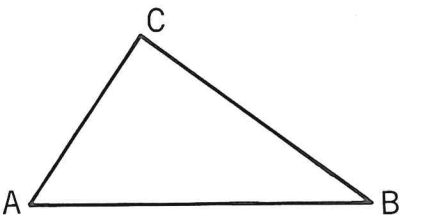
16. In Yellowstone National Park there are 2,213,000 acres of land; Mt. McKinley National Park has 1,939,000 acres and Glacier National Park has 1,009,000 acres. What is the average number of acres for these three National Parks?

17. The sixth grade sold 327 tickets at $.75 each to their play.
    a. How much money did they take in?
    b. They spent $27.56 for paint, $35.27 for costume material, $7.34 for lumber, $15.00 for advertising, and $75.00 for the hall. How much did they spend?
    c. How much money did they make?

Part 18 Geometry and Sets

Geometric figures are sets of points.

Look at triangle ABC. How can we describe it as a set of points? The triangle is made up of segments AB, BC, and AC. Each of these segments is a set of points. The triangle is the union of these sets of points.



Triangle ABC = $\overline{AB} \cup \overline{BC} \cup \overline{AC}$.

Match each set description to a figure.

1. $\overline{AB} \cup \overline{AC}$
2. $\overline{AB} \cup \overline{AC} \cup \overline{BC}$
3. $\overline{AB} \cup \overline{AC} \cup \overline{BC} \cup \overline{AD}$
4. $\overline{AB} \cup \overline{AC} \cup \overline{CD}$
5. $\overline{AB} \cup \overline{BC} \cup \overline{CD} \cup \overline{AD}$
6. $\overline{AB} \cup \overline{BC} \cup \overline{AC} \cup \overline{BD}$

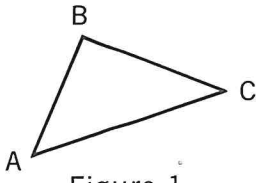


Figure 1

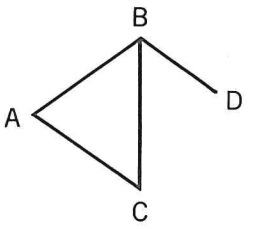


Figure 2

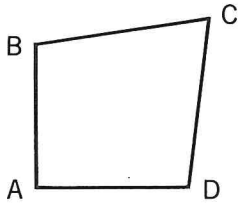


Figure 3

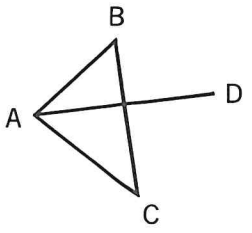


Figure 4

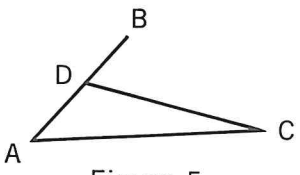


Figure 5

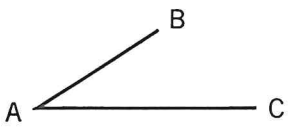
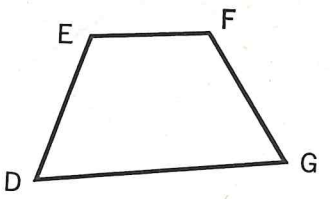


Figure 6

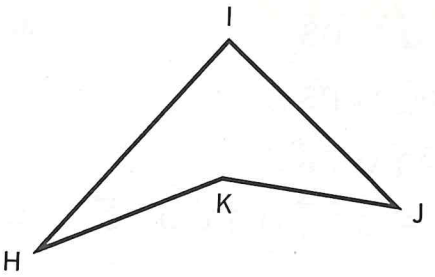
7. Quadrilateral ABCD = $\overline{AB} \cup \overline{BC} \cup \overline{CD} \cup$ ▩.

Describe each of the following figures as a union of line segments.

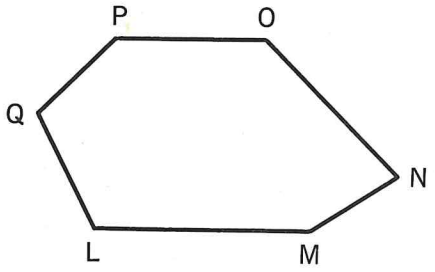
1.



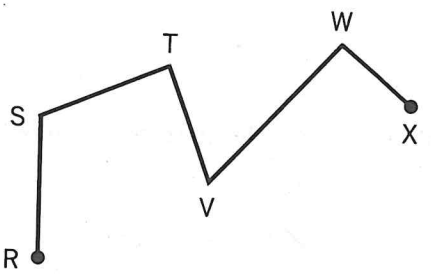
2.

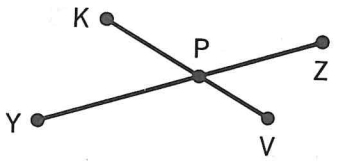


3.



4.

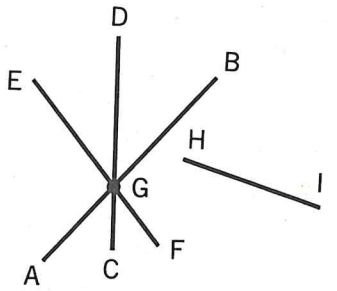




Point P is the point of intersection of segments KV and YZ. In terms of sets,

$\overline{KV} \cap \overline{YZ} = \{P\}$

Which of the following sentences are true about the line segments shown on the right?

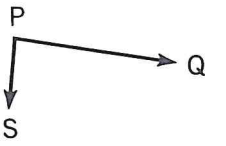


5. $\overline{AB} \cap \overline{EF} = \{G\}$
6. $\overline{AB} \cap \overline{DC} \cap \overline{EF} = \{\ \}$
7. $\overline{EF} \cap \overline{DC} \neq \{\ \}$
8. $\overline{HI} \cap \overline{AB} = \{G\}$
9. $\overline{AB} \cap \overline{DC} = \{G\}$
10. $\overline{AB} \cap \overline{DC} \cap \overline{HI} = \{\ \}$

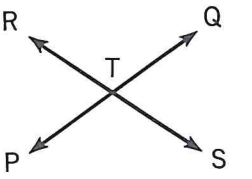
Match each set description to a figure.

1. $\overrightarrow{PQ} \cup \overrightarrow{QS}$
2. $\overline{PQ} \cup \overline{RS}$
3. $\overrightarrow{PQ} \cup \overrightarrow{PS}$
4. $\overrightarrow{TP} \cup \overrightarrow{TQ} \cup \overrightarrow{TR} \cup \overrightarrow{TS}$

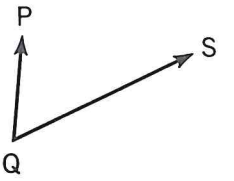
a.



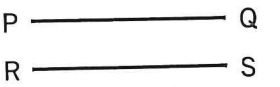
b.



c.



d.



5. Write a set description for each figure.

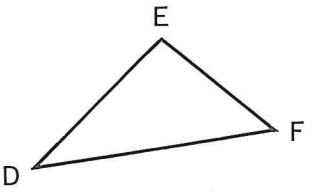


Figure 1

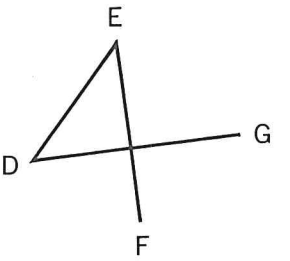


Figure 2

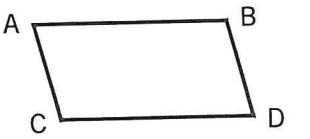


Figure 3

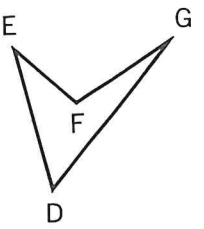


Figure 4

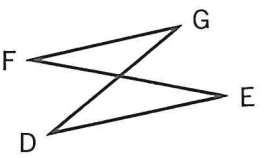


Figure 5

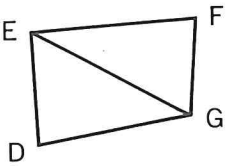


Figure 6

Draw a figure to match each set description.

6. $\overline{XY} \cup \overline{YZ}$
7. $\overline{XY} \cup \overline{YZ} \cup \overline{XZ}$
8. $\overline{XY} \cup \overline{YZ} \cup \overline{ZW} \cup \overline{WX}$
9. $\overline{XY} \cup \overline{YZ} \cup \overline{XW} \cup \overline{ZX}$
10. $\overline{XY} \cup \overline{YZ} \cup \overline{ZW} \cup \overline{WY}$
11. $\overline{XY} \cup \overline{YZ} \cup \overline{YW} \cup \overline{WX}$

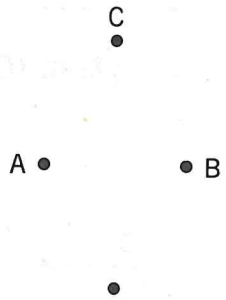
We have used set descriptions to talk about sets of numbers.

$$\{n: 201 < n < 205\} = \{202, 203, 204\}$$

Find the sets.

1. $\{n: 969 < n < 975\}$
2. $\{n: 998 < n < 1003\}$
3. $\{n: 3 < n + 1 < 5\}$
4. $\{n: 15 < n + 10 < 20\}$

We have also used set descriptions to talk about sets of points. The segment AB is a set of points.
We write "$\overline{AB}$" for segment AB.
We write "$m\overline{AB}$" for the length or measure of $\overline{AB}$.
In the figure at the right, $m\overline{AB} = m\overline{AC} = m\overline{BC}$ and $m\overline{AB} = m\overline{AD} = m\overline{BD}$.
$\{C, D\}$ is the set of points that are the distance $m\overline{AB}$ from both A and B.

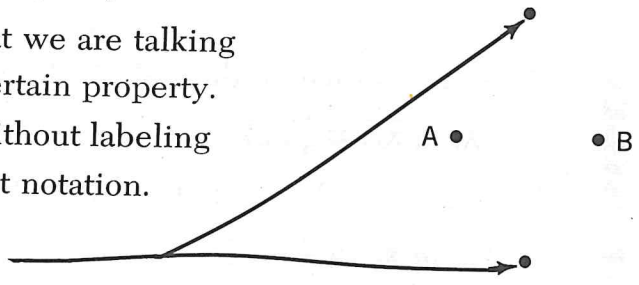


In set notation we may write:

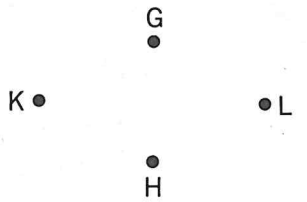
$$\{X: m\overline{XA} = m\overline{XB} = m\overline{AB}\} = \{C, D\}$$

The letter "X" just tells us that we are talking about all points that have a certain property.
We may also picture the set without labeling the points described by the set notation.

$$\{X: m\overline{XA} = m\overline{XB} = m\overline{AB}\}$$



5. Look at the figure at the right. What set of points is $\{X: m\overline{XG} = m\overline{XH} = m\overline{GH}\}$?

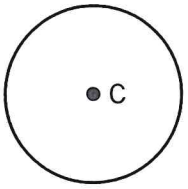


Any figure may be described as a set of points.

The circle with center C and radius equal in length to $\frac{1}{2}$ in. is the set of all points X such that $m\,\overline{XC} = \frac{1}{2}$ in.

In set notation, we then have:

$\{X: m\,\overline{XC} = \frac{1}{2}\text{ in.}\}$ = the circle with center C and radius whose measure is $\frac{1}{2}$ in.



The points X are the points of the circle.

1. Match the set description to the colored part of the figure.

a. $\{X: m\,\overline{XC} = \frac{1}{2}\text{ in.}\}$

b. $\{X: m\,\overline{XC} = \frac{1}{4}\text{ in.}\}$

c. $\overline{AB} \cup \overline{BD} \cup \overline{AC}$

d. $\{X: \frac{1}{4}\text{ in.} < m\,\overline{XC} < \frac{3}{8}\text{ in.}\}$

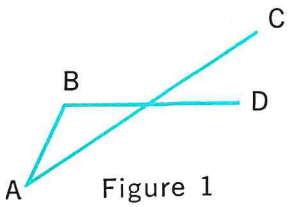


Figure 1

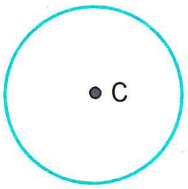


Figure 2

Figure 3

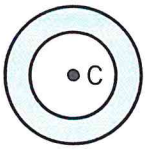


Figure 4

The circle shown on the right is described as

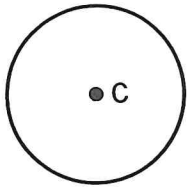
$\{X: m\,\overline{XC} = \frac{1}{2}\text{ in.}\}$.

Points outside the circle belong to the set

$\{X: m\,\overline{XC} > \frac{1}{2}\text{ in.}\}$.

Points inside the circle belong to the set

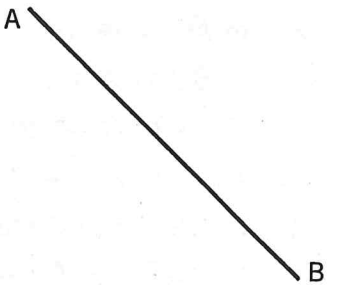
$\{X: m\,\overline{XC} < \frac{1}{2}\text{ in.}\}$.



2. a. In set notation describe the points inside a circle whose radius is 2 inches.

b. In set notation describe the points on a circle whose radius is 2 inches.

1. Some sets of points are empty sets. Which of the following describe empty sets when A and B are 2 inches apart?



   a. $\{X: m\,\overline{AX} = m\,\overline{BX} = \frac{1}{2}\text{ in.}\}$

   b. $\{X: m\,\overline{AB} = m\,\overline{AX} = m\,\overline{BX}\}$

   c. $\{X: m\,\overline{AX} = m\,\overline{BX} = 1\text{ in.}\}$

   d. $\{X: m\,\overline{AX} < \frac{1}{2}\text{ in. and } m\,\overline{BX} < \frac{1}{2}\text{ in.}\}$

   e. $\{X: m\,\overline{AX} < 1\text{ in. and } m\,\overline{BX} < 1\text{ in.}\}$

2. Match each set description to the shaded portion of one of the figures.

   a. $\{X: m\,\overline{XB} < \frac{1}{4}\text{ in.}\}$

   b. $\{X: m\,\overline{XB} < \frac{1}{4}\text{ in.}\} \cap \{X: m\,\overline{XC} < \frac{1}{4}\text{ in.}\}$

   c. $\{X: m\,\overline{XB} < \frac{1}{4}\text{ in.}\} \cup \{X: m\,\overline{XC} < \frac{1}{4}\text{ in.}\}$

   d. $\{X: m\,\overline{XB} < \frac{1}{4}\text{ in.}\} - \{X: m\,\overline{XC} < \frac{1}{4}\text{ in.}\}$

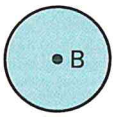


Figure 1

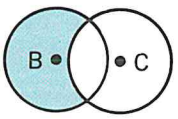


Figure 2

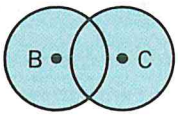


Figure 3

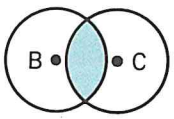


Figure 4

3. If points C and D are $\frac{1}{2}$ in. apart, draw a shaded figure to match each set description.

   a. $\{X: m\,\overline{XC} < 1\text{ in.}\} \cup \{X: m\,\overline{XD} < 1\text{ in.}\}$

   b. $\{X: m\,\overline{XC} < 1\text{ in.}\}$

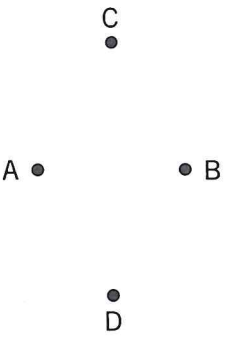
   c. $\{X: m\,\overline{XC} < 1\text{ in.}\} \cap \{X: m\,\overline{XD} < 1\text{ in.}\}$

   d. $\{X: m\,\overline{XC} < 1\text{ in.}\} - \{X: m\,\overline{XD} < 1\text{ in.}\}$

C •

• D

1. Look at the figure below. Use your ruler to measure, in order to solve the exercises.



a. $m\,\overline{AB} =$ ▢ in.

b. $m\,\overline{AC} =$ ▢ in.

c. $m\,\overline{BC} =$ ▢ in.

d. Does $m\,\overline{AC} = m\,\overline{AD}$?

e. Does $m\,\overline{AB} = m\,\overline{CD}$?

f. $m\,\overline{AA}$ is the distance from point A to itself. $m\,\overline{AA} =$ ▢ in.

g. If $A = X$ then $m\,\overline{AX} =$ ▢ in. (Here "X" is just another name for point A.)

h. $\{X: m\,\overline{AX} = 0 \text{ in.}\} =$ ▢

i. $\{X: m\,\overline{AX} = m\,\overline{BX} = m\,\overline{AB}\} =$ ▢

2. Match the set description to the figure.

a. $\{X: m\,\overline{AX} = \frac{1}{4} \text{ in.}\}$

b. $\{X: m\,\overline{AX} = m\,\overline{BX}\}$

c. $\{X: m\,\overline{AB} = m\,\overline{AX} = m\,\overline{BX}\}$

d. $\overline{AB} \cup \overline{AC}$

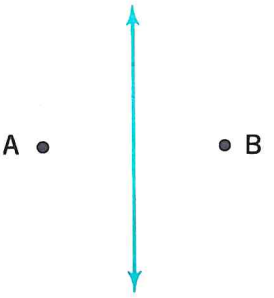


Figure 1

Figure 2

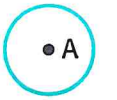


Figure 3

Figure 4

3. Draw a figure to show $\{X: m\,\overline{XC} = \frac{3}{4} \text{ in.}\}$.

4. Draw a shaded figure to show $\{X: m\,\overline{XC} < \frac{3}{4} \text{ in.}\}$.

1. Let A and B be two points which are 3 in. apart. Then $m\ \overline{AB} = 3$ in. Consider the following sets of points.

$\{X: m\ \overline{AX} = m\ \overline{BX}\}$ $\{X: m\ \overline{AX} = m\ \overline{BX} = 1.2 \text{ in.}\}$

$\{X: m\ \overline{AX} = m\ \overline{BX} = 1.5 \text{ in.}\}$ $\{X: m\ \overline{AB} = m\ \overline{AX} = m\ \overline{BX}\}$

a. Which set of points is empty?
b. Which set of points has exactly one member?
c. Which set of points has exactly two members?
d. Which set of points has an infinite number of members?

2. Write a set description to describe the colored part of each figure. Where necessary, use your ruler to measure.

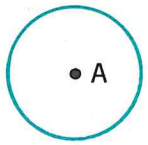


Figure 1

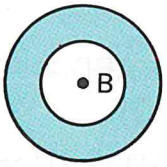


Figure 2

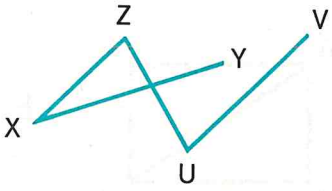


Figure 3

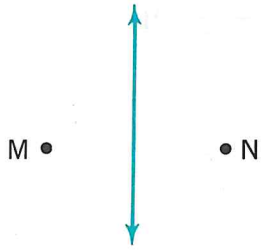


Figure 4

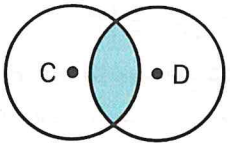


Figure 5

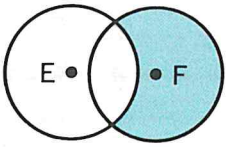


Figure 6

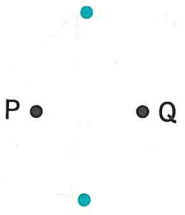


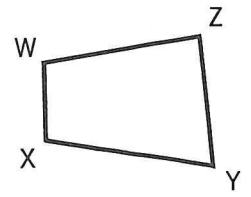
Figure 7

3. If points D and E are $\frac{1}{2}$ inch apart, draw a shaded figure to show:

a. $\{X: m\ \overline{XD} < \frac{3}{4} \text{ in.}\} \cup \{X: m\ \overline{XE} < 1 \text{ in.}\}$

b. $\{X: m\ \overline{XD} < \frac{3}{4} \text{ in.}\} \cap \{X: m\ \overline{XE} < 1 \text{ in.}\}$

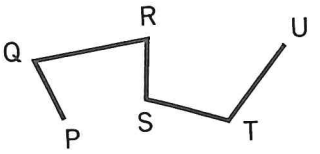
For each problem, which statements are true about the given figure?

1. 

a. Figure WXYZ = $\overline{WX} \cup \overline{XY} \cup \overline{YZ} \cup \overline{WZ}$

b. $\overline{WX} \cap \overline{ZY} = \{W, X, Y, Z\}$

c. $\overline{WX} \subset$ figure WXYZ

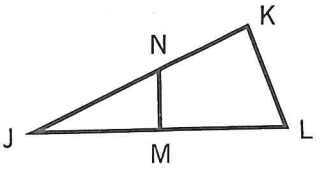
2. 

a. $\overline{QR} \cap \overline{ST} = \{\ \}$

b. $\overline{QP} \cap \overline{QR} = \overline{QR}$

c. $\overline{ST} - \overline{QP} = \overline{QR}$

d. $\overline{UT} \cap \overline{TS} = \{T\}$

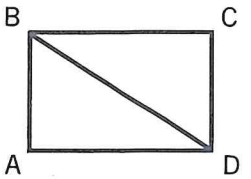
3. 

a. $\overline{MN} \cap \overline{JL} = \{N\}$

b. $\overline{JM} \subset \overline{JL}$

c. m $\overline{JM}$ + m $\overline{ML}$ = m $\overline{JL}$

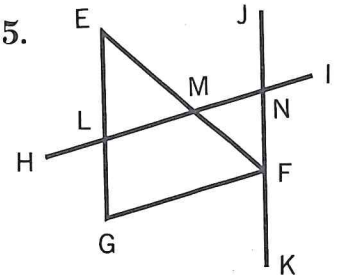
d. m $\overline{JN}$ + m $\overline{NM}$ > m $\overline{JM}$

4. 

a. triangle ABD = $\overline{AB} \cup \overline{BD} \cup \overline{AD}$

b. $\overline{BD} \subset \overline{AD}$

c. $\overline{AD} \cap \overline{BD} \cap \overline{CD} = \{D\}$

5. 

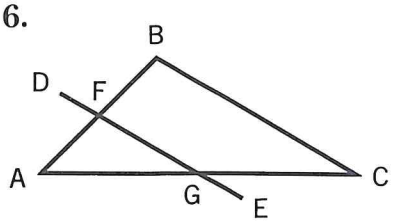
a. m $\overline{HL}$ + m $\overline{LM}$ + m $\overline{MN}$ + m $\overline{NI}$ = m $\overline{HI}$

b. m $\overline{LM}$ + m $\overline{MN}$ > m $\overline{LI}$

c. $\overline{EF} \cap \overline{GF} \cap \overline{JK} = \{F\}$

d. $\overline{EG} \cup \overline{GF} \subset$ triangle EGF

e. $\{M, N, F\} \subset$ triangle MNF

6. 

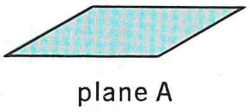
a. $\overline{DG} \cup \overline{FE} = \overline{DG}$

b. $\overline{BC} \cap \overline{DE} = \{\ \}$

c. $\overline{DE} \cap \overline{AB} = \{F\}$

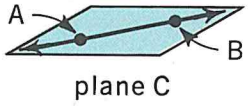
d. $\overline{FG} \subset \overline{DE}$

A plane is a set of points. It has no thickness. It is a flat surface that extends an infinite distance.

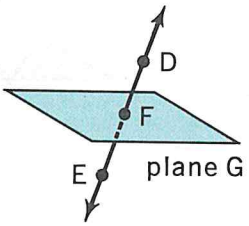


We represent a plane like this.

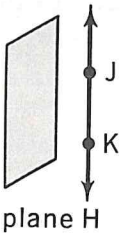
1. Points A and B lie in plane C, so line AB lies in plane C.
We can write:
$\overleftrightarrow{AB} \cap$ plane C $= \overleftrightarrow{AB}$



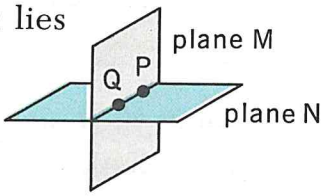
2. Points D and E do not lie in plane G. Line DE intersects plane G in point F.
We can write:
$\overleftrightarrow{DE} \cap$ plane G $= \{\blacksquare\}$



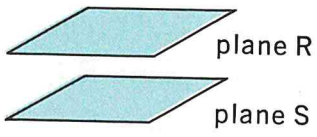
3. Line JK and plane H are parallel. No matter how far we extend the picture they do not intersect.
We can write:
plane H $\cap \overleftrightarrow{JK} = \blacksquare$



4. Plane M intersects plane N in line PQ. Line PQ lies in plane M and also in plane N.
We can write:
plane M $\cap$ plane N $= \blacksquare$



5. Plane R and plane S are parallel.
They do not intersect.
We can write:
plane R $\cap$ plane S $= \blacksquare$



1. Pair each statement with a figure.

a. $\overleftrightarrow{BC} \cap \overleftrightarrow{CD} \cap$ plane $A = \{C\}$

b. $\overleftrightarrow{CD} \cap$ plane $A = \{B\}$

c. $\overleftrightarrow{CD} \cap$ plane $A \cap$ plane $B = \overleftrightarrow{CD}$

d. $\overrightarrow{CD} \cap$ plane $A \cap$ plane $B = \{C\}$

Figure 1

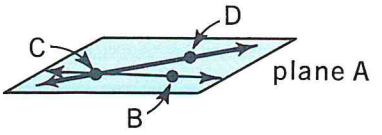


Figure 2

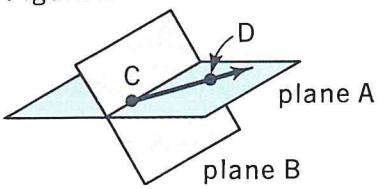


Figure 3

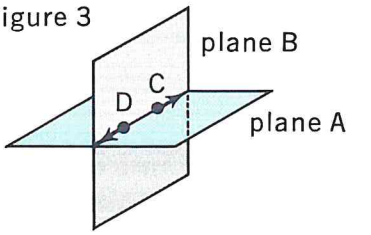
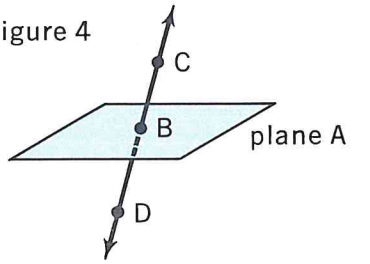
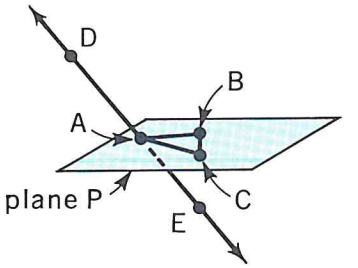


Figure 4



2. Points A, B, and C lie in plane P. Points D and E do not lie in plane P. Use this information and the diagram to complete the equations.



a. $\overleftrightarrow{DE} \cap$ plane $P =$ ■

b. $\overline{AB} \cap$ plane $P =$ ■

c. $\overline{BC} \cap \overline{AB} =$ ■

d. $\overline{AC} \cap$ plane $P =$ ■

e. $\overline{AC} \cap \overleftrightarrow{DE} =$ ■

3. Points V, W, and X lie in plane R. Points V, W, X, Y, and Z lie in plane S. Tell whether each statement is true or false.

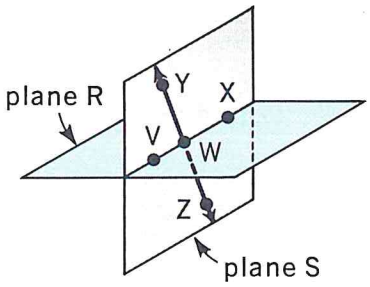
a. $\overline{VX} \cap$ plane $R = \overline{VX}$

b. plane $S \cap$ plane $R = \overline{YZ}$

c. $\overline{YZ} \cap$ plane $S = \overline{VX}$

d. plane $R \cap$ plane $S = \overleftrightarrow{VX}$

e. plane $R \cap \overline{YZ} = \{W\}$



Pair each statement with a figure.

1. Plane M ∩ plane N = $\overleftrightarrow{CF}$
2. $\overline{CF}$ ∩ plane M = {D}
3. $\overline{CF}$ ∩ plane M = {E}
4. $\overline{CF}$ ∩ $\overline{DE}$ = { }

Figure 1

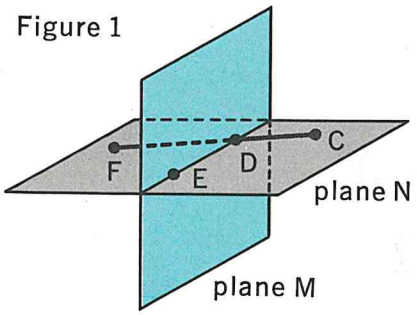


Figure 2

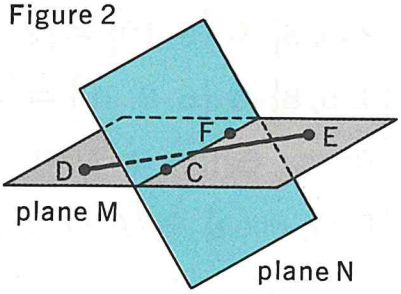


Figure 3

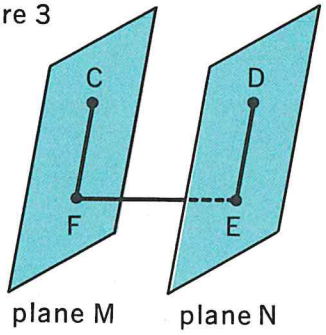
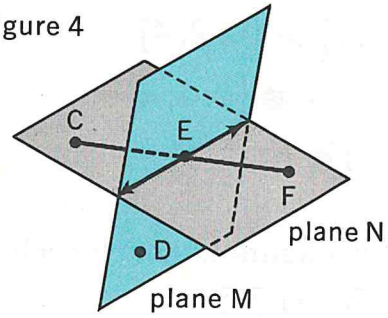
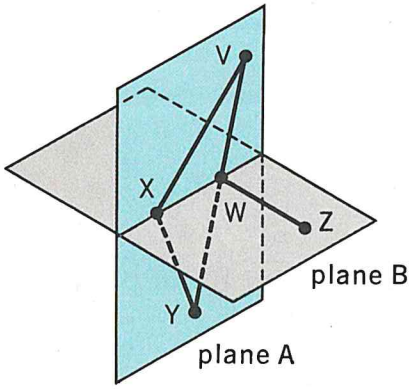


Figure 4



5. Complete each statement. Points V, W, X, and Y lie in plane A. Points X, W, and Z lie in plane B.



a. Plane A ∩ plane B = ■
b. $\overline{VY}$ ∩ plane B = ■
c. $\overline{XW}$ ∩ plane A = ■
d. $\overline{WZ}$ ∩ plane B = ■
e. $\overline{XV}$ ∩ $\overline{ZW}$ = ■

Checkout

Choose $\cap$ or $\cup$ or $-$.

1. $\{3, 6, 9\} \bigcirc \{2, 4, 6, 8\} = \{6\}$
2. $\{2, 5, 8\} \bigcirc \{5, 10\} = \{2, 5, 8, 10\}$
3. $\{4, 6, 8\} \bigcirc \{6, 8, 10\} = \{4\}$
4. $\{\text{Donna, Betty}\} \bigcirc \{\text{Betty, Karen}\} = \{\text{Betty}\}$
5. $\{\text{Paul, Allen}\} \bigcirc \{\text{Allen, Donald}\} = \{\text{Paul}\}$

Choose $\subset$ or $\in$.

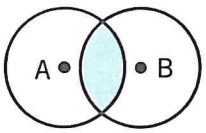
6. $\{0\} \bigcirc \{0, 2, 4\}$
7. $\{\ \} \bigcirc \{0, 2, 4\}$
8. $0 \bigcirc \{0, 2, 4\}$

Draw a figure to match each set description.

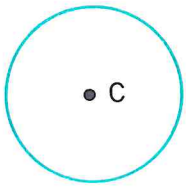
9. $\overrightarrow{AB} \cup \overrightarrow{AC}$
10. $\overline{AB} \cup \overline{AC} \cup \overline{CA}$
11. $\overline{AB} \cup \overline{BC} \cup \overline{CD} \cup \overline{DB}$

Write a set description for each figure.

12.



13.

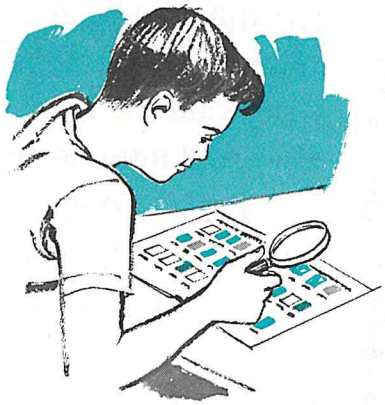


14. Points A and B lie in plane D. Point C does not lie in plane D.
    a. Does line AB lie in plane D?
    b. Does line AC lie in plane D?

Checkout

Choose = or ≠.

1. $3^5 \times 6^8 \bigcirc 6^8 \times 3^5$
2. $(.7)^2 + (.3)^2 \bigcirc (.3)^2 + (.7)^2$
3. $(-3) - 16 \bigcirc 16 - (-3)$
4. $5^4 \div 3^2 \bigcirc 3^2 \div 5^4$



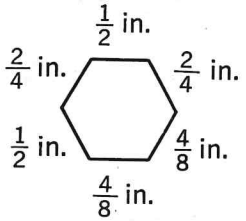
Solve each equation.

5. $1\frac{2}{3} \times 6 = n$
6. $2\frac{1}{2} \times 1\frac{1}{3} = n$
7. $9 \times 2\frac{2}{3} = n$
8. $4\frac{1}{2} \div 3 = n$
9. $3\frac{1}{3} \div 1\frac{1}{4} = n$
10. $10 \div 2\frac{2}{3} = n$

Find each answer.

11. $\begin{array}{r} 84.6 \\ \times\ .92 \\ \hline \end{array}$
12. $\begin{array}{r} 9.87 \\ \times .025 \\ \hline \end{array}$
13. $\begin{array}{r} 6.04 \\ \times 7.23 \\ \hline \end{array}$
14. $\begin{array}{r} 29.64 \\ \times\ 10.8 \\ \hline \end{array}$

15. Jim measured the lengths of the sides of a hexagon. The picture shows his results. Can you find a better way to show the results?



16. Max's garden measures 7 ft. by 9.1 ft. Bob's garden measures 2 yd. by 3.5 yd. Which has the greater area?

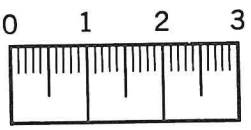
17. Mark has $6.45. His sister Ruth has $3.80. Do they together have enough money to buy a gift that costs $10.20?

18. Last week Mr. Clark worked 38.5 hours and earned $3.50 an hour. How much did he earn for the week?

Part 19 More Measuring

The metric system of measurement is widely used in scientific work. The different units are multiples of 10, so it is easy to convert from one unit to another. Some of the units for measuring distance in the metric system are the kilometer (km.), meter (m.), decimeter (dm.), centimeter (cm.), and millimeter (mm.).

1 m. = 10 dm.
= 100 cm.
= 1000 mm.



1 mm. = .1 cm.
= .01 dm.
= .001 m.

3 cm. = 30 mm.

1 km. = 1000 m. or .001 km. = 1 m.

1. Arrange by size from smallest to largest.
   a. 1 m. b. 1 cm. c. 1 km. d. 1 mm. e. 1 dm.

2. Convert to decimeters, centimeters, and millimeters.

.12 m. = 10 × .12 dm. = 1.2 dm.
.12 m. = 100 × .12 cm. = 12 cm.
.12 m. = 1000 × .12 mm. = 120 mm.

   a. 3.1 m. b. .02 m. c. 8.23 m. d. .317 m. e. .008 m.

3. Convert to centimeters, decimeters, and meters.

18 mm. = .1 × 18 cm. = 1.8 cm.
18 mm. = .01 × 18 dm. = .18 dm.
18 mm. = .001 × 18 m. = .018 m.

   a. 9 mm. b. 12 mm. c. 103 mm. d. 152 mm. e. 700 mm.

The metric system uses the **square centimeter** (sq. cm.) instead of the square inch as the basic unit for measuring area.

Fig. A shows a square measuring 1 cm. on each side. Its area is 1 sq. cm.

Fig. B shows a rectangle measuring 2 cm. by 3 cm. Its area is 6 sq. cm.

Fig. A

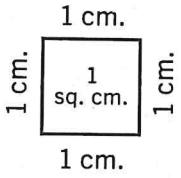
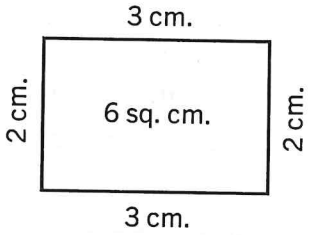


Fig. B



A square measuring 1 m. by 1 m. has an area of 1 square meter (sq. m.).

1 m. = 100 cm.

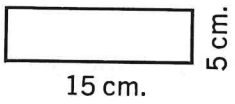
1 sq. m. = 100 × 100 sq. cm.

1 m. = 10 dm.

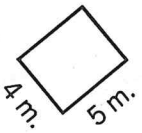
1 sq. m. = 10 × 10 sq. dm.

1. Write the area measure of these figures.

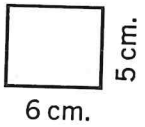
a.



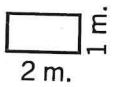
b.



c.



d.



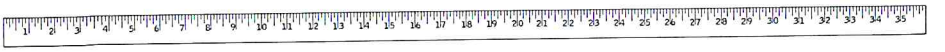
2. Find the area of each rectangle described below.

a. 45 cm. by 25 cm.
b. 10 cm. by 27 cm.
c. 92 cm. by 30 cm.
d. 14 m. by 12 m.
e. 80 m. by 60 m.
f. 4.4 m. by 19 m.
g. 5.4 cm. by 32 cm.
h. 14 m. by 6.4 m.
i. 90 m. by 100 m.

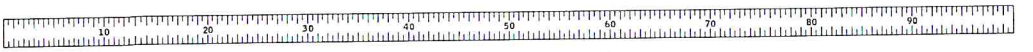
3. First express the length of both sides in centimeters. Then give the area of the rectangles in square centimeters.

a. 70 cm. by 1 m.
b. 45 cm. by 1 m.
c. 24 cm. by 2 m.
d. 2 m. by 50 cm.
e. 1 m. by 92 cm.
f. 4 m. by 20 cm.
g. 40 cm. by 1.4 m.
h. 50 cm. by 1.5 m.
i. 60 cm. by 1.1 m.

We can convert between systems



1 yard = 36 inches



1 meter = 100 centimeters

1 inch is approximately 2.54 centimeters
1 meter is approximately 39.4 inches
1 kilometer is approximately .621 miles

1. Choose < or = or > .

a. 1 meter ● 1 yard	d. 2 km. ● 2 mi.	g. 200 km. ● 3 yd.
b. 3 yd. ● 2 m.	e. 4 m. ● 9 ft	h. 100 cm. ● 1 yd.
c. 1 ft. ● 1 m.	f. 5 in. ● 10 cm.	i. 300 mi. ● 400 km.

2. One rectangle measures 6 cm. by 9 cm. Another measures 2 in. by 5 in. Which has the greater perimeter?

3. A sign says "75 km. to Greenwich". Approximately how many miles is this?

4. Mike's French pen pal's garden measures 3 m. by 5 m.
   a. What are the approximate dimensions in yards?
   b. What is the area in square meters?
   c. What is the approximate area in square yards?

5. Each side of a hexagon measures 13.72 cm. What is its approximate perimeter in inches?

6. The length of a rectangle is 12.63 m. and its width is 8.72 m. What is its approximate perimeter in yards?

Countries using the metric system measure the volume of liquids in **liters.** A cube of width 10 cm., length 10 cm. and height 10 cm. has volume 1 liter. A liter is approximately 1.06 quarts.

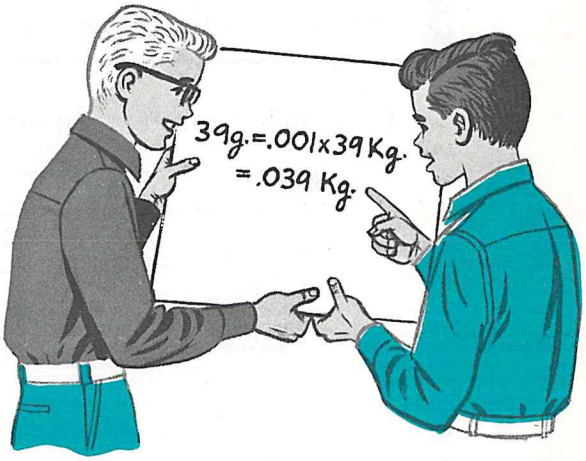
1. Choose $<$ or $=$ or $>$.

a.	1 qt. ● 1 liter	**c.**	1 liter ● 3 pt.	**e.**	3 gal. ● 3 liters
b.	1 pt. ● 1 liter	**d.**	1 gal. ● 3 liters	**f.**	5 gal. ● 4 liters

Countries using the metric system use the **gram** (g.) and **kilogram** (kg.) instead of the ounce and pound as basic units of measure. Under standard conditions one liter of water weighs one kilogram.

1 kg. = 1000 g. or .001 kg. = 1 g.

1 kg. is approximately 2.2 lb.



2. Convert the grams to kilograms.

a.	3 g.	**c.**	24 g.	**e.**	308 g.
b.	16 g.	**d.**	154 g.	**f.**	1932 g.

3. Convert the kilograms to pounds.

2 kg. = 2 × 2.2 lb. = 4.4 lb.

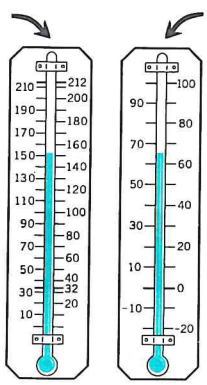
a.	16 kg.	**c.**	107 kg.	**e.**	827 kg.
b.	38 kg.	**d.**	98 kg.	**f.**	1000 kg.

4. Ellen weighed 88 lb. before leaving the U. S. to visit friends in Italy. A scale in Italy measured her weight at 40 kg. Has she gained or lost any weight?

5. About how many ounces equal 1 kilogram? (16 oz. = 1 lb.)

Measuring Temperature

We often use the **Fahrenheit** scale to measure temperature. On this thermometer the freezing point of water is 32°F and the boiling point of water is 212°F.



The **Centigrade** scale is also used to measure temperature. On this scale the freezing point of water is 0°C and the boiling point of water is 100°C.

On both scales, temperatures below 0° are written with a minus sign. Every 1° rise in temperature on the Centigrade scale is equivalent to a 1.8° rise on the Fahrenheit scale.

1. Copy and complete this table.

	Centigrade Scale	Fahrenheit Scale
boiling point of water		
freezing point of water		32°F
number of degrees between boiling and freezing points of water	100	

2. To convert from Centigrade to Fahrenheit, first multiply the Centigrade reading by 1.8, then add 32°. Convert these temperatures to Fahrenheit.

Convert −6°C to Fahrenheit.

$-6 \times 1.8 = -10.8$

$-10.8 + 32 = 21.2$

So $-6°C = 21.2°F$

Convert 0°C to Fahrenheit.

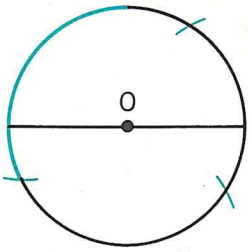
$0 \times 1.8 = 0$

$0 + 32 = 32$

So $0°C = 32°F$

a. 70°C.
b. 40°C.
c. 50°C.
d. 3°C.
e. 65°C.
f. −10°C.
g. 85°C.
h. −40°C.
i. −8°C.
j. −16°C.
k. 98°C.
l. −23°C.

Circumference of a Circle



1. **a.** On your paper construct a circle having a 2-inch radius and label the center "O".
   b. Draw a diameter of the circle.
   c. Cut a piece of string whose length is the length of the diameter of the circle.
   d. Use the string as a unit of measure and find the circumference of the circle. Mark unit lengths as shown.

2. **a.** On your paper construct a circle having a radius of 1 inch.
   b. Draw a diameter.
   c. On a long piece of string, mark off lengths equal to the length of the diameter.
   d. Wrap the string around the circle and find the circumference.
   e. Is the circumference longer than 3 diameters?
   f. Is the circumference longer than 4 diameters?

Let d be the length of a diameter of a circle. The circumference of the circle is greater than 3d and less than 4d. If we measured more accurately we would find that the circumference is about 3.14d. To find the circumference we multiply d by a number called "pi".
"Pi" is written "π".
The number π is approximately equal to 3.14. It is more nearly equal to 3.1416. π is even more nearly equal to 3.14159265358979323846, but this number is still not exactly equal to π.

3. Letting $\pi = 3.14$, find the circumference of each of these circles whose diameters are given.

   a. $d = 10$ in.
   b. $d = 12$ ft.
   c. $d = 18$ yd.
   d. $d = 20$ ft.
   e. $d = 42$ ft.
   f. $d = 16$ in.

1. A diameter is ▒ times as long as a radius.

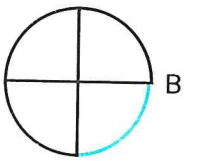
2. Find the circumference of each of these circles whose radii are given. Remember $d = 2r$.

   a. $r = 7$ in.
   b. $r = 15$ in.
   c. $r = 8$ in.
   d. $r = 50$ ft.
   e. $r = 16$ ft.
   f. $r = 30$ yd.

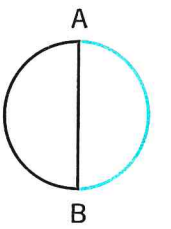
3. We sometimes let $\pi = 3\frac{1}{7}$ or $\frac{22}{7}$. Letting $\pi = 3\frac{1}{7}$, find the circumference of each of these circles whose diameters are given.

   a. $d = 21$ ft.
   b. $d = 56$ ft.
   c. $d = 49$ yd.
   d. $d = 70$ yd.
   e. $d = 14$ in.
   f. $d = 7$ in.

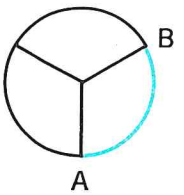
4. The length of arc AB is $\frac{1}{4}$ the circumference of the circle. If $r = 8$ ft., what is the circumference? What is the length of arc AB?



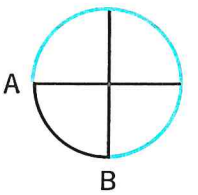
5. If $r = 3$ in., what is the circumference? What is the length of arc AB?



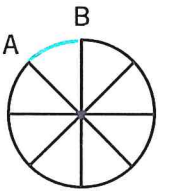
6. For each circle, find the length of arc AB.

a.


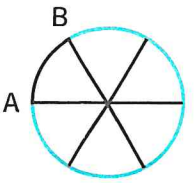
r = 3 in.

b.


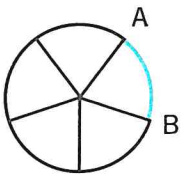
r = 1 m.

c.


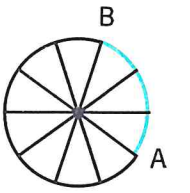
r = 1 mi.

d.


r = 10 ft.

e.


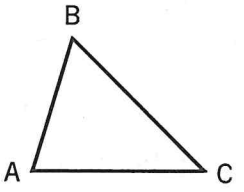
r = 5 cm.

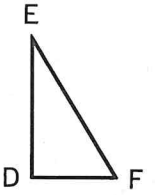
f.


r = 18 yd.

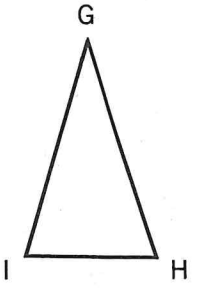
Base and Height of a Triangle

The **base** of a triangle can be any side of the triangle. Often we think of the base of the triangle as the horizontal side on which the triangle rests. Then we say that the base of triangle ABC is $\overline{AC}$ and the base of triangle DEF is $\overline{DF}$.



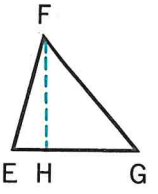


In an isosceles triangle with two congruent sides, we often think of the third side as the base. Then we say that the base of triangle GHI is $\overline{HI}$.

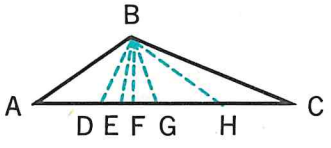


An **altitude** of a triangle is a line segment whose endpoints are a vertex and a point on the opposite side or the opposite side extended, and which is perpendicular to the opposite side. A **height** of a triangle is the length of an altitude.

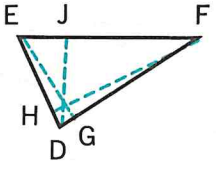
The distance from F to H is the height of triangle EFG.



1. If $\overline{AC}$ is the base of triangle ABC, which dotted line segment shows an altitude?

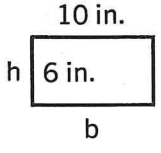


2. Tell what sides could be the base and what the corresponding altitudes would be.

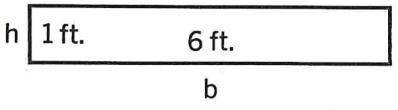


3. Find the area of each rectangle.

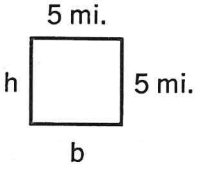
a.



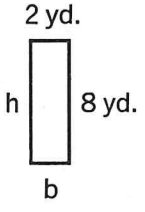
b.



c.

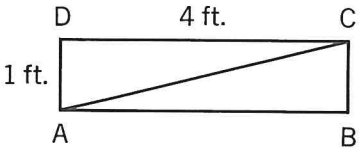


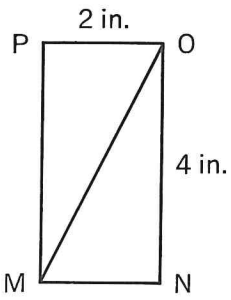
d.



Area of a Triangle

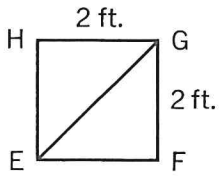
1. a. Find the area of rectangle ABCD.
   b. Triangle ABC has the same area as triangle ACD. What is the area of triangle ABC?



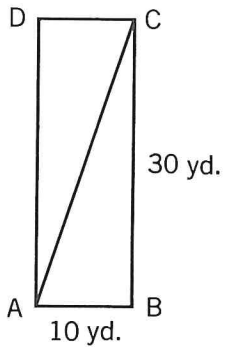


2. a. Find the area of rectangle MNOP.
   b. Find the area of triangle PMO.
   c. Find the area of triangle MNO.

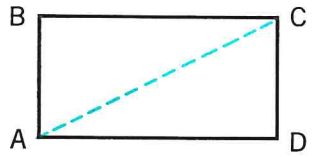
3. a. Find the area of square EFGH.
   b. Find the area of triangle EHG.
   c. Find the area of triangle EGF.



4. Find the area of triangle ABC. (Hint: First find the area of rectangle ABCD.)



We write "m $\overline{AB}$" for the length or measure of $\overline{AB}$.



Area of ABCD $=$ m $\overline{AB}$ $\times$ m $\overline{BC}$
$=$ m $\overline{CD}$ $\times$ m $\overline{AD}$

Area of triangle ABC $= \frac{1}{2} \times$ (m $\overline{AB}$ $\times$ m $\overline{BC}$)

because the inside of triangle ABC is one-half the inside of rectangle ABCD

Area of triangle ADC $= \frac{1}{2} \times$ (m $\overline{CD}$ $\times$ m $\overline{AD}$)

Area of a triangle $= \frac{1}{2} \times$ length of base $\times$ height.

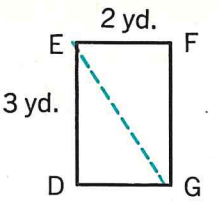
1. Find the area of:

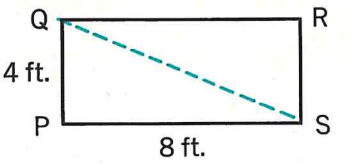
a. triangle DEG (area in sq. yd.)

b. triangle EFG (area in sq. yd.)

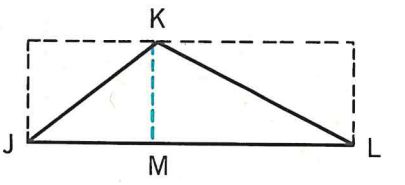
c. triangle PQS (area in sq. ft.)

d. triangle QRS (area in sq. ft.)





To find the area of triangle JKL:



area of triangle JKM $= \frac{1}{2} \times (m\,\overline{KM} \times m\,\overline{JM})$

area of triangle KML $= \frac{1}{2} \times (m\,\overline{KM} \times m\,\overline{ML})$

area of triangle JKL = the area of JKM + the area of KML

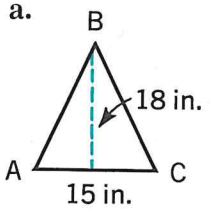
$= \left(\frac{1}{2} \times m\,\overline{KM} \times m\,\overline{JM}\right) + \left(\frac{1}{2} \times m\,\overline{KM} \times m\,\overline{ML}\right)$

$= \frac{1}{2} \times m\,\overline{KM} \times (m\,\overline{JM} + m\,\overline{ML})$ (because of the distributive law)

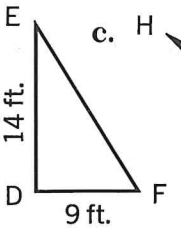
$= \frac{1}{2} \times m\,\overline{KM} \times m\,\overline{JL}$ (because $m\,\overline{JM} + m\,\overline{ML} = m\,\overline{JL}$)

2. Using the given dimensions, find the area of each triangle.

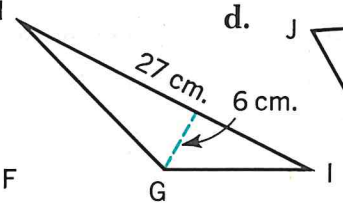
a.



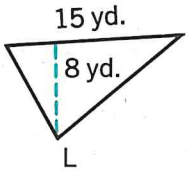
b.



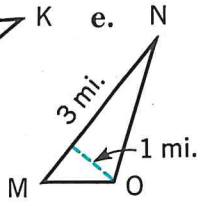
c.



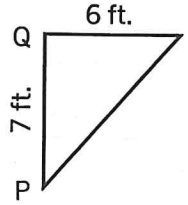
d.



e.



f.



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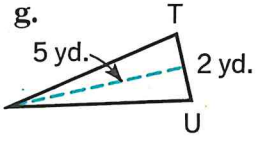

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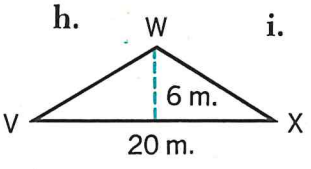
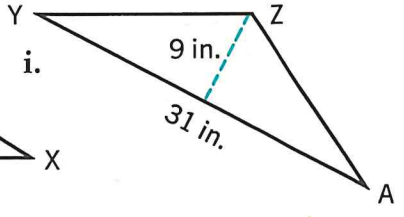
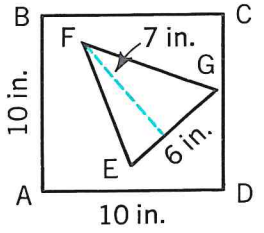


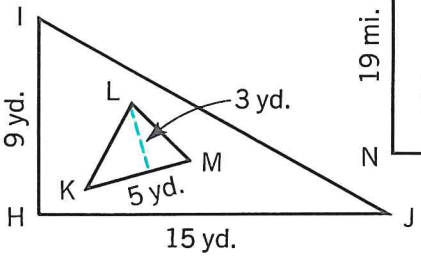
i.

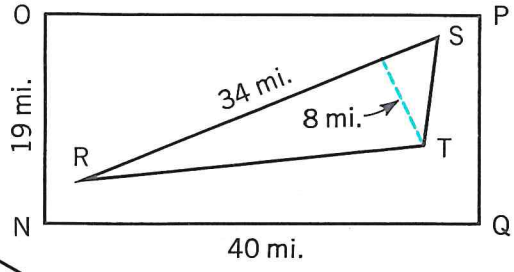


1. Find the area of:

   a. square ABCD
   b. triangle EFG
   c. triangle HIJ
   d. triangle KLM
   e. rectangle NOPQ
   f. triangle RST



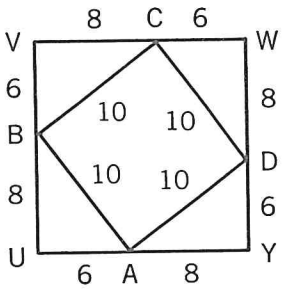




2. Use the results of Exercise 1 to answer these questions.

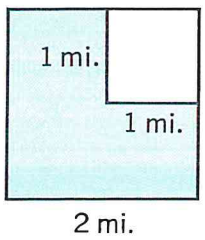
   a. How much greater is the area of square ABCD than the area of triangle EFG?
   b. How much greater is the area of triangle HIJ than the area of triangle KLM?
   c. How much greater is the area of rectangle NOPQ than the area of triangle RST?

3. a. Find the area of triangles BVC, CWD, DYA, AUB.
   b. Find the area of squares UVWY, ABCD. (Find the area of ABCD in 2 ways.)
   c. How much greater is the area of square UVWY than the area of square ABCD?

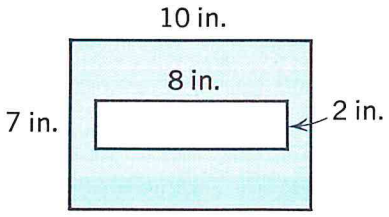


4. Find the area of the shaded part.

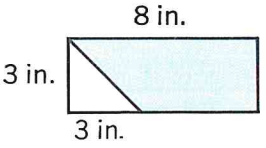
a.

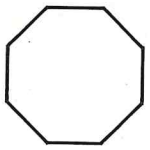


b.



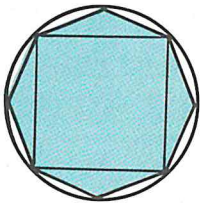
c.



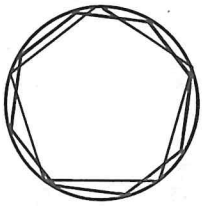


A figure with 8 sides is called an **octagon.**

Choose **greater than** or **less than.**



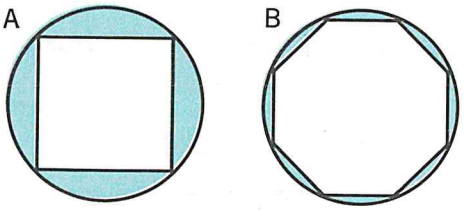
1. **a.** The area of the octagon is ■ the area of the square.
   b. The area of the octagon is ■ the area of the circle.
   c. The area of the circle is ■ the area of the square.



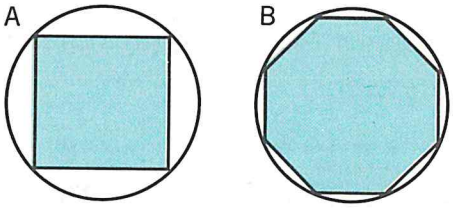
2. **a.** The area of the octagon is ■ the area of the circle.
   b. The area of the pentagon is ■ the area of the circle.
   c. The area of the octagon is ■ the area of the pentagon.

Which shaded part has the greater area?

3.

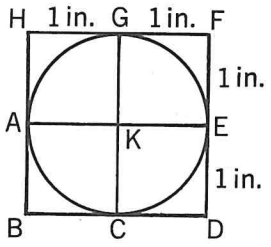


4.



Complete each exercise.

5. **a.** Area square AHGK = ■ sq. in.
   b. Area square KEFG = ■ sq. in.
   c. Area square BDFH = ■ sq. in.
   d. Is the area of circle K greater or less than the area of square BDFH?

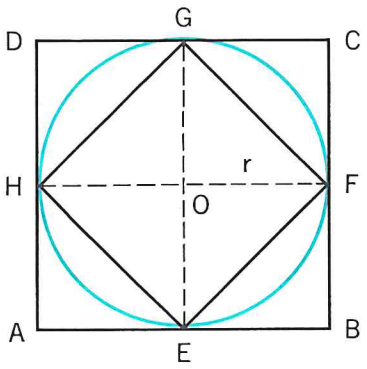


6. **a.** Construct the figure in Exercise 5 and draw $\overline{AG}$, $\overline{GE}$, $\overline{EC}$, and $\overline{AC}$.
   b. Area of square ACEG = ■ sq. in.
   c. Is the area of circle K greater or less than the area of square ACEG?

Area of a Circle

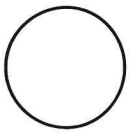
The radius of the circle is r. Complete each problem.

1. Area square GOFC = ▢
2. Area square ABCD = ▢
3. Area triangle GOF = ▢
4. Area square EFGH = ▢
5. Area circle > ▢
6. Area circle < ▢



The area of a circle is greater than $2r^2$ and less than $4r^2$.
The area is slightly greater than $3r^2$.

Area of a circle = πr^2



Letting $\pi = 3.14$ approximately, find the area of these circles whose radii are given.

7. r = 10 in.
8. r = 7 cm.
9. r = 100 m.
10. r = 15 in.
11. r = 20 ft.
12. r = 64 yd.

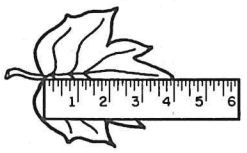
Letting $\pi = 3\frac{1}{7}$ or $\frac{22}{7}$ approximately, find the area of each of these circles whose radii are given.

13. r = 49 in.
14. r = 7 ft.
15. r = $3\frac{1}{2}$ in.
16. r = 42 yd.
17. r = 56 cm.
18. r = 35 m.

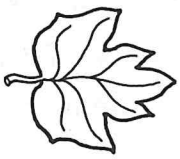
19. a. If you know the radius of a circle, how do you find the area of the circle?
    b. If you know the area of a circle, how do you find the area of half of the circle?

In science class, Sara, Jack, and Scott tried to find the average length of a maple leaf. They chose 4 leaves and each child measured the length of each leaf, using a ruler marked to show sixteenths. Here are their results:

measurement made by	length of leaves			
	leaf 1	leaf 2	leaf 3	leaf 4
Sara	$3\frac{7}{8}$ in.	4 in.	$3\frac{15}{16}$ in.	$3\frac{15}{16}$ in.
Jack	$3\frac{15}{16}$ in.	$4\frac{1}{16}$ in.	4 in.	4 in.
Scott	$3\frac{13}{16}$ in.	$3\frac{7}{8}$ in.	$3\frac{15}{16}$ in.	4 in.

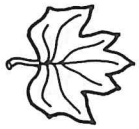


leaf 1



leaf 2

leaf 3



leaf 4

1. What might be some reasons that different measurements were obtained for the length of the same leaf?

2. The children agreed to use the average measurement of each leaf. What is the average measurement of each leaf?

leaf 1 : $\dfrac{3\frac{7}{8} + 3\frac{15}{16} + 3\frac{13}{16}}{3} = 3\frac{7}{8}$ The average measurement of leaf 1 is $3\frac{7}{8}$ in.

3. Using the average lengths found in problem 2, what is the average length of a maple leaf?

4. Jack used the 4 measurements he made and found the average length of a maple leaf to be 4 in. Sara and Scott said by first finding the average length of each leaf, and then finding the average length of the 4 leaves, they got a more accurate result. Do you agree? Why or why not?

Scale Drawings

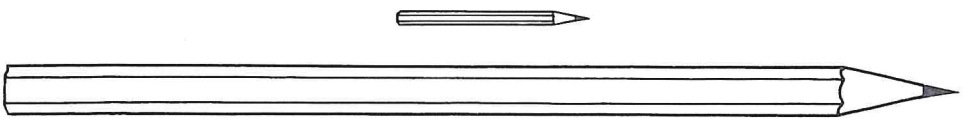
When drawings are used in a book or when maps are drawn, they are drawn to a **scale.** On a map the scale may be described as:

1 inch = 200 mi.

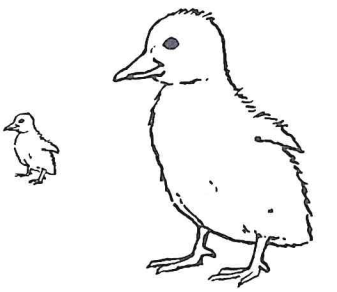
This tells us that every inch on the map represents 200 miles.

Pictures of insects, animals, and fish found in many books are drawn to a scale. The scale is usually written as a fraction such as $\frac{1}{4}$. This tells us that the drawing, of an insect, for example, is $\frac{1}{4}$ the actual size of the insect.

1. a. Here are two drawings of a pencil which is 5 inches long.
   a. The small drawing was made to a scale of 1 in. = ▢ in.
   b. The large drawing was made to a scale of 1 in. = ▢ in.



2. Here are two drawings of a baby duck which is 7.2 cm. tall.
   a. If the large drawing is 3.6 cm. tall, it is made to a scale of $\frac{1}{▢}$.
   b. If the small drawing is .9 cm. tall, it is made to a scale of $\frac{1}{▢}$.



3. These goldfish are drawn to different scales. Fish A was drawn to a scale of $\frac{1}{5}$. Fish B was drawn to a scale of $\frac{1}{2}$.

Fish A

Fish B

If the drawing of fish A is 1.4 cm. long and the drawing of fish B is 3.9 cm. long, which goldfish is longer?

Ratios

When we express the scale that we are using as $\frac{1}{4}$, we are actually expressing it as a **ratio**.

We write: 1:4 or $\frac{1}{4}$ We read: The ratio of 1 to 4.

We use ratios when we compare the measures of two objects or the number of objects in two sets.

1. **a.** Karen has 2 skirts and 3 sweaters. The ratio of skirts to sweaters is 2 : ■.

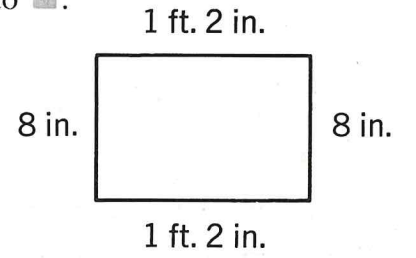
b. The ratio of sweaters to skirts is ■.

2. There are 16 boys and 12 girls in Miss Clark's room.

a. The ratio of boys to girls is 16 to 12 or 4 to ■.

b. The ratio of girls to boys is ■.

3. The ratio of the length to the width of this rectangle is ■.



4. **a.** The ratio of the circumference of a circle to the diameter is π : ■.

b. The ratio of the diameter of a circle to the circumference is ■.

5. Measure the lengths of these line segments to the nearest half inch.

a. _____

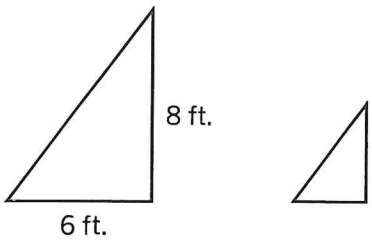
b. _____

c. _____

6. Using the lengths of the line segments in Exercise 5, find each ratio.

a. a : b **b.** a : c **c.** b : c **d.** c : a

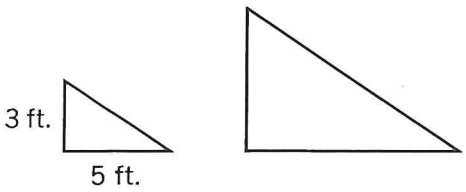
1. The sides of the large triangle are twice as long as the sides of the small triangle.



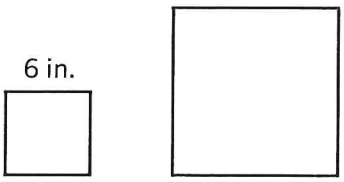
   a. What is the area of the small triangle?
   b. What is the area of the large triangle?
   c. Is the area of the large triangle twice the area of the small triangle?
   d. The ratio of the area of the large triangle to the area of the small triangle is ▢.

2. Each side of the larger figure is 2 times as long as the side of the smaller figure. Find the area of each figure.

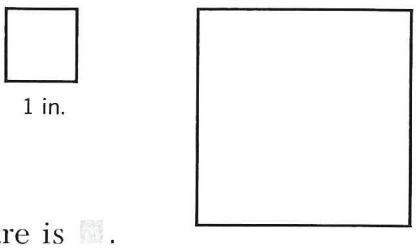
   a.



   b.



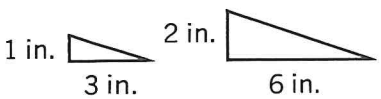
3. The ratio of the sides of the large square to the sides of the small square is 3 : 1.



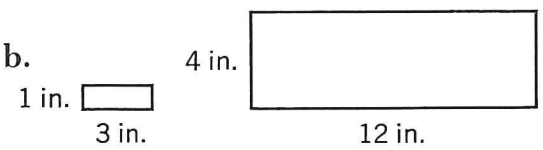
   a. The area of the large square is ▢ times the area of the small square.
   b. The ratio of the area of the large square to the area of the small square is ▢.
   c. The ratio of the area of the small square to the area of the large square is ▢.

4. The area of the large figure is how many times the area of the small figure?

   a.



   b.



Checkout

Compute each power.

1. 3^4
2. 9^3
3. 6^2
4. 4^1
5. 1^{15}
6. 2^5

Divide. Check by multiplication.

7. $8\overline{)1389}$
8. $6\overline{)7866}$
9. $20\overline{)98080}$
10. $17\overline{)9211}$
11. $6.4\overline{)24.32}$
12. $.018\overline{)1.71}$

Express each number in the indicated base.

13. $1011_{two} = \square_{ten}$
14. $213_{five} = \square_{ten}$
15. $1211_{three} = \square_{ten}$
16. $36_{ten} = \square_{eight}$
17. $18_{ten} = \square_{two}$
18. $42_{ten} = \square_{five}$

Find each sum or difference.

19. $8\frac{1}{3} + 2\frac{4}{5}$
20. $7\frac{3}{4} + 8\frac{3}{8}$
21. $38\frac{1}{2} - 14\frac{5}{8}$
22. $29\frac{2}{3} - 14\frac{3}{4}$

Copy and complete each table so that the sum of the numbers expressed in each row, column, and diagonal is the same.

23.

80		20	
25		50	40
45	35		
20		70	5

24.

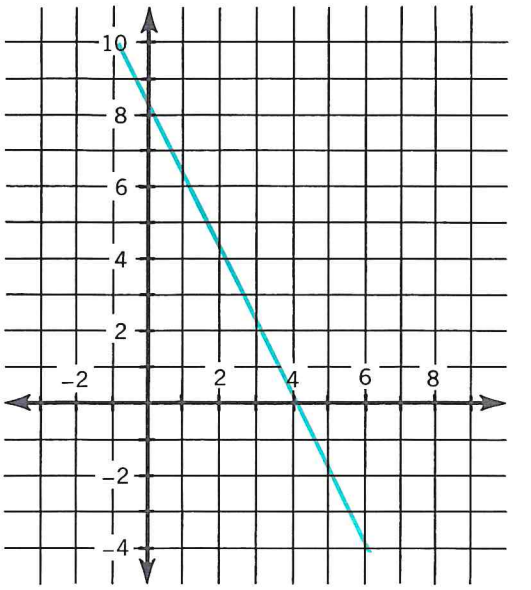
34			16	30
	10		28	
8		26	40	44
20	24		42	6
22	36			

Part 20 Graphs and Functions

Any figure or region drawn on a coordinate system is called a **graph.**

A graph is a set of points.

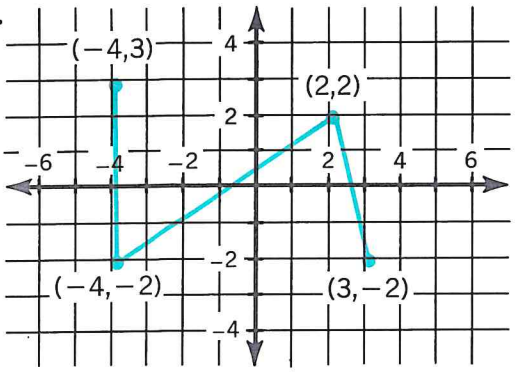
The point (2, 4) is on the graph shown at the right.
The point (5, 2) is not on the graph shown at the right.



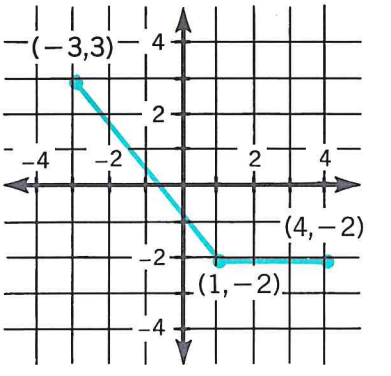
1. Which of these points lie on the graph shown above?
   a. (0, 8) b. (6, 1) c. (−1, −3) d. (4, 0)

2. On which graphs below are these points?
   a. (2, 2) c. (1, −2) e. (−1, 0) g. (3, −2)
   b. (4, −2) d. (−4, 3) f. (−4, 1) h. (2, −2)

A.



B.



Using Tables

Instead of listing points $(-6, -2)$, $(-4, 1)$, $(0, -1)$, $(2, 2)$, $(3, -4)$, we can write the coordinates in a **table:**

x	−6	−4	0	2	3
y	−2	1	−1	2	−4

Row x gives the horizontal coordinates.

Row y gives the vertical coordinates.

1. Make a table for each set of points.

a. $\{(-7, -3), (-4, 5), (-1, 0), (3, 6), (4, -5), (6, 0)\}$

b. $\{(-6, -6), (-5, -3), (-6, 4), (-6, 0), (-6, -7), (-6, 8)\}$

c. $\{(-8, 7), (-5, 7), (0, 7), (4, 7), (5, 7), (7, 7)\}$

2. List the points given in each table.

g	−8	−5	−3	−2	0	1
h	1	2	4	5	7	8

$\{(-8, 1), (-5, 2), (-3, 4), (-2, 5), (0, 7), (1, 8)\}$

a.

r	−7	−5	−2	0	2	3
s	−4	−2	1	3	5	6

b.

a	−2	−2	−2	−2	−2	−2
b	−8	−6	−3	0	4	7

c.

j	−7	−6	−4	−1	0	6
k	3	3	3	3	3	3

d.

p	−6	−5	−2	3	5	7
q	−4	−3	0	5	7	9

3. For each problem in Exercise 2 draw a coordinate system and plot the points.

Rules

Some rules may be written as English sentences or mathematical sentences.

English sentence	Mathematical sentence
Multiply each number by 4.	$n = 4 \times m$
Square each number.	$y = x^2$
Subtract 3 from each number	$q = p - 3$

1. Match each rule written as a mathematical sentence with one written as an English sentence.

$v = \ell \times w \times h$ $\quad$ $i = 36y$ $\quad$ $a = \ell \times w$

$v = s$ $\quad$ $y = x \div 4$ $\quad$ $n - n = 0$

a. If we subtract a number from itself, we get zero.

b. To convert yards to inches, multiply the number of yards by 36.

c. The number of vertices of a polygon is equal to the number of sides.

d. To convert quarts to gallons, divide the number of quarts by 4.

e. To find the area of a rectangle, multiply length by width.

f. The volume of a rectangular solid equals length times width times height.

2. Match each rule with a table. Copy and complete each table.

$d = c$ $\quad$ $d = 2c$ $\quad$ $d = 3c$ $\quad$ $d = c + 2$

a.

c	−3	−2	−1	0	1	2
d		−6			3	6

b.

c	−7	−5	−2	0	3	6
d	−7	−5			3	

c.

c	−9	−6	−2	−1	0	4
d			0		2	

d.

c	−4	−2	−1	0	2	3
d		−4	−2		4	

Satisfying a Rule

A pair of coordinates may satisfy a rule.

Example 1:

Does $(-3, 7)$ satisfy $y = 4 - x$? Yes, because $7 = 4 - (-3)$

$y = 7$ $x = -3$

Example 2:

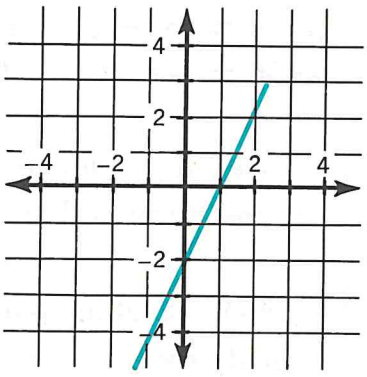
Does $(1, 4)$ satisfy $q = 2p + 3$? No, because $4 \neq (2 \times 1) + 3$

$q = 4$ $p = 1$

1. For each exercise, which pairs of coordinates satisfy the given rule?

a.	$c = d + 4$	$(1, 5)$ $(3, 7)$ $(-1, 5)$ $(-3, -7)$ $(-6, -2)$
b.	$k = j - 3$	$(4, 1)$ $(2, 5)$ $(0, -3)$ $(-2, -5)$ $(-4, -7)$
c.	$n = 5 - m$	$(3, 8)$ $(1, 4)$ $(-1, 6)$ $(0, -5)$ $(-2, 3)$
d.	$s = -3$	$(2, 2)$ $(6, -3)$ $(2, -3)$ $(1, 4)$ $(-2, -3)$
e.	$w = 2v$	$(3, 6)$ $(4, 2)$ $(-1, 4)$ $(-2, -4)$ $(-3, -5)$
f.	$z = 1 + 2y$	$(4, 9)$ $(0, 1)$ $(-2, 5)$ $(-3, -5)$ $(-4, -7)$
g.	$b = 3a - 2$	$(1, 1)$ $(3, 2)$ $(-1, 5)$ $(-2, -8)$ $(-3, -5)$
h.	$x = 4y$	$(4, 16)$ $(1, 4)$ $(0, 4)$ $(-1, -4)$ $(-3, -12)$
i.	$p = 2q + 2$	$(2, 6)$ $(1, 4)$ $(0, 2)$ $(-1, 0)$ $(-2, -2)$

2. a. Choose 4 points on the graph and give their ordered pairs.
   b. Do the pairs of coordinates satisfy $y = 2x - 2$?
   c. Choose 3 points not on the graph and give their ordered pairs.
   d. Do their coordinates satisfy $y = 2x - 2$?
   e. Is the point $(0, 0)$ on the graph?



Making Tables

Given a rule, we can make a table of coordinates.

1. Choose horizontal coordinates.
2. Find the corresponding values for the vertical coordinates.

Example: $s = r - 2$ We wish to find ordered pairs (r, s). Suppose we choose -5, 0, 2, and 4 as values for "r."

If $r = -5$, then $s = -5 - 2$
$= -7$

If $r = 2$, then $s = 2 - 2$
$= 0$

If $r = 0$, then $s = 0 - 2$
$= -2$

If $r = 4$, then $s = 4 - 2$
$= 2$

Our table looks like this:

r	−5	0	2	4
s	−7	−2	0	2

1. Make a table for each of these rules. Choose six numbers between -6 and 6 for the horizontal coordinate.

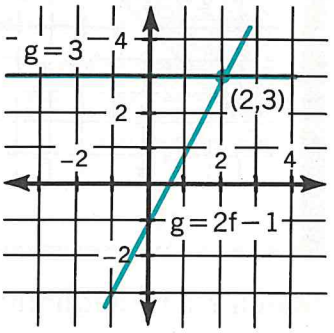
a. $d = c + 3$	d. $w = v + 4$	g. $z = 5$	j. $c = 2b + 1$
b. $n = m - 1$	e. $r = s$	h. $f = -2x$	k. $j = 8 - 3p$
c. $k = -3$	f. $q = 4 - p$	i. $y = 3x$	l. $u = 2t - 4$

2. For each problem in Exercise 1, graph each rule. (Plot the points and draw a line through them.)

3. Choose 2 other points on each graph and find out if their coordinates satisfy the rule.

Solving Equations by Graphing

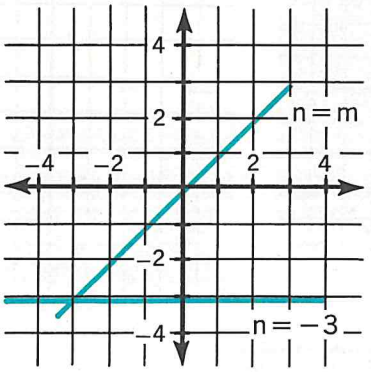
We can draw two graphs on the same coordinate system.
Sometimes the graphs cross each other.
The pair of coordinates of the point of intersection is the **solution** for the two rules. The coordinates satisfy both rules because the point lies on both graphs.



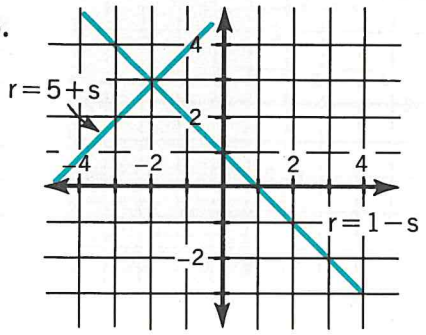
The solution of the rules $g = 3$ and $g = 2f - 1$ is (2, 3).
The coordinates satisfy both rules: $3 = 3$ and $3 = (2 \times 2) - 1$
If the graphs never cross, there is no solution for the two rules.

1. Is there a solution for each pair of rules? If so, what?

a.



b.



2. Graph each pair of rules on one coordinate system. Is there a solution for the pair? If so, what is it?

a. $y = x$
$y = 6 - x$

b. $b = a - 2$
$b = 7$

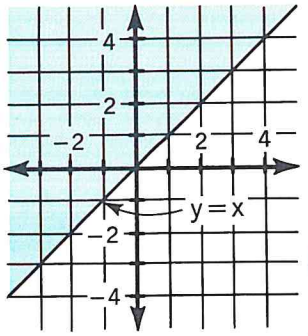
c. $n = m + 4$
$n = 4 - m$

d. $s = r + 3$
$s = r - 2$

e. $e = -9$
$e = d - 7$

f. $k = j + 2$
$k = -j$

Inequalities



The line is the graph of $y = x$.
The shaded part is the graph of the inequality $y > x$.
The coordinates of points in the shaded part satisfy $y > x$.

$-1 > -3$ $\quad$ $0 > -1$ $\quad$ $3 > 0$

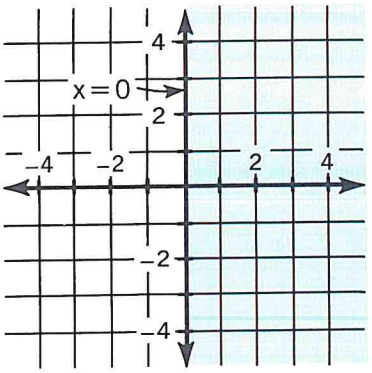
$y = -1$ $\quad$ $x = -3$ $\quad$ $y = 0$ $\quad$ $x = -1$ $\quad$ $y = 3$ $\quad$ $x = 0$

So $(-3, -1)$, $(-1, 0)$ and $(0, 3)$ are solutions to $y > x$.

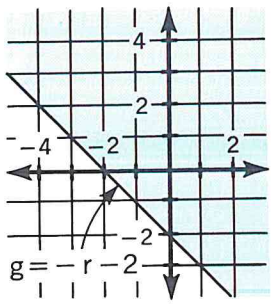
Match each inequality with a graph. Give 3 solutions to each inequality.

1. $x > 1$
2. $y < 0$
3. $q > 3 - p$
4. $x > 0$
5. $x < -1$
6. $g > -r - 2$

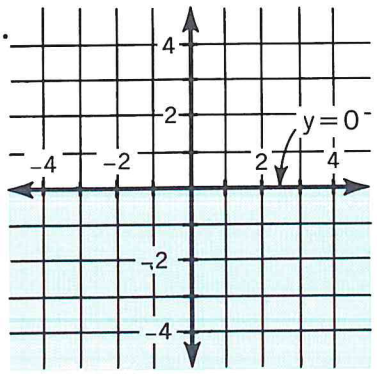
A.



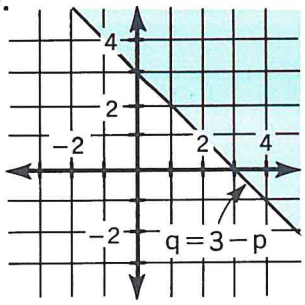
B.



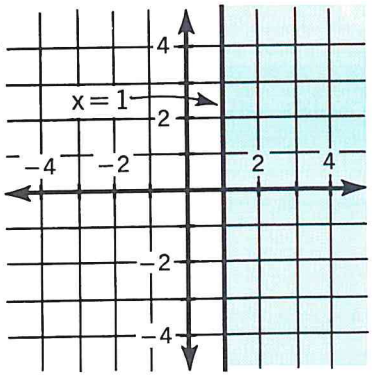
C.



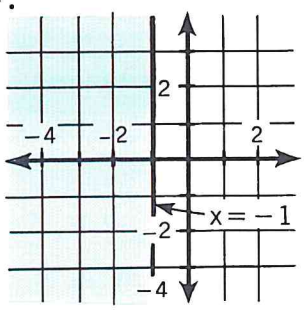
D.



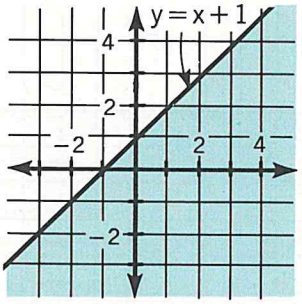
E.



F.



The black line is the graph of $y = x + 1$. So the shaded part is the graph of either $y < x + 1$
or $y > x + 1$.

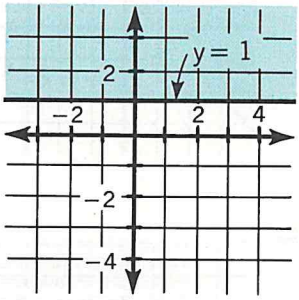


The point $(2, -1)$ is on the colored graph.
Does $(2, -1)$ satisfy $y > x + 1$?
No. -1 is not greater than $2 + 1$.
Does $(2, -1)$ satisfy $y < x + 1$?
Yes. $-1 < 2 + 1$.

$(2, -1)$ is on the graph and satisfies $y < x + 1$. This tells us that the colored graph must be the graph of $y < x + 1$.

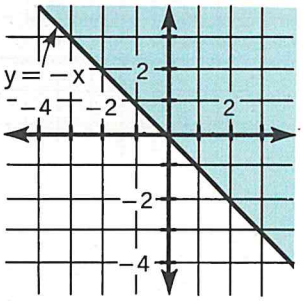
1. a. Does each ordered pair satisfy
$y < 1$ or $y = 1$ or $y > 1$?
$(2, 0)$ $(2, 1)$ $(2, 2)$
$(0, 3)$ $(-1, -3)$ $(-3, 1)$

b. What inequality is the rule for the graph?



2. a. Does each ordered pair satisfy
$y < -x$ or $y = -x$ or $y > -x$?
$(1, -4)$ $(0, 0)$ $(2, -1)$
$(3, 2)$ $(1, -2)$ $(3, -3)$

b. What inequality is the rule for the graph?

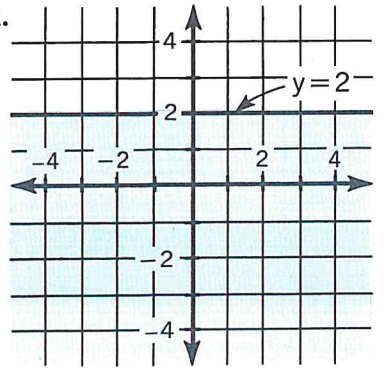


3. a. Does $(0, 0)$ satisfy the inequality $y < 2x$?

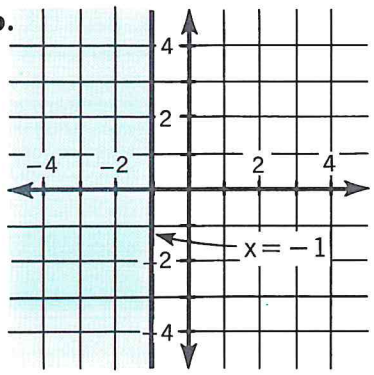
b. Does $(1, 1)$ satisfy the inequality $y < 2x$?

1. What inequality is the rule for each graph? Give 3 solutions to each inequality. (Give 3 pairs of coordinates that satisfy each inequality.)

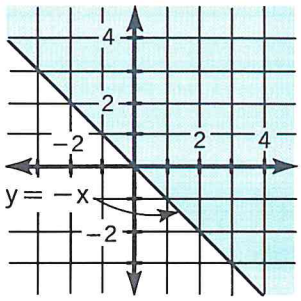
a.



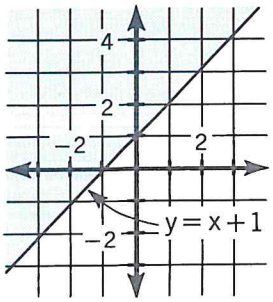
b.



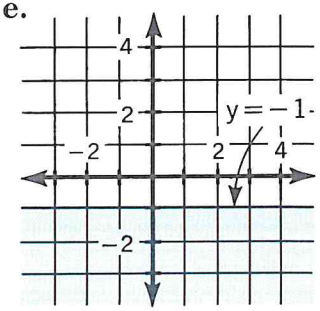
c.



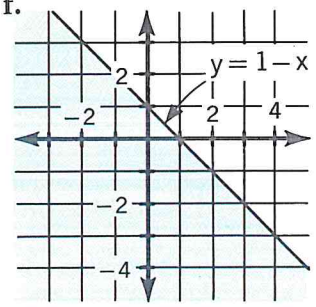
d.



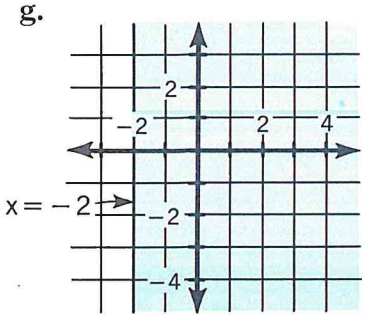
e.



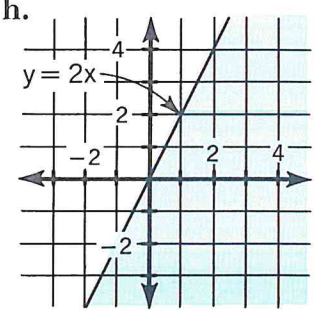
f.



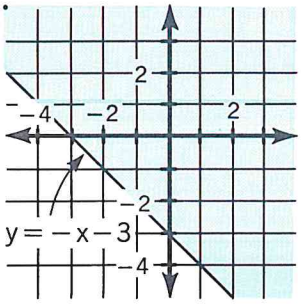
g.



h.



i.



2. Draw a coordinate system and draw the graph of each inequality. (Hint: Before shading, graph each equation.) Give 3 solutions to each inequality.

a. $x < 2$

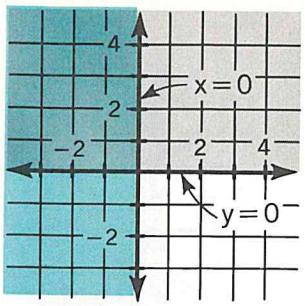
b. $y > 0$

c. $y < x$

d. $y > x + 2$

e. $y > x - 3$

f. $y < 4 - x$



We can solve pairs of inequalities by graphing. The blue shading shows the graph of $x < 0$. The gray shading shows the graph of $y > 0$. The part where the two colors overlap shows the graph of $x < 0$ and $y > 0$. We can check that the coordinates of points in the part shaded blue and gray satisfy both inequalities.

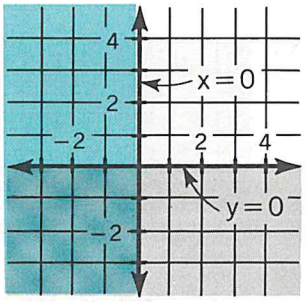
$(-2, 3)$ satisfies $x < 0$ and $y > 0$: $-2 < 0$ and $3 > 0$

$x = -2$ $y = 3$

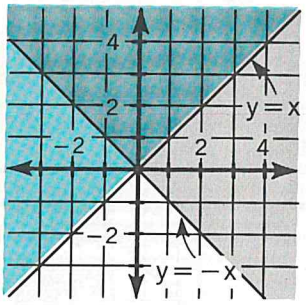
Match each pair of inequalities with a graph. Give 3 solutions to each pair of inequalities.

1. $y > x$ and $x < 1$
2. $y > -x$ and $y < x$
3. $x > 0$ and $y > 0$
4. $y < 0$ and $x < 0$
5. $y > -x$ and $x > 1$
6. $y > -x$ and $y > x$

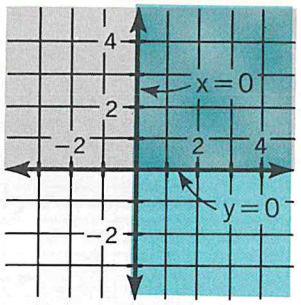
a.



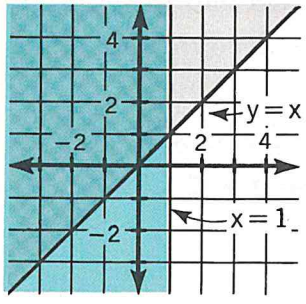
b.



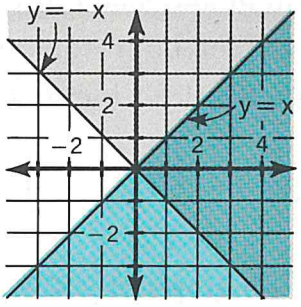
c.



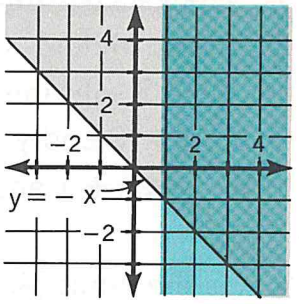
d.



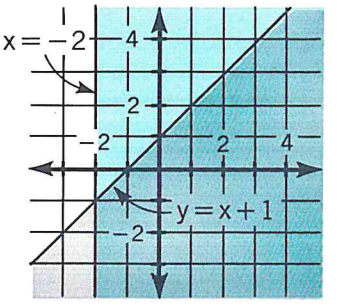
e.



f.



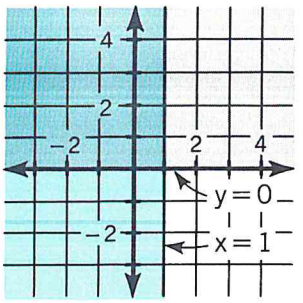
1. $(-1, 3)$ $(2, 1)$ $(-2, 3)$ $(0, 2)$
   $(-3, -1)$ $(0, 1)$ $(-3, -3)$ $(-1, 1)$



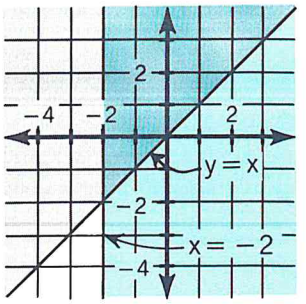
   a. Which of the ordered pairs satisfy $x < -2$?
   b. Which satisfy $x = -2$?
   c. Which satisfy $x > -2$?
   d. The part shaded gray shows the graph of what inequality?
   e. Which ordered pairs satisfy $y < x + 1$?
   f. Which satisfy $y = x + 1$?
   g. Which satisfy $y > x + 1$?
   h. The part shaded blue shows the graph of what inequality?
   i. The part shaded both gray and blue shows the graph of what pair of inequalities?

2. What pair of inequalities describes the part shaded both gray and blue? Give 3 solutions to each pair of inequalities.

a.



b.



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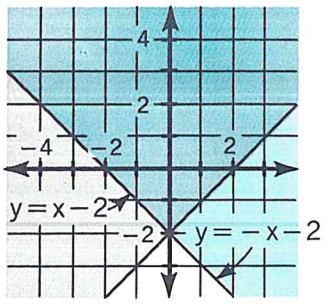


3. Draw a coordinate system and shade it to show each pair of inequalities. (Hint: Before shading, plot both equations. Use 2 colors for shading.) Give 3 solutions for each pair of inequalities.
   a. $x > 0$ and $y < 0$
   b. $y < x + 1$ and $y < 3$
   c. $y > x - 2$ and $x > -3$
   d. $x > 2$ and $y < x$

4. Is the point $(4, 8)$ on the graph of $y < 2x$?

Functions

A set of ordered pairs is a **function** if no two ordered pairs have the same horizontal coordinate.

$\{(-8, 2), (-4, -1), (0, 6), (3, 7), (4, 7)\}$
This set of ordered pairs is a function.

same horizontal coordinate

$\{(-6, 3), (-1, 0), (3, -1), (2, 4), (3, 0)\}$
This set of ordered pairs is **not** a function.

1. Which sets of ordered pairs are functions?
   a. $\{(-7, -3), (-5, -1), (-1, 6), (0, 3), (1, 5), (2, 6), (4, -8), (7, -2)\}$
   b. $\{(-3\frac{1}{2}, -1), (-1, 2\frac{1}{2}), (0, 2\frac{1}{2}), (0, 3\frac{1}{2}), (2\frac{1}{2}, 4\frac{1}{2}), (5, -3)\}$
   c. $\{(-6, -4), (-3, -4), (0, -4), (4, -4), (7, -4), (9, -4)\}$
   d. $\{(-\frac{1}{2}, \frac{1}{2}), (-\frac{1}{3}, \frac{1}{3}), (-\frac{1}{4}, \frac{1}{4}), (0, 0), (\frac{1}{4}, -\frac{1}{4}), (\frac{1}{3}, -\frac{1}{3}), (\frac{1}{2}, -\frac{1}{2})\}$
   e. $\{(3, \frac{1}{3}), (5, \frac{1}{5}), (6, \frac{1}{6}), (-7, -\frac{1}{7}), (-5, -\frac{1}{5}), (-2, -\frac{1}{2})\}$
   f. $\{(-8, -9), (-8, -7), (-7, -8), (-7, -6), (-6, -7), (-6, -5)\}$
   g. $\{(\frac{1}{2}, 1), (\frac{1}{3}, \frac{2}{3}), (\frac{1}{4}, \frac{2}{4}), (\frac{1}{2}, 0), (\frac{1}{3}, 0), (\frac{1}{4}, 0)\}$
   h. $\{(-4, 8), (-2, 4), (0, 0), (1, 1), (2, 4), (3, 6), (5, 10)\}$

2. For each problem in Exercise 1, plot the points on a coordinate system. Label each graph "function" or "not function".

3. On a vertical line do the points have the same horizontal coordinate?

4. On a horizontal line do the points have the same vertical coordinate?

Graphs of Functions

On the graph in Figure 1, no two points have the same horizontal coordinate. This is the **graph of a function.**

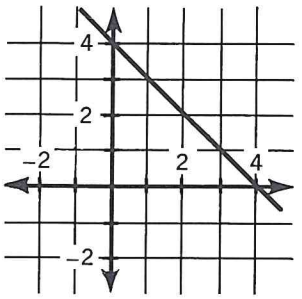


Figure 1

On the graphs in Figures 2 and 3, there are two points with the same horizontal coordinate. These are not graphs of functions.

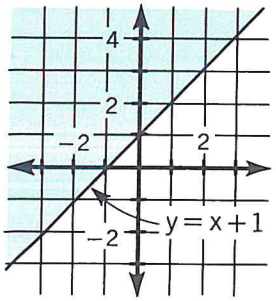


Figure 2

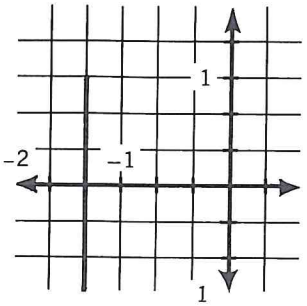
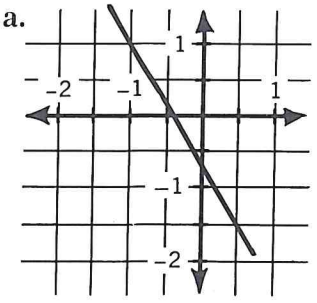


Figure 3

1. Which graphs are graphs of functions?

a.



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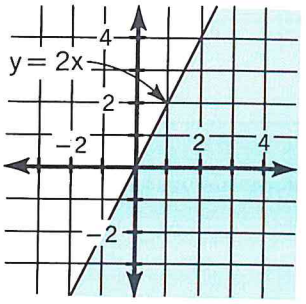

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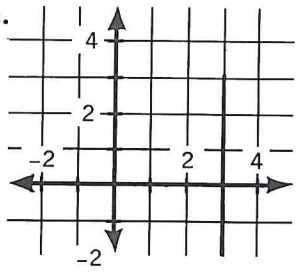


2. Draw the graph for each rule. Have you drawn the graph of a function?

a. $y > x$ **b.** $y = 2x$ **c.** $y = 1 - x$ **d.** $y < 3x$

A linear function determines a straight line.

1. Which tables show linear functions? To find out, graph the given ordered pairs and see if a straight line can be drawn through them. Remember, if we plot a vertical line, we have not plotted a function.

x	−9	−6	−3	0	1	2	4
y	−4	−1	2	5	6	7	9

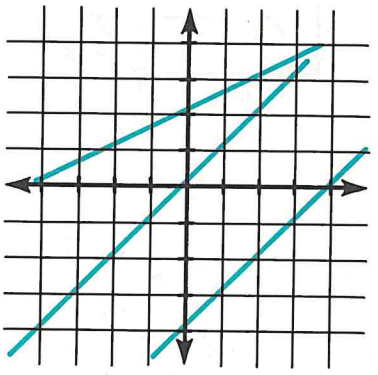
r	−5	−3	−1	2	4	7	9
s	−9	−7	−5	−2	0	3	5

m	−8	−5	−2	0	1	3	6
n	8	8	8	8	8	8	8

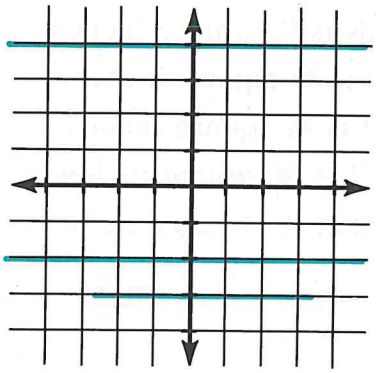
p	−6	−3	−1	0	2	4	8
q	9	6	4	3	1	−1	−5

j	−2	−1	0	1	2	3	4
k	9	6	3	0	−3	−6	−9

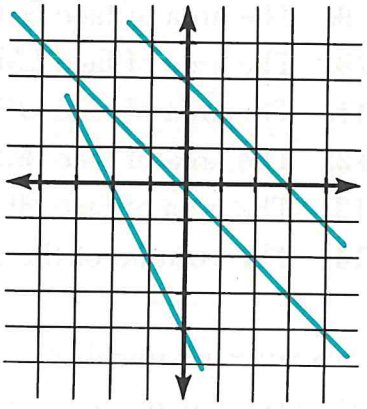
a	−2	−2	−2	−2	−2	−2	−2
b	−7	−4	−1	0	2	4	8



When we plot points on a line that slants up to the right, we graph an increasing function.



When we plot points on a horizontal line, we graph a constant function.



When we plot points on a line that slants down to the right, we graph a decreasing function.

2. Use the linear functions you graphed in Exercise 1 to answer these questions.
   a. Which functions are increasing? constant? decreasing?
   b. May a linear function be an increasing function? a constant function? a decreasing function?

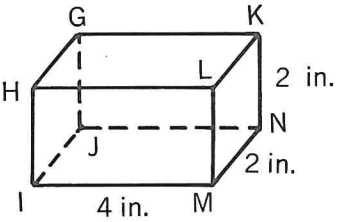
Checkout

Write a mathematical sentence for each English sentence.

1. Bob has 2 more pets than Steve.
2. Darlene has saved more money than her sister Rita.
3. Kent has saved twice as much money as his sister Carol.
4. Jack has 3 fewer stamps than Alan.
5. Joanne has the same number of stamps as Laura.

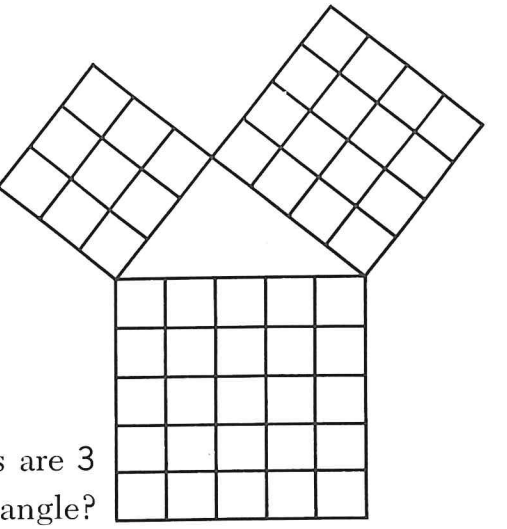
Complete each sentence.

6. This rectangular prism has ▢ faces.
7. Each face is in the shape of a ▢.
8. The area of face IMLH is ▢ square inches.
9. The area of face GJNK is ▢ square inches.
10. The area of face LMNK is ▢ square inches.
11. The area of face IJGH is ▢ square inches.
12. The area of face IMNJ is ▢ square inches.
13. The area of face HLKG is ▢ square inches.
14. The volume of the prism is ▢ cubic inches.



Answer each question.

15. What is the area of the square whose side is 3 units?
16. What is the area of the square whose side is 4 units?
17. What is the area of the square whose side is 5 units?
18. Does $3^2 + 4^2 = 5^2$?
19. Is a triangle, the length of whose sides are 3 units, 4 units, and 5 units, a right triangle?



Checkout

1. Give all the factors of each number.
   a. 36 b. 75 c. 90

2. Find the Greatest Common Factor for each set of numbers.
   a. $\{15, 25\}$ b. $\{14, 21, 28\}$ c. $\{18, 24, 48\}$

3. Find the Least Common Multiple for each set of numbers.
   a. $\{12, 20\}$ b. $\{8, 32, 96\}$ c. $\{9, 15, 45\}$

4. Give twelve primes less than 50.

Write each problem in column form and find the sum or difference.

5. $2\frac{1}{2} + 4\frac{1}{8}$
6. $3\frac{1}{3} + 5\frac{1}{4}$
7. $6\frac{4}{7} + 3\frac{2}{3}$
8. $9\frac{3}{4} - 5\frac{5}{9}$
9. $8\frac{1}{6} - 6\frac{2}{7}$
10. $4\frac{1}{7} - 2\frac{2}{5}$

Write each problem in column form. Find the sum, difference, or product.

11. 17.38 + 2.99
12. .73 + .4
13. 54.8 + .03
14. 16.51 − 3.24
15. 7.95 − .86
16. 29.08 − 13.79
17. .3 × 176
18. 5 × .24
19. 95 × .68
20. 36 × .25

Find each answer.

21. 17% of 59
22. 8% of 77
23. 32% of 85
24. 64% of 203
25. 5% of 198
26. 86% of 50
27. 29% of 15
28. 44% of 398

29. Kay made a punch of 4 qt. of ginger ale, 2 qt. of grape juice, 3 qt. of lemonade, and 1 qt. of orange juice. What per cent of the punch was ginger ale?

Part 21 Logic

These are atomic sentences.

John is here. $x + y = 6$

Mavis has come home. $n - 2 = 25$

We use words like "and", "or", and "if . . . , then . . ." to combine atomic sentences to form molecular sentences. These combining words are called connectives. The word "not" is a connective that changes one atomic sentence to a molecular sentence.

If the sun shines, then it will be a hot day. $y = 4$ or $y > 4$.

It is not raining. $m < 3$ and $n > 3$.

The connectives are not part of the atomic sentences.
Atomic sentences do not contain any connectives.

1. What connectives are used in the following sentences?
   a. Mac is home and Susan is at school.
   b. If the Tigers win the game, then Bill will be happy.
   c. Adam is not at school.
   d. Sue is babysitting or Sarah is babysitting.
   e. If $x = 2$, then $y = 3$.
   f. $3 < 5$ and $n > 0$.
   g. $A \subset B$ and $A = \{\ \}$.
   h. $x + y = 2$ or $x + y = 6$.
2. The connective "if . . . , then . . ." is used to form a **conditional sentence.** Which sentences in Exercise 1 are conditional sentences?
3. The connective "and" is used to form a **conjunction.** Which sentences in Exercise 1 are conjunctions?
4. The connective "or" is used to form a **disjunction.** Which sentences in Exercise 1 are disjunctions?
5. The connective "not" is used to form a **denial.** Which sentences in Exercise 1 are denials?

The IF Rule

Often from given statements we can infer other statements.
The given statements are the **premises.** The statement we infer is the **conclusion.**

Premise: If Jerry is late, then his mother is angry.
Premise: Jerry is late.
Conclusion: His mother is angry.

What conclusion can we infer from each pair of premises?

1. (1) If Jim is early, then Jane is early.
(2) Jim is early.

2. (1) If it is snowing, then it is cold.
(2) It is snowing.

3. (1) If $2n = 24$, then $n = 12$.
(2) $2n = 24$

The conclusions drawn from premises in Exercise 1 use the **IF** rule.
To apply the **IF** rule, one premise must be a conditional sentence and another premise must be the **antecedent** or first part of the conditional sentence.

Use the **IF** rule to draw a conclusion from each set of premises.
(In problems 5 and 7 you need not use all the premises.)

4. (1) If Sally is not late, then Jack is not late.
(2) Sally is not late.

5. (1) If it is cloudy, then it is raining.
(2) If it is sunny, then it is not raining.
(3) It is cloudy.

6. (1) If $2n + 1 \neq 3$, then $n \neq 1$.
(2) $2n + 1 \neq 3$

7. (1) If $3n + 2 = 11$, then $n = 3$.
(2) If $3n + 2 = 17$, then $n = 5$.
(3) $3n + 2 = 11$

We may use the **IF** rule more than once to draw a conclusion.

Premise (1) If $3n - 7 = 11$, then $3n = 18$.

Premise (2) If $3n = 18$, then $n = 6$.

Premise (3) $3n - 7 = 11$

(4) $3n = 18$ from (1) and (3)

(5) $n = 6$ from (2) and (4)

Our conclusion is: $n = 6$. We write the conclusion as line (5). Note that on the right we show to what lines we apply the **IF** rule.

Use the **IF** rule twice to draw a conclusion from each set of premises.

1. (1) If Linda is away, then Helen is playing records.
   (2) If Helen is playing records, then Nancy is listening.
   (3) Linda is away.

2. (1) If Carol is not here, then Jay is not here.
   (2) Carol is not here.
   (3) If Jay is not here, then Sue is at home.

3. (1) If $3y + 4 = 1$, then $3y = -3$.
   (2) If $3y = -3$, then $y = -1$.
   (3) $3y + 4 = 1$

4. (1) If point A is on the circle, then point B is inside the circle.
   (2) If point B is inside the circle, then point C is inside the circle.
   (3) Point A is on the circle.

5. (1) If the square is inside the rectangle, then the square is inside the pentagon.
   (2) If the square is inside the pentagon, then the square is inside the circle.
   (3) The square is inside the rectangle.

Denials

"$n \neq 5$" is the denial of "$n = 5$".
"$n = 5$" is the denial of "$n \neq 5$".
"Jill is here" is the denial of "Jill is not here".

Write the denial of each sentence.

1. Mavis is here.
2. Garth is not here.
3. Sam is lost.
4. The bird is not in the cage.
5. $n = 3$
6. $x + y = 7$
7. $n + 2 \neq 4$
8. Harold is not at home.
9. My car is blue.
10. It is cold.
11. Jill is not sad.
12. $m < 7$
13. $n + 5 = 6$
14. $y - 3 \neq 8$

Use the **IF** rule more than once to infer a conclusion.

15. (1) If $m > n$, then $n > p$.
    (2) If $n > p$, then $p > 5$.
    (3) $m > n$

16. (1) If $m + 2 > 17$, then $m > 15$.
    (2) If $m > 15$, then $m = 41$.
    (3) $m + 2 > 17$

17. (1) If $3m > 21$, then $m > 7$.
    (2) If $m > 7$, then $m \neq 4$.
    (3) $3m > 21$

18. (1) If n is odd, then $n = 3$.
    (2) If $n = 3$, m is odd.
    (3) n is odd.

19. (1) If $x = y$, then $a \neq b$.
    (2) If $a \neq b$, then $c \neq y$.
    (3) $x = y$

20. (1) If $A \cup B = B$, then $A \subset B$.
    (2) If $A \subset B$, then $A \neq B$.
    (3) $A \cup B = B$

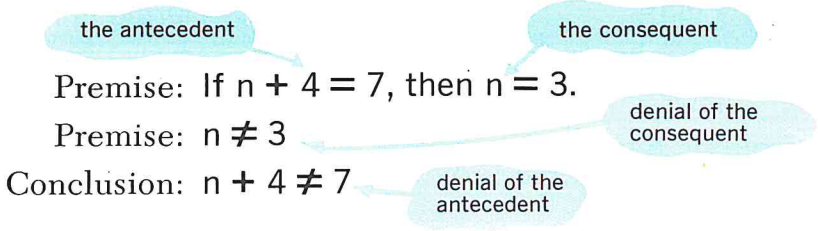
21. (1) If $m \neq n$, then $c = d$.
    (2) If $c = d$, then $m \neq c$.
    (3) $m \neq n$

22. (1) If $2m < 6$, then $m < 3$.
    (2) If $m < 3$, then $m \neq 4$.
    (3) $2m < 6$

The IFN Rule

Another rule of inference is the "If-then-not" rule, which we shall call the **IFN** rule.

In this case, one premise is a conditional sentence and the other premise is the denial of the **consequent**, the last part of the conditional sentence. We infer the denial of the antecedent, the first part of the conditional sentence.


the antecedent
the consequent
Premise: If n + 4 = 7, then n = 3.
Premise: n ≠ 3
denial of the consequent
Conclusion: n + 4 ≠ 7
denial of the antecedent

Use the **IFN** rule to draw a conclusion from each set of premises. (In problems 5 and 6 you need not use all the premises.)

1. (1) If it is raining, then it is cloudy.
   (2) It is not cloudy.
2. (1) If John is absent from school, then he will miss the test.
   (2) John will not miss the test.
3. (1) If this figure is a quadrilateral, then it is a square.
   (2) It is not a square.
4. (1) If this triangle is equilateral, then all three sides are congruent.
   (2) All three sides are not congruent.
5. (1) If Figure A has four sides, then it is a quadrilateral.
   (2) If Figure A has five sides, then it is a pentagon.
   (3) It is not a pentagon.
6. (1) If $n + 9 = 5$, then $n = -4$.
   (2) If $n + 8 = 2$, then $n = -6$.
   (3) $n \neq -4$

We may also use the **IFN** rule more than once to draw a conclusion.

Premise (1) If $n = 30$, then $n \div 3 = 10$.
Premise (2) If n is even, then $n = 30$.
Premise (3) $n \div 3 \neq 10$
(4) $n \neq 30$ from (1) and (3)
(5) n is not even from (4) and (2)

Use the **IFN** rule twice to draw a conclusion from each set of premises. Follow the example.

1. (1) If John is here, then Sally is here.
(2) If Sally is here, then Mom is glad.
(3) Mom is not glad.

2. (1) If Mary is at school, then Tim is gone.
(2) If Tim is gone, then Sam is mad.
(3) Sam is not mad.

3. (1) If $2n + 4 = 16$, then $2n = 12$.
(2) If $2n = 12$, then $n = 6$.
(3) $n \neq 6$

4. (1) If $x + y = 11$, then $x - y = 3$.
(2) If $x - y = 3$, then $x = 7$.
(3) $x \neq 7$

Use the **IFN** rule three times to draw a conclusion from each set of premises.

5. (1) If it is below 0°F, then it is very cold.
(2) If it is very cold, then the lake is frozen.
(3) The lake is not frozen.
(4) If my toes are frostbitten, then it is below 0°F.

6. (1) If $a + b = a$, then $b = 0$.
(2) If $b = 0$, then $a > -6$.
(3) a is not greater than -10.
(4) If $a > -6$, then $a > -10$.

Giving Reasons

To show exactly what is happening in longer proofs, we give a reason for each step.

Statement	Reason
(1) If $x = y$, then $x - y = 0$.	P
(2) If $x \neq y$, then $x^2 \neq y^2$.	P
(3) $x - y \neq 0$	P
(4) $x \neq y$	IFN 1, 3
(5) $x^2 \neq y^2$	IF 2, 4

"P" means that this statement is a premise.

"IFN 1, 3" means that we inferred line (4) from lines (1) and (3) by using the IFN rule.

"IF 2, 4" means that we inferred line (5) from lines (2) and (4) by using the IF rule.

In each of the proofs below the premises are marked. Copy the proof and give the reason for each line that is not a premise. Follow the example.

1. (1) p is even. P
   (2) If p is even, then $p \neq 7$. P
   (3) $p \neq 7$

2. (1) If $u = v - 1$, then $u \neq v$. P
   (2) If $u \neq v$, then $u - v \neq 0$. P
   (3) $u - v = 0$ P
   (4) $u = v$
   (5) $u \neq v - 1$

3. (1) If $f = g$, then $f = h$. P
   (2) If $f \neq g$, then $f - g = 3$. P
   (3) $f \neq h$ P
   (4) $f \neq g$
   (5) $f - g = 3$

4. (1) If $n < p$, then $n < p + 1$. P
   (2) If $n < p + 1$, then $n < p + q$. P
   (3) $n < p$ P
   (4) $n < p + 1$
   (5) $n < p + q$

We use the **Rule of Adjunction** to combine two separate premises as a conjunction.

From:

(1) John is here.

(2) Marie is eating.

We may infer:

Conclusion (1) John is here and Marie is eating.

or, Conclusion (2) Marie is eating and John is here.

Use the **Rule of Adjunction** to draw a conclusion from each pair of premises.

1. (1) The clouds are dark.
   (2) It will soon rain.

2. (1) The temperature is 20°F.
   (2) The lake is frozen.

3. (1) $n > m + 1$
   (2) $m < n$

4. (1) $x + y = 7$
   (2) $x - y = 3$

We use the **Rule of Simplification** to infer one of the statements of a conjunction as a conclusion.

From:

$x > z + 5$ and $y > x - z$

We may infer:

Conclusion (1) $x > z + 5$

or, Conclusion (2) $y > x - z$

Use the **Rule of Simplification** to draw a conclusion from each conjunction.

5. Ted is gone and Marie is not here.
6. $x + y = 15$ and $x - y = 13$.
7. $x = 7$ and $y > 4$.
8. Betty is happy and John is sad.

The connective "or" is used to form a **disjunction.**

To draw a conclusion from a disjunction we need a second premise that denies one of the **disjuncts,** or one part of the disjunction. We infer the other disjunct.

From:

(1) "Joe is at home or Joe is at school"

(2) "Joe is not at home"

We may infer by using the **OR** rule:

"Joe is at school".

Use the **OR** rule to draw a conclusion from each pair of premises.

1. (1) He asked for ice cream or he asked for pie.
   (2) He did not ask for pie.

2. (1) p is odd or p is even.
   (2) p is not odd.

3. (1) The temperature is high or the humidity is high.
   (2) The humidity is not high.

4. (1) $x + y = 6$ or $x + y = 10$.
   (2) $x + y \neq 6$

5. (1) $m > 5$ or $m = n$.
   (2) m is not greater than 5.

6. (1) Jim is at the dentist's or he is at home.
   (2) Jim is not at home.

7. (1) I am walking south or I am walking east.
   (2) I am not walking south.

8. (1) $x = 0$ or $x = 1$.
   (2) $x \neq 1$

9. (1) It is raining or it is snowing.
   (2) It is not raining.

Give a reason for each statement that is not a premise. Use:

A for Rule of Adjunction
S for Rule of Simplification
OR for the Or Rule
IF for If-then Rule
IFN for If-then-not Rule

1. (1) The wind is blowing and the lake is not frozen. **P**
(2) If the sky was clear last night, then the lake is frozen. **P**
(3) The lake is not frozen.
(4) The sky was not clear last night.

2. (1) If the weather is cold and the sky is cloudy, then it will snow. **P**
(2) The weather is cold. **P**
(3) The sky is cloudy. **P**
(4) The weather is cold and the sky is cloudy.
(5) It will snow.

3. (1) If n is odd, then $m + n$ is even. **P**
(2) If m is odd, then n is odd. **P**
(3) m is odd and $m - n$ is even. **P**
(4) m is odd.
(5) n is odd.
(6) $m + n$ is even.

4. (1) If $m = n$, then $8m = 64$. **P**
(2) $m = 9$ or $m = n$. **P**
(3) $8m \neq 64$ **P**
(4) $m \neq n$
(5) $m = 9$

5. (1) If $u = 4$ and $v = 6$, then $u + v = 10$. **P**
(2) $u = 4$ or $y = 3$. **P**
(3) $v = 6$ **P**
(4) $y \neq 3$ **P**
(5) $u = 4$
(6) $u = 4$ and $v = 6$.
(7) $u + v = 10$

6. (1) $m - n$ is even or $m + n$ is odd. **P**
(2) If $m + n$ is odd, then n is not odd. **P**
(3) $m - n$ is not even. **P**
(4) $m + n$ is odd.
(5) n is not odd.

Checkout

What logical connectives are used in the following sentences?

1. Mary is tall and Pam is short.
2. The earth is not round.
3. He is Bob or he is John.
4. If $p = 3$, then $q = 4$.
5. $v^2 > 0$ and $v < 0$.
6. $A = B$ or $A \subset B$.

Give the denial of each sentence.

7. Mavis is not here.
8. Jan is older than Jerry.
9. $p + 2 > 5$
10. $A \cap B \neq \{\ \}$

Use the IF rule to draw a conclusion from each pair of premises.

11. (1) You are Pat.
    (2) If you are Pat, then I am Joanne.
12. (1) $c < 0$
    (2) If $c < 0$, then $c^3 < 0$.

Use the IFN rule to draw a conclusion from each pair of premises.

13. (1) $b \neq 3$
    (2) If $c = 5$, then $b = 3$.
14. (1) If $A \neq B$, then $A \cup B \neq A$
    (2) $A \cup B = A$

Use the OR rule to draw a conclusion from each pair of premises.

15. (1) It is Feb. 29 or it is March 1.
    (2) It is not March 1.
16. (1) $p = -3$
    (2) $p \neq -3$ or $q = 2$.

Copy the premises, infer the desired conclusion by adding any necessary lines, and give the reason for each line that is not a premise.

17. (1) r is not even and r + 1 is even.
    (2) If r is odd, then 3r is odd.
    (3) r is even or r is odd.
    Therefore, 3r is odd.
18. (1) If $a = b$, then $c = d$.
    (2) If $c = d$, then $d = 7$.
    (3) $d \neq 7$
    Therefore, $a \neq b$.

Checkout

Find each quotient. Check by multiplication.

1. $9\overline{)3060}$ 2. $7\overline{)24603}$ 3. $49\overline{)8653}$ 4. $327\overline{)954033}$

Copy and complete each problem.

5. $4\frac{2}{3} + 6\frac{1}{2} =$ ■
6. $8\frac{1}{3} - 3\frac{5}{9} =$ ■
7. $4\frac{2}{3} \times 6 =$ ■
8. $5\frac{1}{4} \div 1\frac{1}{6} =$ ■
9. $16 \div \frac{2}{3} =$ ■
10. $7\frac{1}{2} - 4\frac{7}{8} =$ ■
11. $3\frac{3}{4} + 6\frac{3}{8} =$ ■
12. $.7 \times .8 =$ ■
13. $.21 \div .3 =$ ■
14. $4\frac{1}{2} \times 2\frac{2}{3} =$ ■

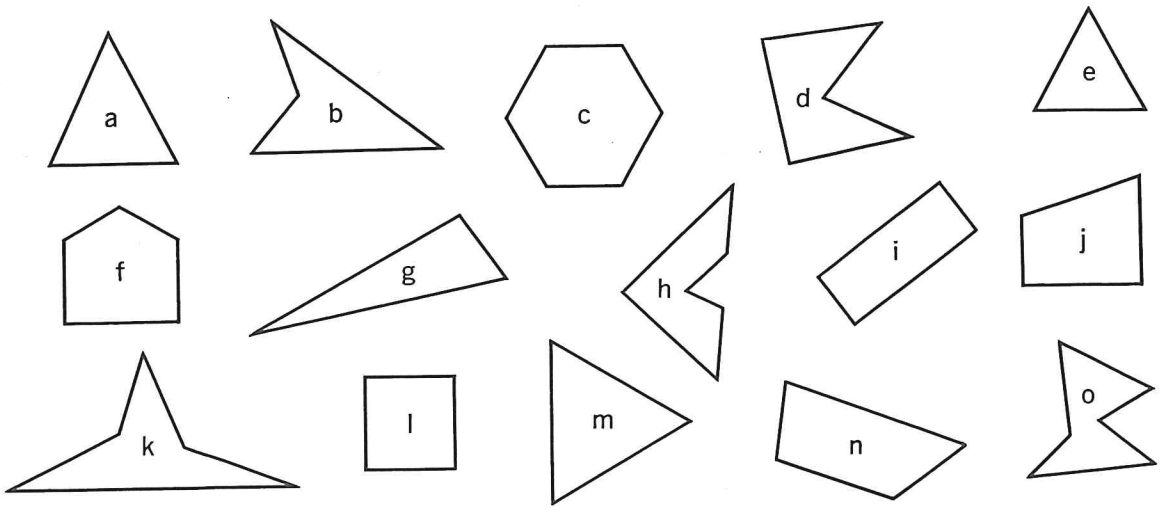
Complete each problem or answer the question.

15. **a.** A cube has ■ faces.
    b. Each face is in the shape of a ■.
    c. If the length of each side is 1 inch, the area of each face is ■ square inches.
    d. The surface area of the cube is ■ square inches.
    e. The volume of the cube is ■ cubic inches.

16. Miss Murray's class was in charge of making box lunches for the 89 children going on the sixth-grade picnic.
    a. If each child eats 2 sandwiches, how many sandwiches should they make?
    b. They have 23 dozen cookies. How many cookies will each child get? How many will be left over?
    c. They buy 6 quarts of lemonade, 5 quarts of grapejuice, $1\frac{1}{2}$ gallons of limeade, and 7 gallons of gingerale to make a punch. They want to serve 2 cups to each child. How much will be left over?

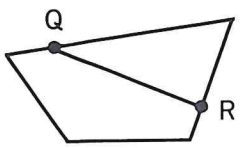
Part 22 Symmetry

2-Dimensional Figures

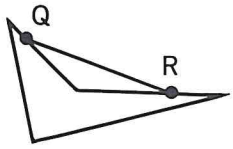


Look at the above figures.

1. Which are triangles?
2. Which are quadrilaterals?
3. Which are pentagons?
4. A **hexagon** is a 6-sided figure. Which of the figures are hexagons?
5. For each of the above figures, can you find 2 points Q and R on the figure so that part of $\overline{QR}$ is outside the figure?

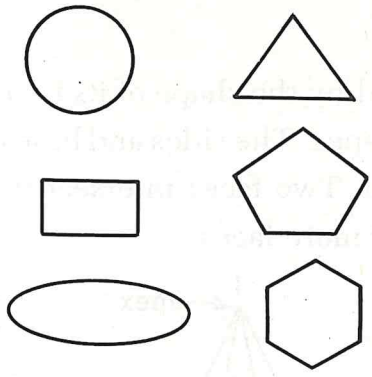


No part of $\overline{QR}$ is outside this quadrilateral.

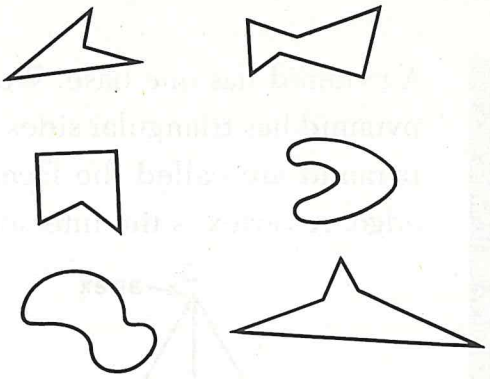


Part of $\overline{QR}$ is outside this quadrilateral.

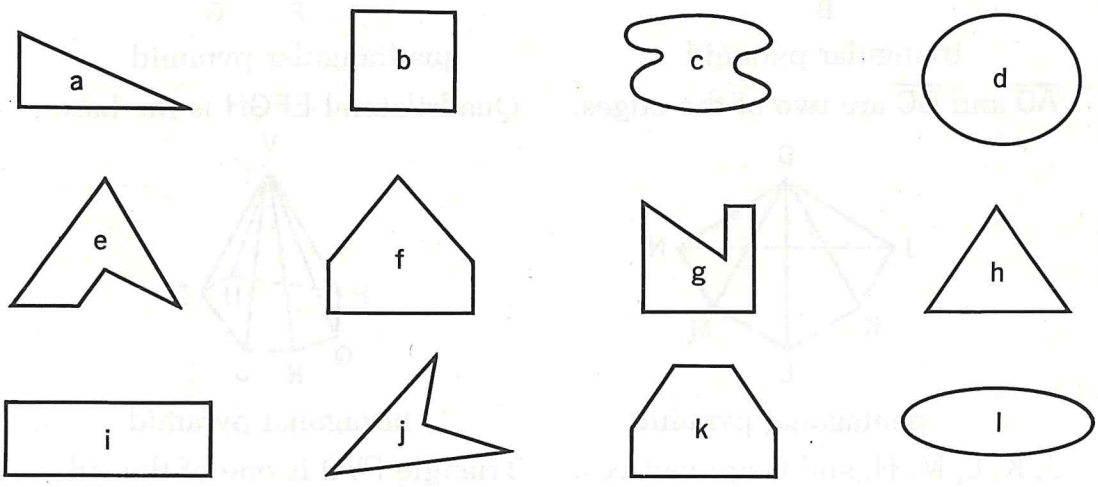
Convex Figures



Concave Figures



1. Tell whether each figure is convex or concave.

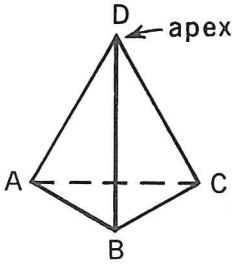


2. Is each statement true or false?

a. All circles are convex.
b. Some squares are concave.
c. Some quadrilaterals are concave.
d. All pentagons are convex.
e. Some hexagons are concave.
f. All triangles are convex.
g. No rectangles are concave.

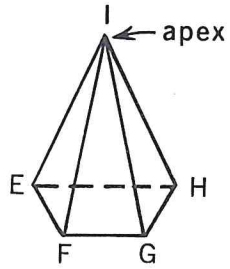
Pyramids

A **pyramid** has one base. We name a pyramid by the shape of its base. A pyramid has triangular sides that meet at the apex. The sides and base of a pyramid are called the faces of the pyramid. Two faces intersect in an edge. A vertex is the intersection of three or more faces.



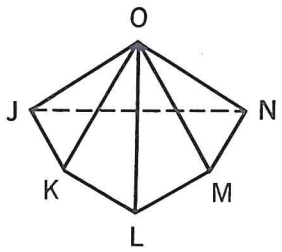
triangular pyramid

$\overline{AD}$ and $\overline{BC}$ are two of the edges.



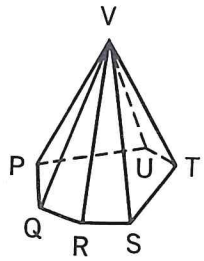
quadrangular pyramid

Quadrilateral EFGH is the base.



pentagonal pyramid

J, K, L, M, N, and O are vertices.



hexagonal pyramid

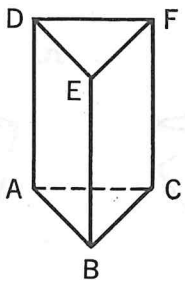
Triangle PVQ is one of the sides.

1. Copy and complete the chart. Use the above pictures for help.

Type of pyramid	Triangular	Quadrangular	Pentagonal	Hexagonal
Shape of base		quadrilateral		hexagon
Number of sides	3			
Number of vertices			6	
Number of edges				12
Number of faces	4			

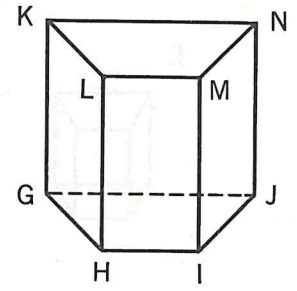
Prisms

A **prism** has two bases that are the same shape and size. We name a prism by the shape of the bases. The sides of a prism are quadrilaterals. The sides and bases of a prism are called the faces of the prism. Two faces intersect in an edge. A vertex of a prism is the intersection of three faces.



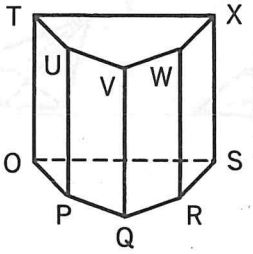
triangular prism

$\overline{AD}$ and $\overline{BC}$ are two of the edges.



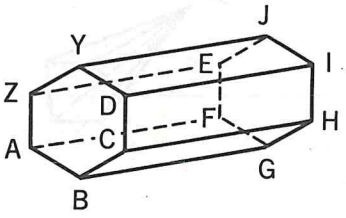
quadrangular prism

Rectangle KGHL is one side.



pentagonal prism

O, P, V, and U are four of the vertices.



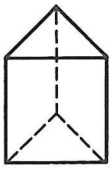
hexagonal prism

Hexagons ABCDYZ and EFGHIJ are the bases.

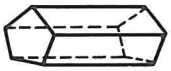
1. Copy and complete the chart. Use the above pictures for help.

Type of prism	Triangular	Quadrangular	Pentagonal	Hexagonal
Shape of bases	triangle			
Number of sides			5	
Number of vertices				12
Number of edges		12		
Number of faces				8

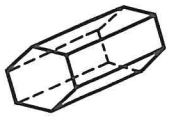
a.



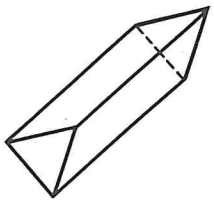
b.



c.

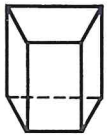


d.

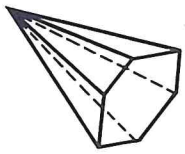


e.

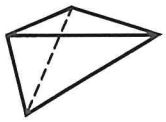
f.



g.

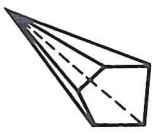


h.



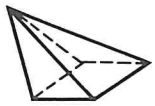
i.

j.

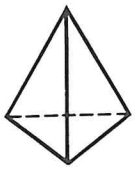


k.

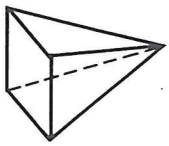
l.



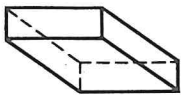
m.



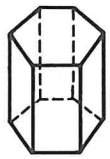
n.



o.



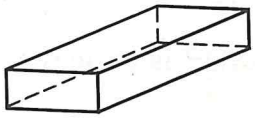
p.

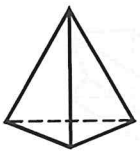


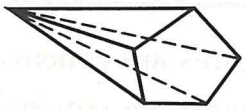
Which figures above are

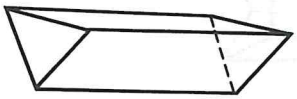
1. prisms?
2. pyramids?
3. triangular prisms?
4. triangular pyramids?
5. quadrangular prisms?
6. quadrangular pyramids?
7. pentagonal prisms?
8. pentagonal pyramids?
9. hexagonal prisms?
10. hexagonal pyramids?

For each picture on the left, find a picture on the right that shows the same 3-dimensional figure in a different position. Write your answer in the form 7-G.

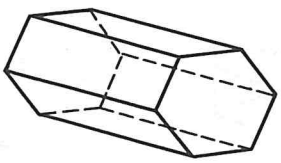
1. 

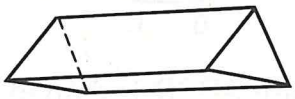
2. 

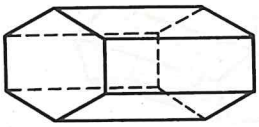
3. 

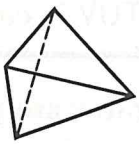
4. 

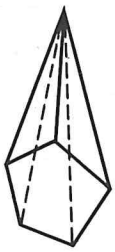
5.

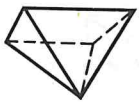
6. 

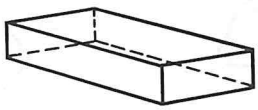
A. 

B. 

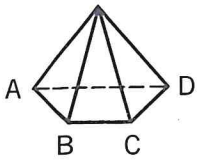
C. 

D. 

E. 

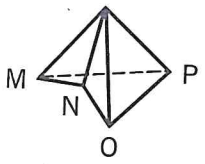
F. 

convex pyramid



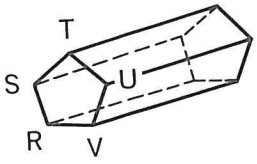
Base ABCD is convex.

concave pyramid



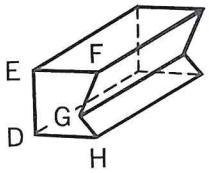
Base MNOP is concave.

convex prism



Base RSTUV is convex.

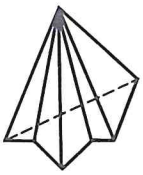
concave prism

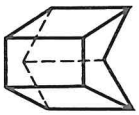


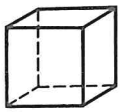
Base DEFGH is concave.

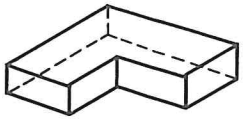
1. Which figures are pyramids?
2. Which figures are prisms?
3. Which figures are concave?
4. Which figures are convex?

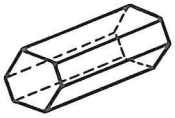
a.

b. 

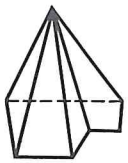
c. 

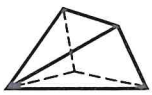
d. 

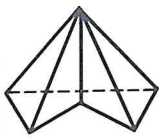
e. 

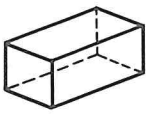
f. 

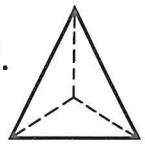
g.

h. 

i. 

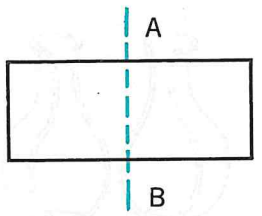
j. 

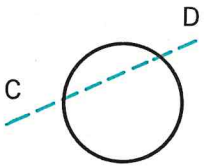
k. 

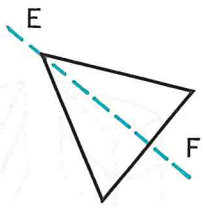
l. 

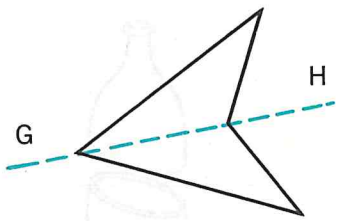
Lines of Symmetry

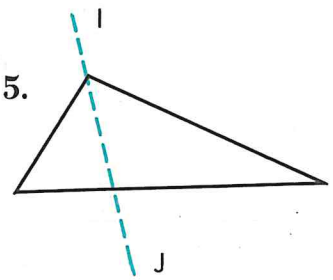
Folding a figure along a line of symmetry makes one part fit the other. A line of symmetry divides a figure into two parts of the same shape and size. Look at the figures below. Which lines shown in color are lines of symmetry?

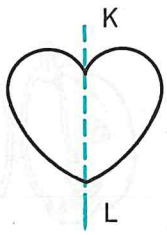
1. 

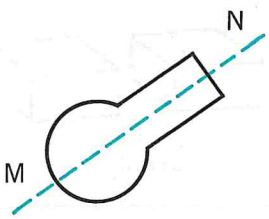
2. 

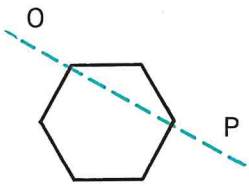
3. 

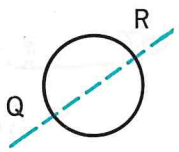
4. 

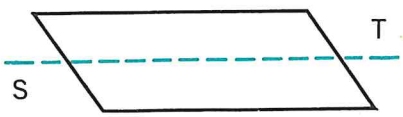
5. 

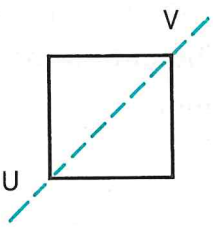
6. 

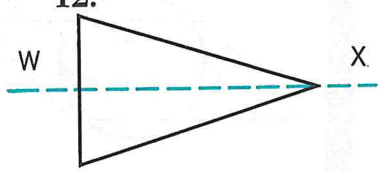
7. 

8. 

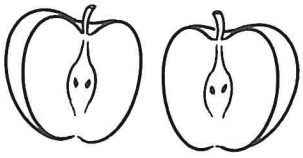
9. 

10. 

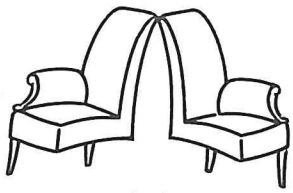
11. 

12. 

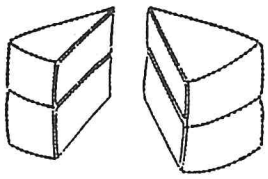
Which pictures show an object cut into two pieces the same size and shape?

1. 

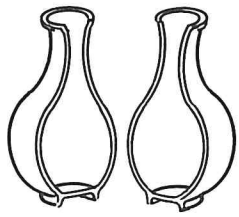
apple

2. 

chair

3. 

cake

4. 

vase

5.

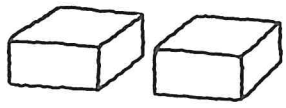
jack-o-lantern

6.

bottle

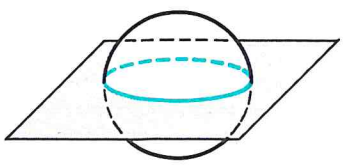
7.

crayon

8. 

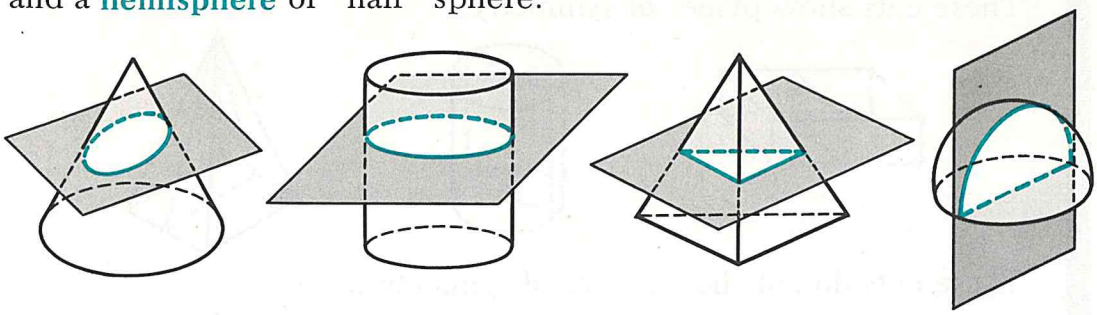
brick

A plane may intersect, or cut an object.

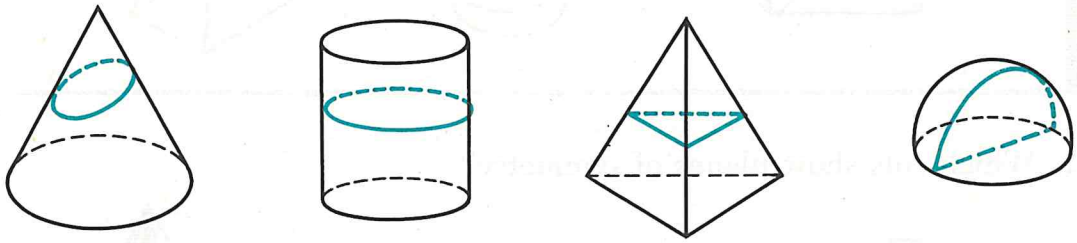


Here is a picture of a plane cutting a **sphere.** The **cut** on the surface of the sphere is shown in color.

Here are pictures of planes cutting a **cone**, a **cylinder**, a triangular pyramid and a **hemisphere** or "half" sphere:



Our pictures are sometimes clearer if we show only the cut, not the picture of the plane.



1. Which picture shows a cut through a
   - **a.** hemisphere?
   - **b.** triangular prism?
   - **c.** cylinder?
   - **d.** cone?
   - **e.** rectangular prism?
   - **f.** quadrangular pyramid?

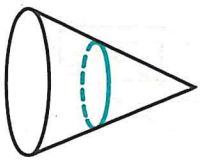


Figure 1

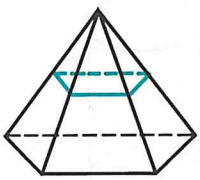


Figure 2

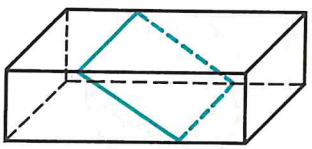


Figure 3

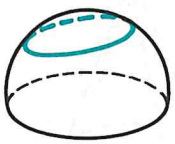


Figure 4

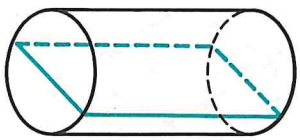


Figure 5

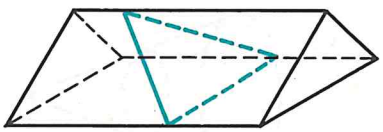
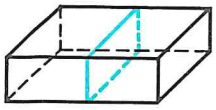
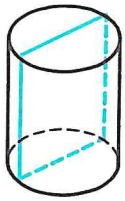
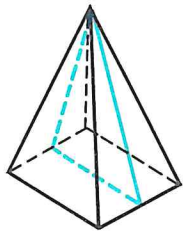
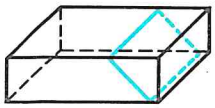
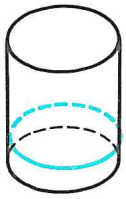
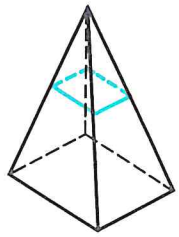


Figure 6

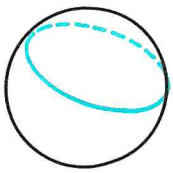
A **plane of symmetry** cuts a figure into two parts the same shape and size.
These cuts show planes of symmetry:

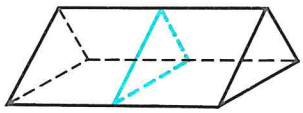




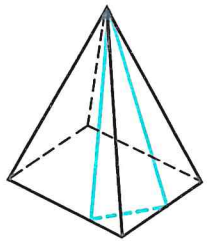
These cuts do not show planes of symmetry:

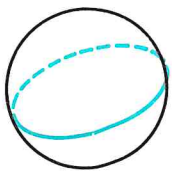




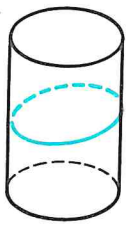
1. Which cuts show planes of symmetry?

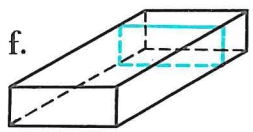
a. 

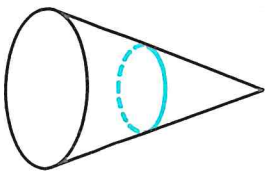
b. 

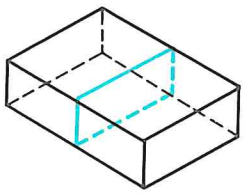
c. 

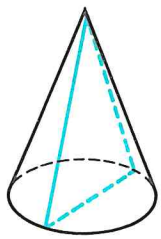
d. 

e. 

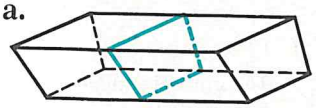
f. 

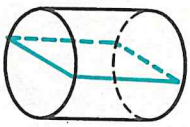
g. 

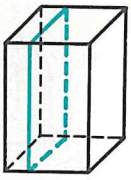
h. 

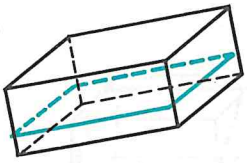
i. 

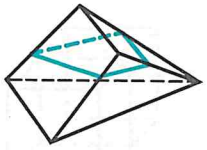
1. Which cuts show planes of symmetry?

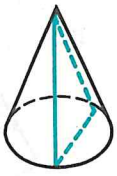
a. 

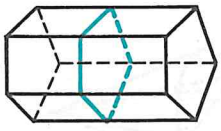
b. 

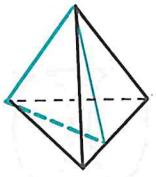
c. 

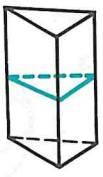
d. 

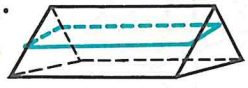
e. 

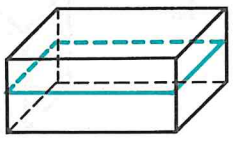
f. 

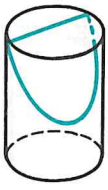
g. 

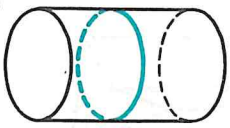
h. 

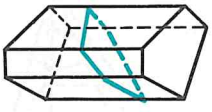
i. 

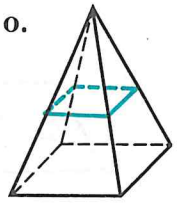
j. 

k. 

l. 

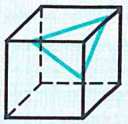
m. 

n. 

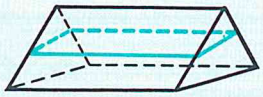
o. 

2. In Exercise 1, name the figure pictured in each problem.

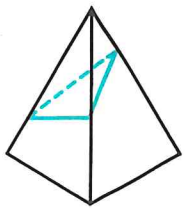
1. What is the plane figure determined by each cut?

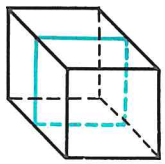


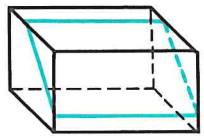
The cut determines a triangle.

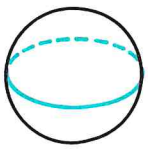


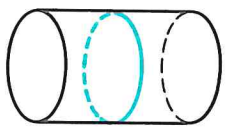
The cut determines a rectangle.

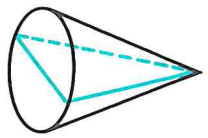
a. 

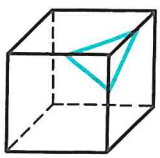
b. 

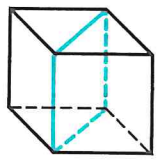
c. 

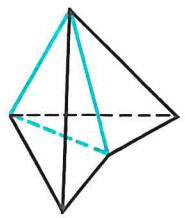
d. 

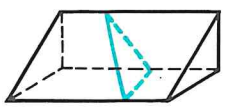
e. 

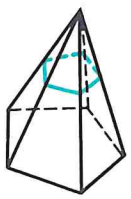
f. 

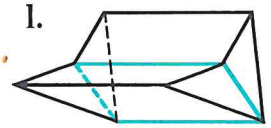
g. 

h. 

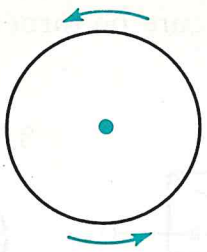
i. 

j. 

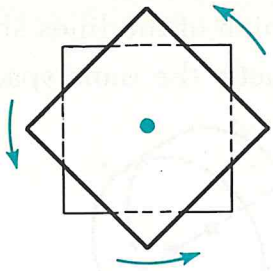
k. 

l. 

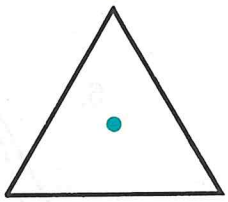
2. In Exercise 1 which cuts are made by planes of symmetry?

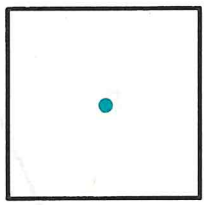


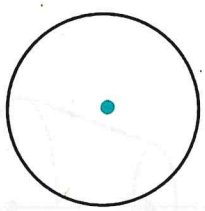
This circular piece of paper will occupy the same space as it turns about its center.



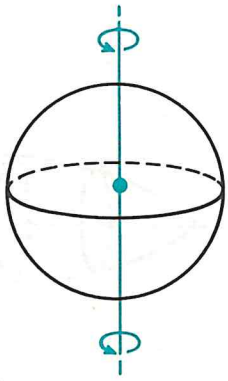
This square piece of paper will not occupy the same space as it turns about its center.



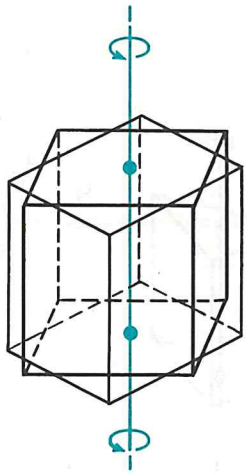




1. Which of the above figures will occupy the same space as it is turned about its center?

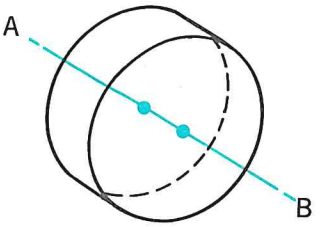


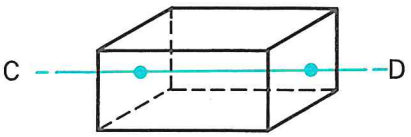
A sphere will occupy exactly the same space as it is turned about a line passing through its center.

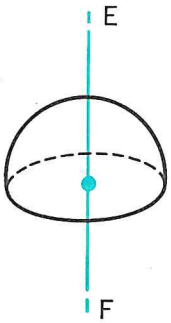


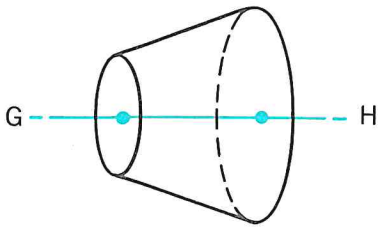
A cube will not occupy exactly the same space as it turns about a line passing through its center.

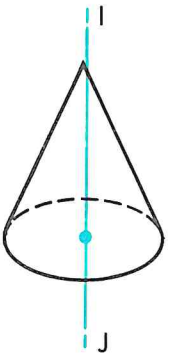
Around which of the lines shown in color may the figure be turned and still occupy exactly the same space?

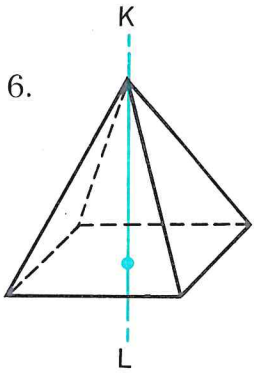
1. 

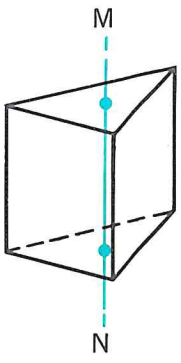
2. 

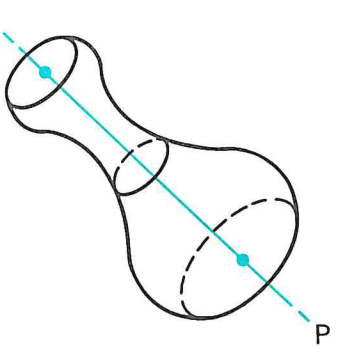
3. 

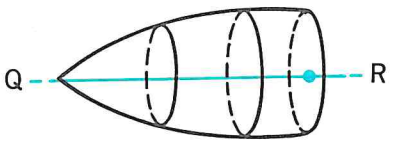
4. 

5. 

6. 

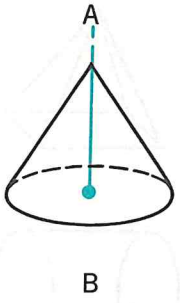
7. 

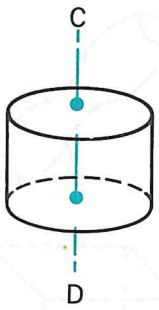
8. 

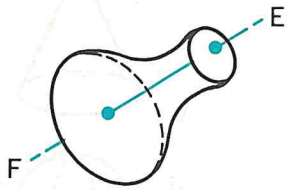
9. 

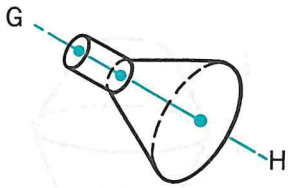
A line around which a figure may be turned and still occupy exactly the same space is an **axis of symmetry.**

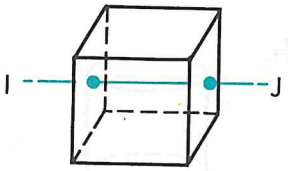
For each figure, is the line shown in color an axis of symmetry?

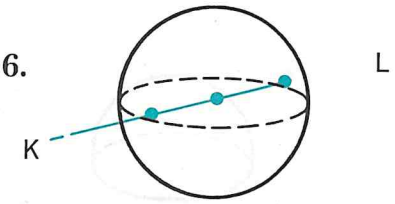
1. 

2. 

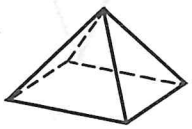
3. 

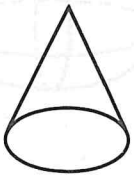
4. 

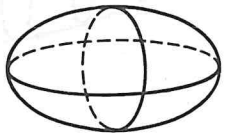
5. 

6. 

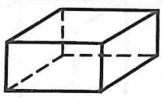
Which of the following figures do not have an axis of symmetry?

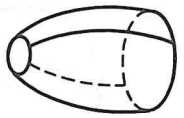
7. 

8. 

9. 

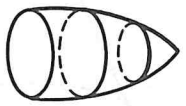
10.

11. 

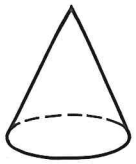
12. 

13.

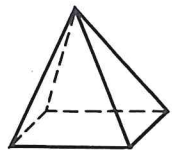
14.

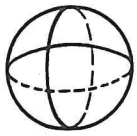
15. 

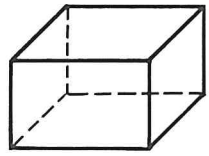
1. Which of the given figures has more than one axis of symmetry?

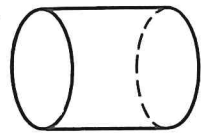
a. 

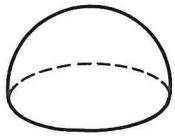
b.

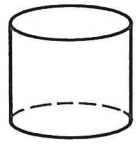
c. 

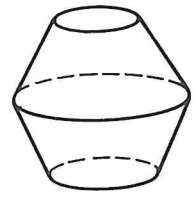
d. 

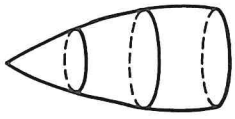
e. 

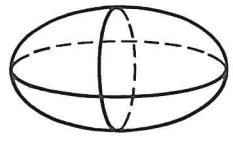
f. 

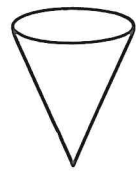
g. 

h. 

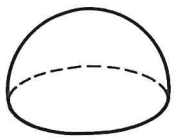
i. 

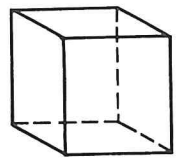
j. 

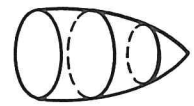
k. 

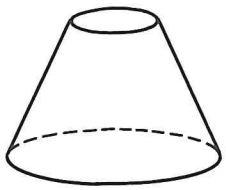
l. 

2. How many axes of symmetry does each figure have?

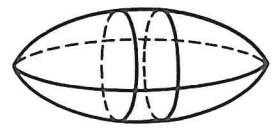
a. 

b. 

c. 

d. 

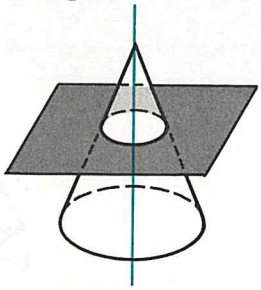
e.

f. 

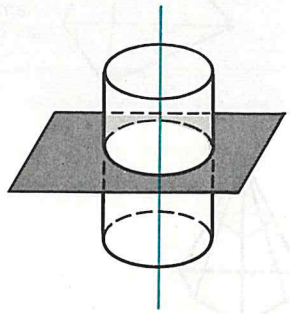
If a line is an axis of symmetry, then a plane perpendicular to the line always intersects the figure in a circle.

1. In each problem what figure is formed by the intersection of the plane and the figure?

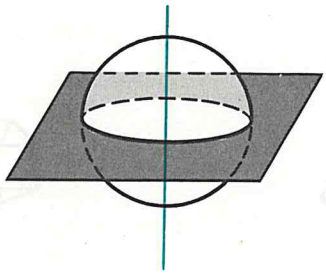
a.



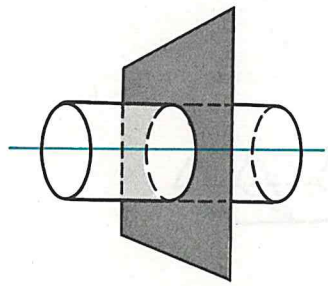
b.



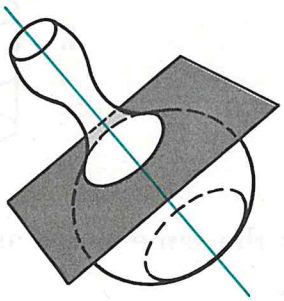
c.



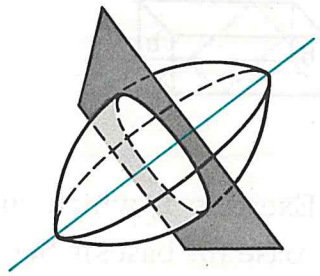
d.



e.



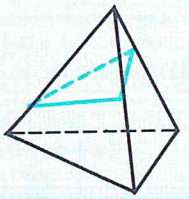
f.



2. The plane intersecting each figure is perpendicular to the line shown in color. Is the line shown an axis of symmetry?

3. Which of the planes shown above are planes of symmetry?

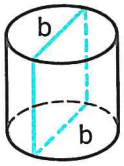
1. Which cuts determine figures the same shape as the base (or bases)? Sometimes the base is labeled b.



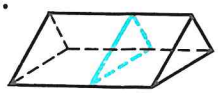
The cut does not determine a triangle the same shape as the base.

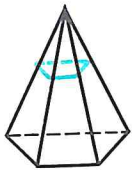
The cut determines a triangle the same shape as the base.

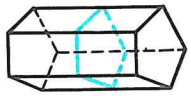
a.

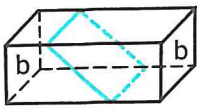
d. 

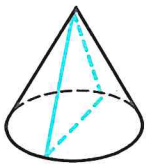
g.

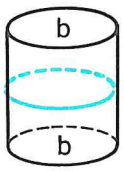
b. 

e. 

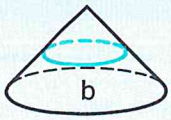
h. 

c. 

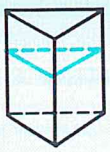
f. 

i. 

2. In Exercise 1, which cuts determine figures the same shape and size as the base (or bases)? See the examples below.



The cut determines a circle the same shape as the base but a different size.



The cut determines a triangle the same shape and size as the bases.

3. In Exercise 1, which cuts show planes of symmetry?

Checkout

Choose $<$ or $=$ or $>$.

1. 10% of \$8 ● 5% of \$16
2. 10% of \$15 ● 8% of \$20
3. 63% of 4 ● 4% of 63
4. 50% of \$.20 ● 5% of \$20
5. 39% of 50 ● 41% of 60
6. 73% of 6 ● 6% of 73

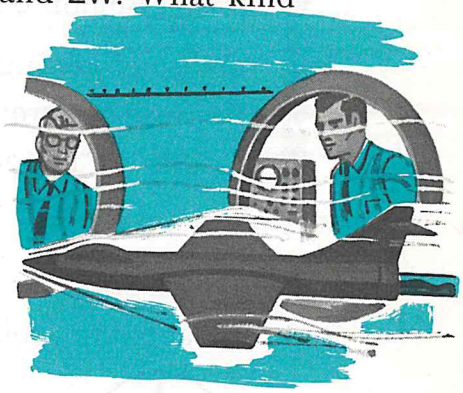
Find each answer.

7. $6\overline{)\$43.75}$
8. $20\overline{)98080}$
9. $37\overline{)\$520.39}$
10. $713\overline{)46998}$

11. \$68.42 + 15.96 + 25.45
12. \$49.68 − 32.55
13. \$82.14 + 9.64 + 31.52
14. \$12.98 × 6

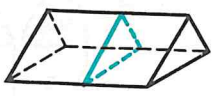
15. Draw a circle with center Q. Draw chords RS and TU passing through Q. Draw segments RT, TS, SU, and UR. What kind of quadrilateral is RTSU?

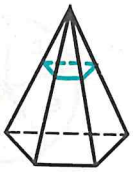
16. Draw a circle with center V. Draw a chord WX passing through V. At V, construct the perpendicular. Label its intersections with the circle "Y" and "Z". Draw segments WY, YX, XZ, and ZW. What kind of quadrilateral is WXYZ?

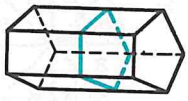


17. A side of a square measures .73.
    a. What is its perimeter?
    b. What is its area?

18. What is the plane figure determined by each cut?

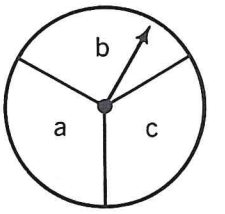
a. 

b. 

c. 

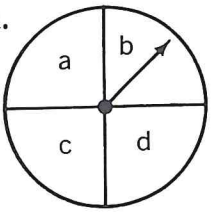
Part 23 Probability

The probability of getting a when spinning this spinner is $\frac{1}{3}$. One of three results can occur: a, b, or c. One of these three results is a. The probability of getting a is one out of three chances or $\frac{1}{3}$.

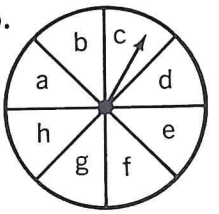


1. What is the probability of getting a when spinning each of the spinners?

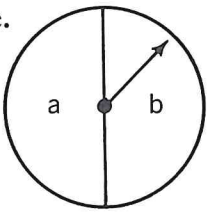
a.



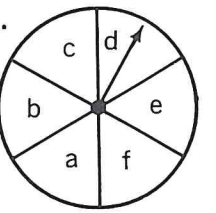
b.



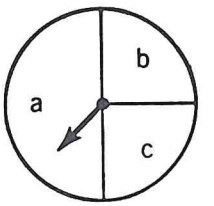
c.



d.



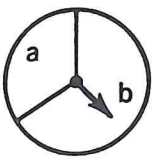
The spinner shown on the right is divided into three regions, but now they are not equal in size. In this case, region a is half of the spinner, so the probability of getting a is $\frac{1}{2}$. Region b is one fourth of the spinner, so the probability of getting b is $\frac{1}{4}$.



Because different results have different probabilities, we say that the results are not equally likely, or that the spinner is not **symmetric.**

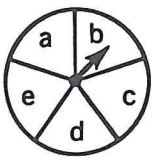
2. Which spinners shown below are symmetric?
3. What is the probability of getting b when spinning each of the spinners?

a.

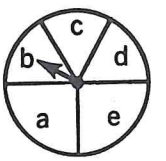


b.

c.

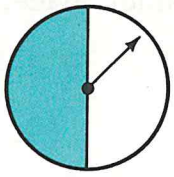


d.



Outcomes and Events

1. Look at the two-color spinner. We may write b for blue when the spinner lands on blue, and w for white.



   a. Suppose we spin this spinner twice. How many different **outcomes** can there be? Complete the table to find out.

	first spin	second spin
Outcome 1	b	b
Outcome 2	b	w
Outcome 3	w	■
Outcome 4	w	w

   b. There may be two results of the first spin, and two results of the second spin. So there are $2 \times 2 =$ ■ possible outcomes of the two spins.

   c. We may think of these four possible outcomes as a set, X. Then we may write:

$$X = \{bb, bw, \blacksquare\blacksquare, ww\}$$

   d. Note that the order in which each color comes up matters. The outcome bw (is, is not) the same as wb.

2. When we spin twice, various events can happen. We may think of these events as sets of outcomes or as subsets of X.

   a. The event A is the event of at least one w. Look at the table for ways A may occur: bw, ■, and ww.

   b. A is a subset of X. $A = \{bw, \blacksquare, \blacksquare\}$.

Pair the events and subsets of X.

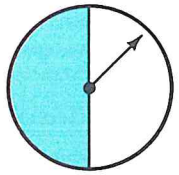
3. The event B of at least one b.
4. The event C of exactly one b.
5. The event D of two w's.
6. The event E of two b's.

a. $\{bw, wb\}$
b. $\{ww\}$
c. $\{bb, bw, wb\}$
d. $\{bb\}$

Probability of Outcomes and Events

We have seen that when we spin our two-color spinner twice, the set X of possible outcomes is

$$X = \{bb, bw, wb, ww\}$$



b = landing on blue
w = landing on white

We may ask what is the probability of each of the four possible outcomes. One outcome is as likely as another because the spinner is symmetric. There are four possible outcomes. So each outcome has 1 out of 4 chances. The probability of each outcome is $\frac{1}{4}$.

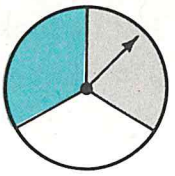
The **probability of an event** is just the sum of the probabilities of the possible outcomes that will make the event occur.

$$A = \{bw, wb, ww\}$$

$$\text{Probability of } A = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4}$$

1. What is the probability of each of these events when we spin the two-color spinner twice?
   - **a.** The event of exactly one spin landing on blue.
   - **b.** The event of both spins landing on blue.
   - **c.** The event of at least one spin landing on white.
   - **d.** The event of exactly one spin landing on white.
   - **e.** The event of both spins landing on the same color.
   - **f.** The event of the two spins landing on different colors.
   - **g.** The event of the first spin landing on white and the second spin landing on blue.

1. Look at this three-color spinner. We spin it twice.



g = landing on gray
b = landing on blue
w = landing on white

   a. The set X of possible outcomes has nine members, because there may be three results of the first spin, three results of the second, and $3 \times 3 = 9$. List the set of nine possible outcomes.

$$X = \{gg, gb, \blacksquare, \blacksquare, \blacksquare, \blacksquare, \blacksquare, \blacksquare, \blacksquare\}$$

   b. One outcome is as likely as another because the spinner is ■ (symmetric, not symmetric).
   c. Each possible outcome of the two spins has 1 out of ■ chances.
   d. The probability of each outcome of two spins is ■.

2. When we spin the three-color spinner twice, each of the following events is what subset of X? What is the probability of each event?

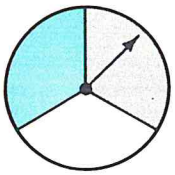
The event Y of landing on gray or white on the second spin.

$$Y = \{gg, gw, bg, bw, wg, ww\}$$

$$\text{Probability of } Y = \frac{1}{9} + \frac{1}{9} + \frac{1}{9} + \frac{1}{9} + \frac{1}{9} + \frac{1}{9} = \frac{6}{9} = \frac{2}{3}$$

   a. The event A of at least one spin landing on blue.
   b. The event B of at least one spin landing on white.
   c. The event C of exactly one spin landing on gray.
   d. The event D of the second spin landing on white.
   e. The event E of the first spin landing on gray.
   f. The event F of the first spin landing on blue or white.
   g. The event G of both spins landing on the same color.
   h. The event H of neither spin landing on gray.
   i. The event I of the two spins landing on different colors.

1. The intersection of two events is also an event.
Let us look again at the three-color spinner.



J = the event of the first spin landing on blue.

$$= \{bg, bb, bw\}$$

K = the event of the second spin landing on gray.

$$= \{gg, bg, wg\}$$

Then:

$$J \cap K = \{bg, bb, bw\} \cap \{gg, bg, wg\}$$

$$= \{\_\_\_\_\}$$

= the event of the first spin landing on blue **and** the second spin landing on gray.

2. Look at the events described at the bottom of the previous page. Find the intersections of the events listed below and describe in words the event that results from the intersection of each pair of events.

a. $A \cap B$	**c.** $A \cap D$	**e.** $C \cap E$	**g.** $H \cap I$
b. $C \cap D$	**d.** $A \cap G$	**f.** $D \cap H$	**h.** $C \cap I$

3. Describe in terms of intersection of sets the following events.

a. The event of the first spin landing on gray and the second spin landing on white.

b. The event of at least one spin landing on blue and neither spin landing on gray.

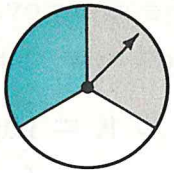
c. The event of at least one spin landing on white and the two spins landing on different colors.

d. The event of the first spin landing on gray and exactly one spin landing on gray.

e. The event of both spins landing on the same color and neither spin landing on gray.

Let us now consider the probability of the event that is the intersection of two events.

Again, look at the three-color spinner and the events A, B, C, D, E, F, G, H, I, J, and K already defined on pages 317 and 318.



The probability of $J \cap K$ = Prob. of $\{bg, bb, bw\} \cap \{gg, bg, wg\}$

= Prob. of $\{bg\}$

$= \frac{1}{9}$

In this case, Prob. of $J \cap K$ = Prob. of J times Prob. of K

$= \frac{1}{3} \times \frac{1}{3}$

$= \frac{1}{9}$

Now consider the probability of $A \cap B$.

Prob. of A = Prob. of $\{gb, bg, bb, bw, wb\}$

$= \frac{5}{9}$

Prob. of B = Prob. of $\{gw, bw, wg, wb, ww\}$

$= \frac{5}{9}$

But Prob. of $A \cap B$ = Prob. of $\{bw, wb\}$

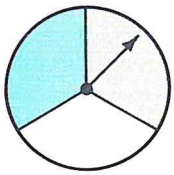
$= \frac{2}{9}$

Prob. of $A \cap B \neq$ Prob. of A times Prob. of B because $\frac{5}{9} \times \frac{5}{9} \neq \frac{2}{9}$

Choose = or $\neq$.

1. Prob. of $J \cap D$ ● Prob. J × Prob. D
2. Prob. of $D \cap E$ ● Prob. D × Prob. E
3. Prob. of $D \cap C$ ● Prob. D × Prob. C
4. Prob. of $K \cap E$ ● Prob. K × Prob. E
5. Prob. of $J \cap E$ ● Prob. J × Prob. E

1. The union of two events is also an event. Once again, look at the three-color spinner, and discuss the same events as before. We may start with J and K, as described two pages earlier.



$J \cup K = \{bg, bb, bw\} \cup \{gg, bg, wg\}$

$= \{\_\_, \_\_, \_\_, \_\_, \_\_\}$

= the event of the first spin landing on blue **or** the second spin landing on gray.

2. Find the union of each pair of events listed below and describe in words the event that results from the union of each pair of events.

a. $A \cup B$ c. $D \cup G$ e. $G \cup I$ g. $E \cup G$

b. $C \cup D$ d. $E \cup H$ f. $C \cup H$ h. $C \cup I$

3. Describe in terms of union of sets the following events.
   a. The event of the first spin landing on blue or the second spin landing on white.
   b. The event of at least one spin landing on blue or at least one spin landing on white.
   c. The event of both spins landing on the same color or the two spins landing on different colors.

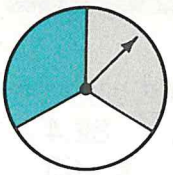
4. Choose $\cap$ or $\cup$. Remember that X is the set of all possible outcomes.

a. $E \bigcirc F = X$ c. $E \bigcirc F = \{\}$ e. $C \bigcirc H = \{\}$

b. $G \bigcirc I = X$ d. $G \bigcirc I = \{\}$ f. $B \bigcirc D = D$

5. An event that cannot happen is an **impossible** event. Such an event is always represented by the empty set. No possible outcome will make it occur. $E \cap J = \{\}$ because the first spin cannot land on both gray and blue. Describe in terms of intersection two other impossible events.

We next want to find the probability of the event that is the union of two events. Once again, consider spinning the three-color spinner twice, and the events A, B, C, D, E, F, G, H, I, J, and K. Sometimes a simple rule of addition works.



$$\begin{aligned}\text{Prob. of E } \cup \text{ J} &= \text{Prob. of } \{\text{gg, gb, gw}\} \cup \{\text{bg, bb, bw}\} \\ &= \text{Prob. of } \{\text{gg, gb, gw, bg, bb, bw}\} \\ &= \tfrac{1}{9} + \tfrac{1}{9} + \tfrac{1}{9} + \tfrac{1}{9} + \tfrac{1}{9} + \tfrac{1}{9} \\ &= \tfrac{6}{9} \\ &= \tfrac{2}{3}\end{aligned}$$

$$\begin{aligned}\text{In this case, Prob. of E } \cup \text{ J} &= \text{Prob. of E plus Prob. of J.} \\ &= \tfrac{1}{3} + \tfrac{1}{3} \\ &= \tfrac{2}{3}\end{aligned}$$

But now consider the probability of J ∪ K.

$$\begin{aligned}\text{Prob. of J } \cup \text{ K} &= \text{Prob. of } \{\text{bg, bb, bw}\} \cup \{\text{gg, bg, wg}\} \\ &= \text{Prob. of } \{\text{bg, bb, bw, gg, wg}\} \\ &= \tfrac{5}{9}\end{aligned}$$

But $\frac{1}{3} + \frac{1}{3} \neq \frac{5}{9}$, so Prob. of J ∪ K ≠ Prob. of J plus Prob. of K.

In this case, the simple rule of addition does not work.

Choose = or ≠.

1. Prob. of A ∪ B ● Prob. A + Prob. B
2. Prob. of E ∪ H ● Prob. E + Prob. H
3. Prob. of D ∪ E ● Prob. D + Prob. E
4. Prob. of E ∪ G ● Prob. E + Prob. G
5. Prob. of C ∪ G ● Prob. C + Prob. G

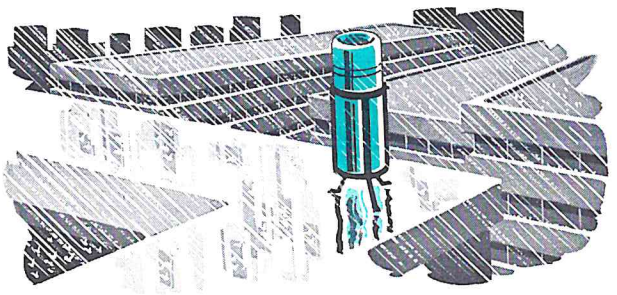
Checkout

Find each product.

1. $\begin{array}{r} 82.4 \\ \times\ .63 \\ \hline \end{array}$
2. $\begin{array}{r} 96.5 \\ \times .083 \\ \hline \end{array}$
3. $\begin{array}{r} 5.26 \\ \times 1.09 \\ \hline \end{array}$
4. $\begin{array}{r} 14.3 \\ \times 260 \\ \hline \end{array}$

Choose $=$ or $\neq$.

5. $8^2 \times 4 \bigcirc 4 \times 8^2$
6. $9^2 + 5^3 \bigcirc 5^3 + 9^2$
7. $(4 \times 3^2) \times 6^2 \bigcirc 4 \times (3^2 \times 6^2)$
8. $6^2 - 4^2 \bigcirc 4^2 - 6^2$

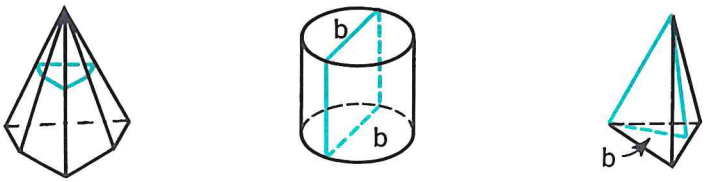


Complete each problem.

9. 3% of \$42 = ■
10. .8 × 46 = ■
11. $12\frac{5}{8} + 4\frac{1}{2} =$ ■
12. 20% of \$80 = ■
13. $7\frac{1}{3} - 4\frac{1}{2} =$ ■
14. $4\frac{2}{3} \div 1\frac{1}{6} =$ ■
15. .3 × .06 = ■
16. .121 ÷ 1.1 = ■
17. $4\frac{1}{2} \div 1\frac{1}{2} =$ ■
18. .26 − .1 = ■
19. $16 \div 1\frac{1}{4} =$ ■
20. $7\frac{1}{2} \times 1\frac{1}{5} =$ ■

21. Jill had 20 problems correct on a test. If there were 25 problems on the test, what per cent did she have correct?

22. What is the plane figure determined by each cut?



23. Find the volume of a box that is 6 ft. by 4 ft. by 3 ft.

24. Find the circumference of a circle that is 10 feet in diameter.

Checkout

Find each quotient. Check by multiplication.

1. $7\overline{)3556}$ 2. $.88\overline{)88.088}$ 3. $50\overline{)4092}$ 4. $76.9\overline{)55.5689}$

Draw a coordinate system and locate these points.

5. A (2, −3) 6. B (4, 6) 7. C (0, 4) 8. D (−3, 0)

Solve each equation for x.

9. $x + 6 = 18$
10. $x - 12 = 20$
11. $3x = 42$
12. $3x + 2x = 40$
13. $2x + x + 3 = 15$
14. $\frac{1}{2}x = 12$

Copy and complete each problem.

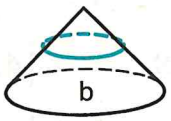
15. $(-6) + (-4) =$ ■
16. $4 + (-8) =$ ■
17. $6 \times (-4) =$ ■
18. $7 - (-4) =$ ■
19. $(-4) + 10 =$ ■
20. $(-8) \times (-3) =$ ■

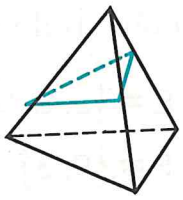
Write a mathematical sentence for each English sentence.

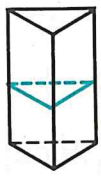
21. Roberta has collected fewer leaves than Karen.
22. Marlene has collected twice as many stamps as Penny.
23. Gerald has 3 more model planes than Wayne.
24. Max has 2 fewer books than David.
25. John has the same number of books as Charles.

Are these cuts of symmetry?

26.

a. 

b. 

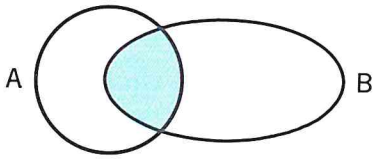
c. 

Part 24 Set Diagrams

We can use diagrams to show operations on sets.

$A = \{1, 2, 3, 4\}$

$B = \{1, 3, 5\}$



$A \cap B = \{1, 3\}$

The shaded part represents $\{1, 3\}$.

For each pair of sets, find the intersection of the two sets. Make a diagram like the one above to show the operation. Tell what the shaded part represents.

1. $C = \{Al, Jo\}$
   $D = \{Jo, Vi\}$

2. $J = \{2, 4, 6\}$
   $K = \{6\}$

3. $M = \{3, 5, 7\}$
   $N = \{6, 4, 2\}$

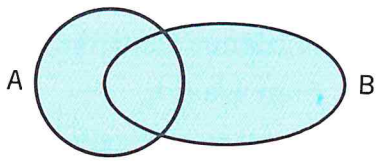
4. $H = \{Kay\}$
   $V = \{Joan, Kay\}$

5. $L = \{Mark, Tony\}$
   $X = \{Tony, Ed\}$

6. $P = \{n: 3 < n < 6\}$
   $Q = \{n: n < 1\}$

$A = \{Joe, Al\}$

$B = \{Ed\}$



$A \cup B = \{Joe, Al, Ed\}$

The shaded part represents $\{Joe, Al, Ed\}$.

For each pair of sets, find the union of the two sets. Make a diagram like the one above to show the operation. Tell what the shaded part represents.

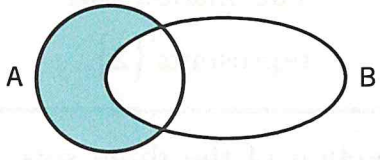
7. $X = \{\ \}$
   $Y = \{Alice\}$

8. $E = \{3, 8, 9\}$
   $F = \{9, 3\}$

9. $V = \{4, 9, 2\}$
   $W = \{n: 1 < n < 5\}$

We can use diagrams to show the difference of two sets.

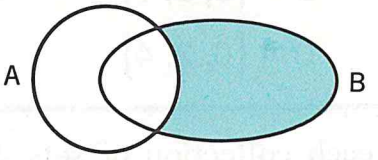
$A = \{May, June, July\}$



$A - B = \{June, July\}$

The shaded part represents $\{June, July\}$.

$B = \{April, March, May\}$



$B - A = \{April, March\}$

The shaded part represents $\{April, March\}$.

For each pair of sets, find the first set difference the second set, and then find the second set difference the first set. Make a diagram to show each operation. For each diagram, tell what the shaded part represents.

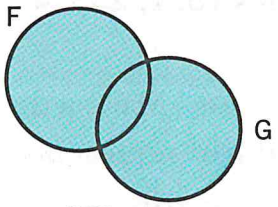
1. $C = \{Iowa, Maine\}$
   $D = \{Oregon, Iowa\}$

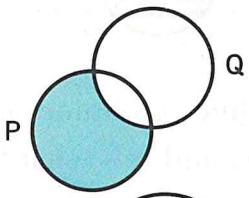
2. $J = \{2, 4, 6, 8\}$
   $K = \{n: 1 < n < 9\}$

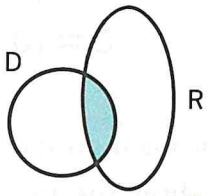
3. $P = \{March, April, May, June\}$
   $Q = \{May, June\}$

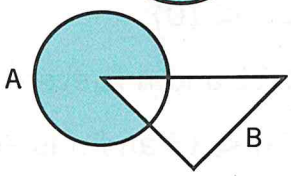
4. $X = \{Al, Tony, Pedro\}$
   $Y = \{\ \}$

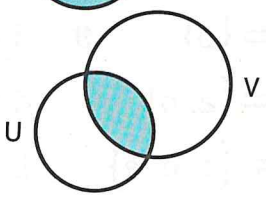
Tell what the shaded part in each diagram represents.

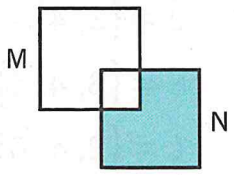
5. 

6. 

7. 

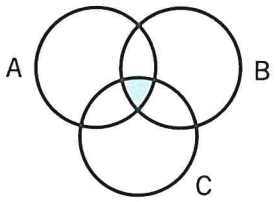
8. 

9. 

10. 

We can use diagrams to show the intersection of three sets.

$A = \{2, 4\}$
$B = \{3, 2, 1\}$
$C = \{0, 2, 4\}$



$A \cap B \cap C = \{2\}$

The shaded part represents $\{2\}$.

For each collection of sets, find the intersection of the three sets. Make a diagram to show the intersection of the sets and tell what the shaded part represents.

1. $R = \{m, n, p\}$
   $S = \{p\}$
   $T = \{m, p\}$

2. $C = \{a, b, c\}$
   $D = \{d, c, x\}$
   $E = \{d\}$

3. $X = \{n: n < 4\}$
   $Y = \{1, 3, 5\}$
   $Z = \{n: n < 6\}$

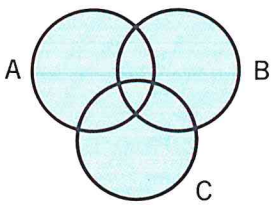
4. $V = \{Kay, Ann\}$
   $W = \{Kay\}$
   $B = \{Ann, Kay\}$

5. $M = \{n: n < 5\}$
   $P = \{n: n < 2\}$
   $R = \{1, 5, 9\}$

6. $J = \{n: n < 10\}$
   $K = \{n: 4 < n < 8\}$
   $L = \{n: 2, 4, 6, 8\}$

We can use diagrams to show the union of three sets.

$A = \{2, 4\}$
$B = \{3, 2, 1\}$
$C = \{0, 2, 4\}$



$A \cup B \cup C = \{0, 1, 2, 3, 4\}$

The shaded part represents $\{0, 1, 2, 3, 4\}$.

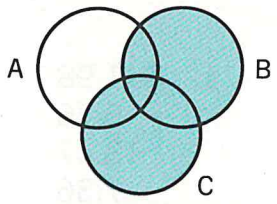
For each collection of sets, find the union of the three sets. Make a diagram to show the union of the sets and tell what the shaded part represents.

7. $J = \{3, 4\}$
   $K = \{3, 4, 5\}$
   $L = \{2, 4, 6\}$

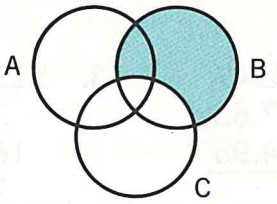
8. $F = \{b\}$
   $G = \{a, b\}$
   $H = \{c, d, e\}$

9. $J = \{n: 2 < n < 10\}$
   $K = \{n: n < 11 \text{ and } n \text{ is even}\}$
   $L = \{n: 3 < n < 13 \text{ and } n \text{ is even}\}$

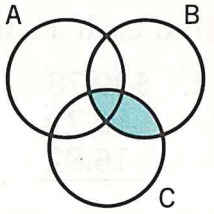
We can shade a diagram to show an operation on 2 of the 3 sets represented.



The shaded part represents B ∪ C.

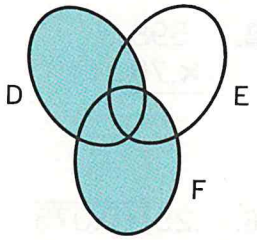


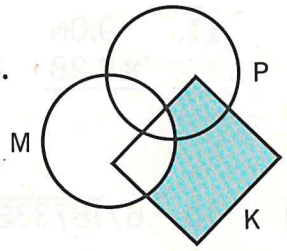
The shaded part represents B − C.

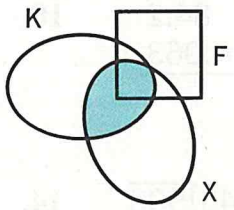


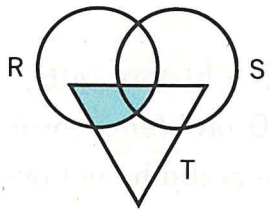
The shaded part represents B ∩ C.

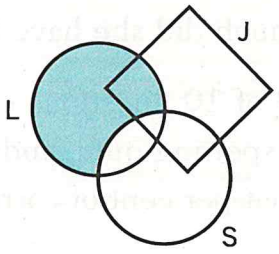
Tell what the shaded part in each diagram represents.

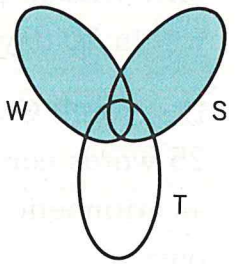
1. 

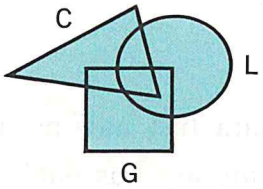
2. 

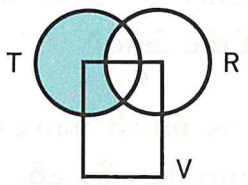
3. 

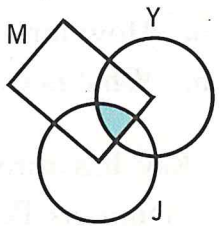
4. 

5. 

6. 

7. 

8. 

9. 

Final Checkout

Find each answer.

1. $39.78 + 9.79 + 16.82$

2. $12.47 + 7.63 + 14.95$

3. $13.97 + 2.48 + 16.35 + 2.76$

4. $41.98 + 5.76 + 13.47 + 7.36$

5. $\$55.32 - 17.98$

6. $\$37.08 - 15.39$

7. $\$28.37 - 19.48$

8. $\$105.75 - 77.88$

9. $84.2 \times .063$

10. $7.28 \times .93$

11. 9.06×3.28

12. $598 \times .75$

13. $43\overline{)903}$

14. $60\overline{)7060}$

15. $67\overline{)87336}$

16. $.25\overline{)10.075}$

17. Mrs. Watson paid a bill for $12.60. If she received 10% off for paying within 10 days, how much did she have to pay?

18. Last week Craig got 8 of 10 questions correct on a history quiz, 19 of 25 words correct in a spelling quiz, and 17 of 20 problems correct on an arithmetic quiz. What per cent of correct answers did he get on each quiz?

19. A rectangular field is 137 feet long and 48 feet wide.
    a. How long must a fence be to surround this field?
    b. What is the area of the field?

20. Kay has three times as much money as Sara. Sara has half as much money as Penny. Penny has $1.26. How much money has each girl?

Final Checkout

Find each answer.

1. $18\frac{3}{5} + 29\frac{7}{10}$

2. $14\frac{2}{9} + 23\frac{1}{2}$

3. $38 - 24\frac{1}{3}$

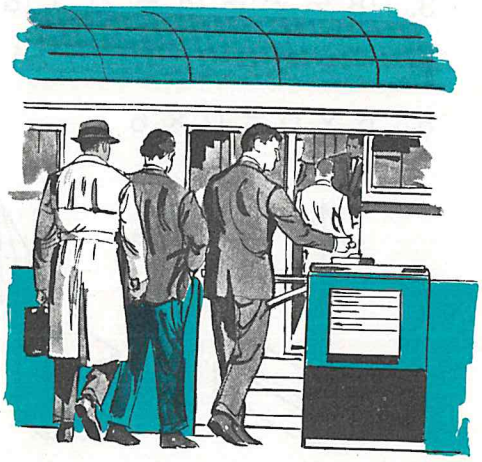
4. $18\frac{1}{4} - 6\frac{2}{3}$

5. $6\frac{2}{3} \times 1\frac{1}{2} =$ ■

6. $8\frac{1}{2} \div 2 =$ ■

7. $4\frac{2}{5} \times 1\frac{1}{4} =$ ■

8. $2\frac{2}{5} \div 6 =$ ■



Complete each problem.

9. $\{2, 8, 64\} \cup \{2, 4, 6, 8\} =$ ■

10. $\{4, 7, 11\} \cap \{3, 5, 7, 9\} =$ ■

11. $\{8, 12, 16\} - \{8, 16, 24\} =$ ■

12. $\{n: n < 3\} \cap \{n: 3 < n < 5\} =$ ■

13. $\{n: 10 < n < 13\} \cup \{n: 11 < n < 13\} =$ ■

Answer each question.

14. Which of these numbers are prime?
    a. 28 b. 37 c. 46 d. 59

15. What is the G.C.F. of each set of numbers?
    a. $\{25, 35, 40\}$ b. $\{24, 36, 60\}$

16. What is the L.C.M. of each set of numbers?
    a. $\{8, 12, 24\}$ b. $\{24, 36, 60\}$

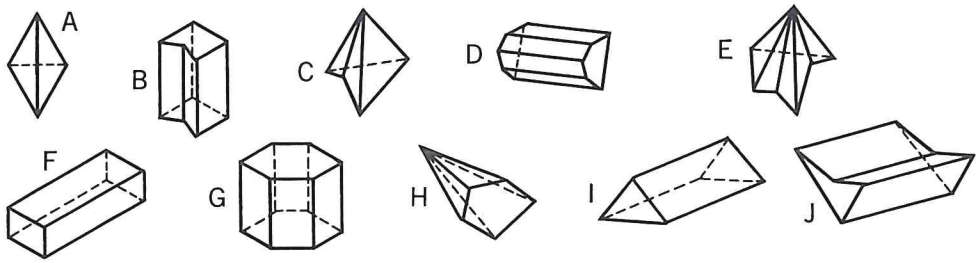
17. Which of these numbers are divisible by 9?
    a. 8296 b. 4572 c. 8298 d. 1457

18. Which of these numbers are divisible by 4?
    a. 423 b. 1492 c. 9342 d. 1040

Final Checkout

What law of arithmetic is illustrated in each sentence?

1. $6(7 + 9) = (6 \times 7) + (6 \times 9)$
2. $(3 + 1) + 2 = 3 + (1 + 2)$
3. $(8 \div 2) + (4 \div 2) = (8 + 4) \div 2$
4. $7 \times (8 \times 2) = (7 \times 8) \times 2$
5. $6 \times 0 = 0 \times 6$



6. Which are pictures of prisms?
7. Which are pictures of pyramids?
8. Which are pictures of convex figures?
9. Which are pictures of concave figures?
10. Which are pictures of quadrangular prisms?

Perform these constructions with straightedge and compass.

11. First, construct a square ABCD with sides about 2 in. in length. Draw diagonal AC.
12. Construct a circle with diameter equal in length to $\overline{AC}$.
13. Construct an equilateral triangle with sides equal in length to $\overline{AC}$.
14. Construct an isosceles triangle with base equal in length to $\overline{AB}$ and the two other sides equal in length to $\overline{AC}$.

Final Checkout

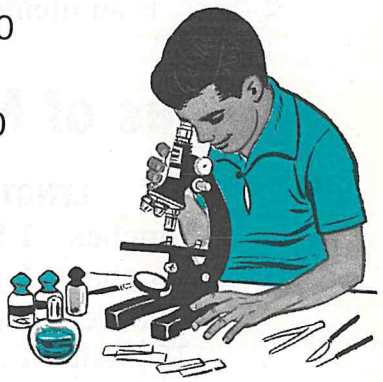
Which of these sentences are atomic? molecular? Which are conjunctions? disjunctions? Which are conditional sentences?

1. p is prime and p is even.
2. Joan is not at home.
3. If you go, then I stay.
4. Accra is the capital of Ghana.
5. $y = 3 + x$
6. $p^2 = 4$ or $p = -2$
7. $y = 2^x$ and $x > 0$
8. $A \cap B \neq A$

Copy the premises, infer the desired conclusion by adding any necessary lines, and give the reason for each line that is not a premise.

9. (1) If $p \neq 3$, then $p = 4$
   (2) $p \neq 4$
   Therefore, $p = 3$

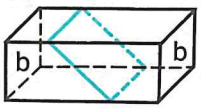
10. (1) If $a^2 = 0$, then $a = 0$
    (2) $a \neq 0$ or $b = 0$
    (3) $b \neq 0$
    Therefore, $a^2 \neq 0$

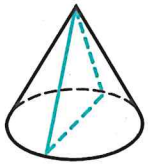


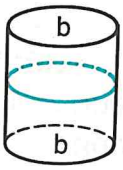
Complete each sentence.

11. ■ cm. = 1 m.
12. 1 km. = ■ m.
13. 40 g. = ■ kg.
14. 200 m. = ■ cm.
15. 1 m. = ■ in.
16. 1 in. = ■ cm.
17. 1 kg. = ■ lb.
18. 1 km. = ■ mi.

What is the plane figure determined by each cut?

19. 

20. 

21. 

Give the base-ten numeral for each of these numerals.

22. 232_{five}
23. 416_{eight}
24. 10011_{two}

Lists of Symbols

$\{1, 2\}$	the set whose members are 1 and 2
$\{n: n < 1\}$	the set of all n such that n is less than 1
$\{\ \}$	the empty set
$\cup$	union of sets symbol
$\cap$	intersection of sets symbol
$-$	difference of sets symbol
$\subset$	subset symbol
$\in$	is an element of
$=$	is equal to
$\neq$	is not equal to
$\approx$	is equivalent to
$>$	is greater than
$<$	is less than
$\overline{AB}$	line segment AB
$\overleftrightarrow{AB}$	line AB
$\overrightarrow{AB}$	ray AB
$\angle ABC$	angle ABC
$\cong$	is congruent to

Tables of Measure

LENGTH

12 inches	1 foot
36 inches	1 yard
3 feet	1 yard
$16\frac{1}{2}$ feet	1 rod
$5\frac{1}{2}$ yards	1 rod
5280 feet	1 mile
1760 yards	1 mile
320 rods	1 mile

LIQUID

3 teaspoons	1 tablespoon
2 tablespoons	1 fluid ounce
8 fluid ounces	1 cup
2 cups	1 pint
2 pints	1 quart
4 quarts	1 gallon

TIME

60 seconds	1 minute
60 minutes	1 hour
24 hours	1 day
7 days	1 week
52 weeks	1 year
365 days	1 year
366 days	1 leap year
12 months	1 year
10 years	1 decade
100 years	1 century
1000 years	1 millenium

WEIGHT

16 ounces	1 pound
100 pounds	1 hundredweight
2000 pounds	1 ton

Note: The last five pages of this book, 333 to 337 to be precise, are left out intentionally (all of them Index pages).

Therefore this concludes the book titled "Sets And Numbers—Grade 6".

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